Noise-induced shallow circuits and absence of barren plateaus

Antonio Anna Mele, **Armando Angrisani**, Soumik Ghosh, Sumeet Khatri, Jens Eisert, Daniel Stilck França, Yihui Quek

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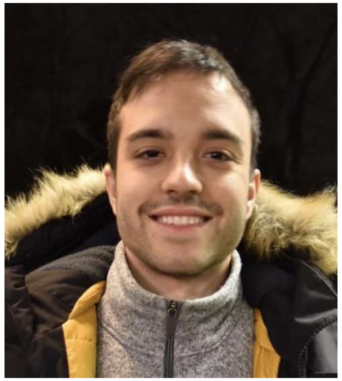
Joint work with:

Antonio Anna Mele, Soumik Ghosh, Sumeet Khatri, Jens Eisert, Daniel Stilck França, Yihui Quek



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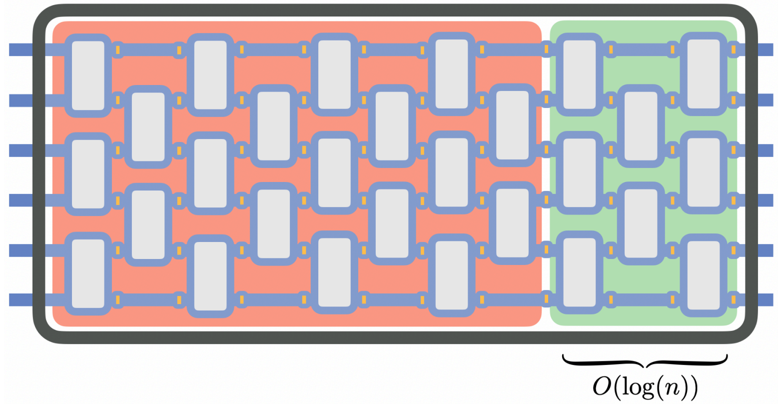
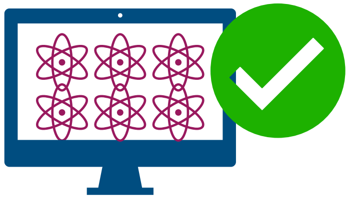
Outline



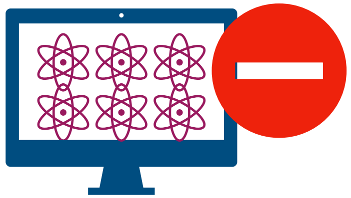
- Noise in quantum circuits
- Effective shallow circuits
- Classical simulation of Pauli expectation values of noisy random circuits
- Barren plateaus

- Understanding **noise impact** in current quantum devices is important for:

Capabilities



Limitations



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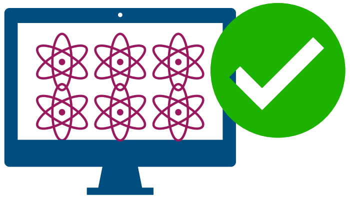

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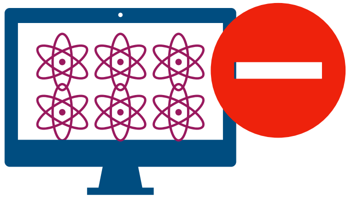

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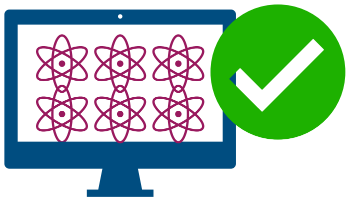

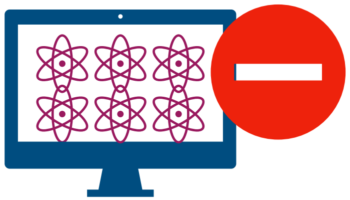

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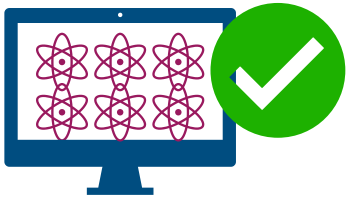

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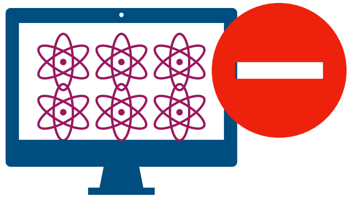

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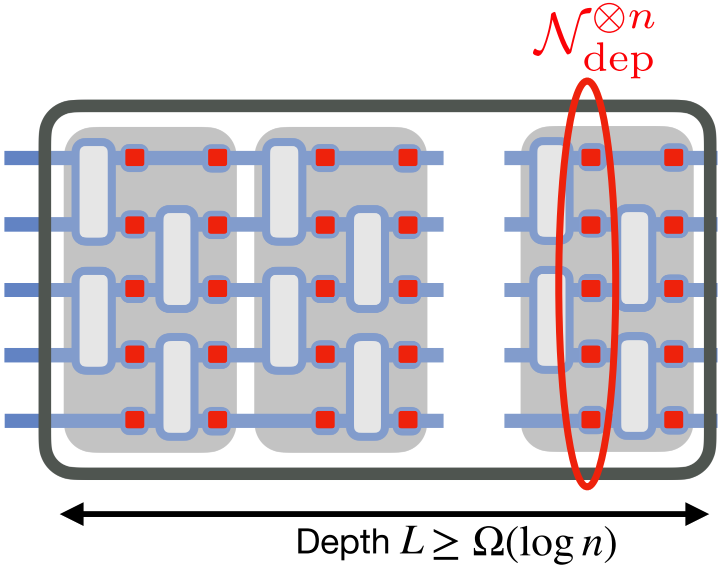

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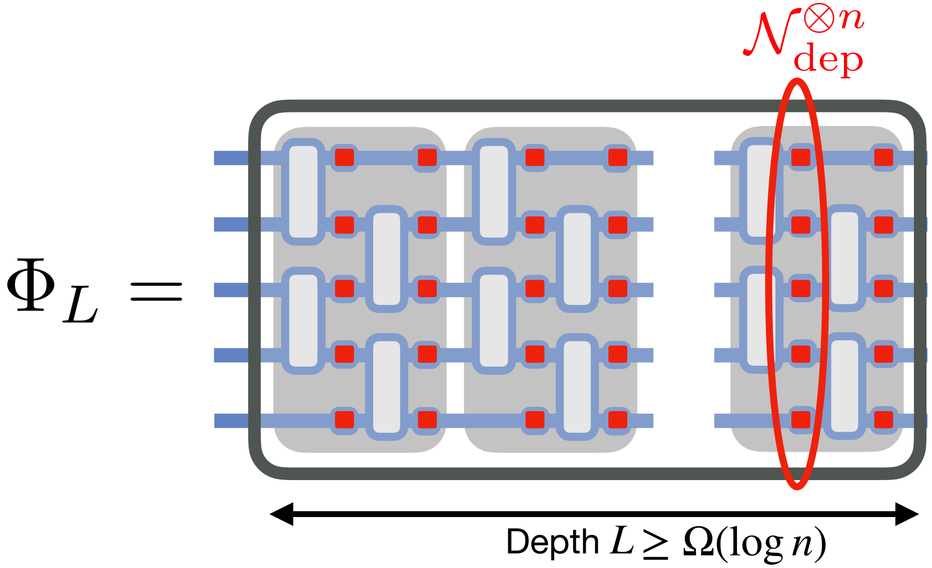

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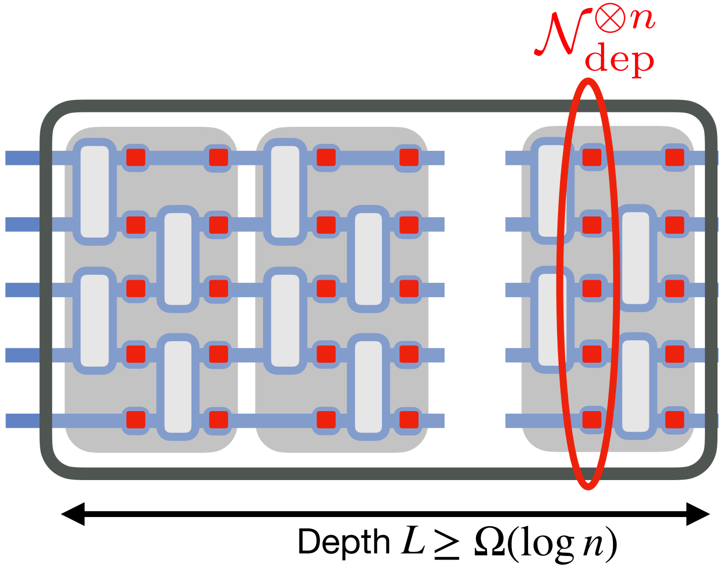

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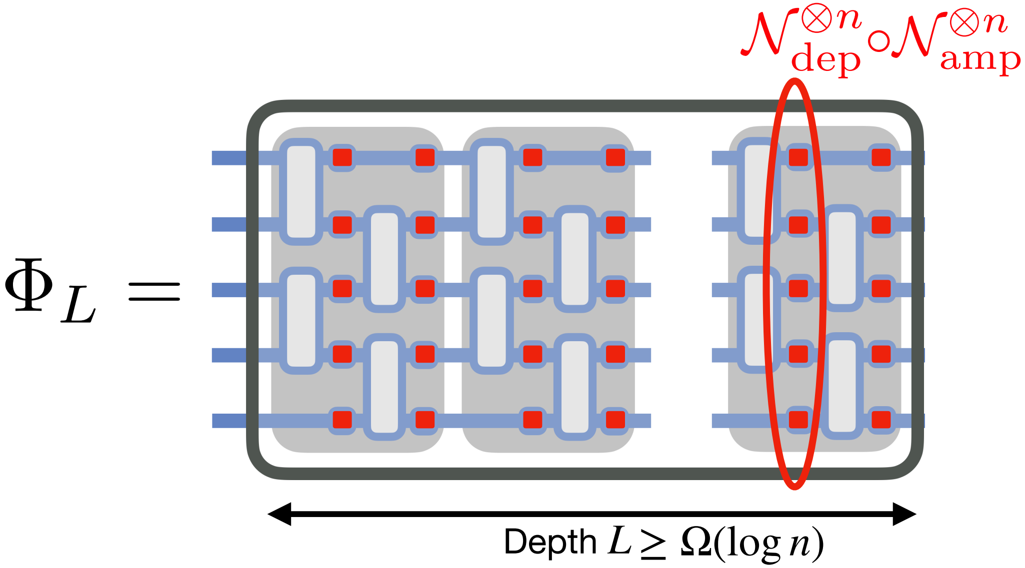

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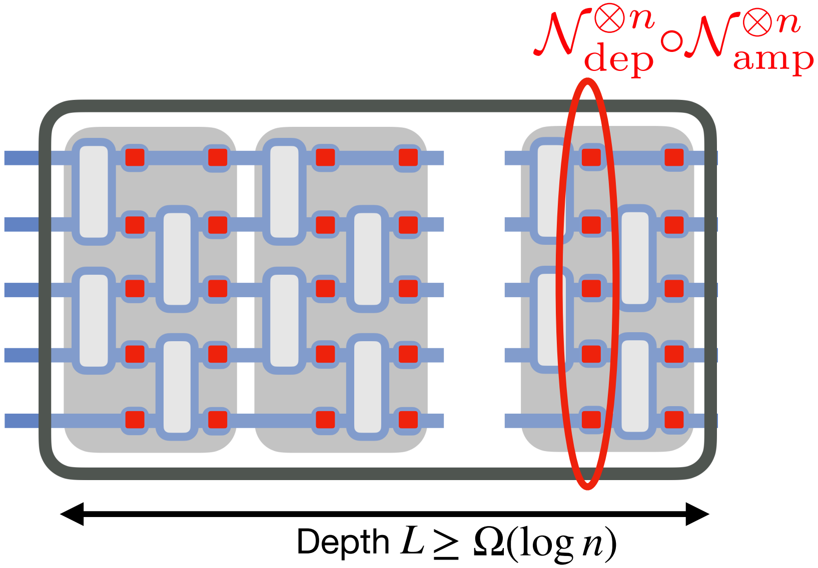

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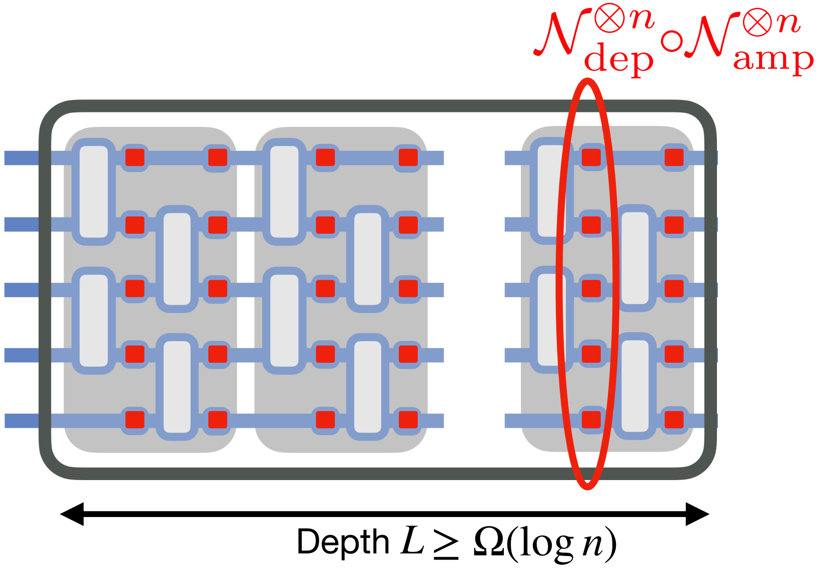

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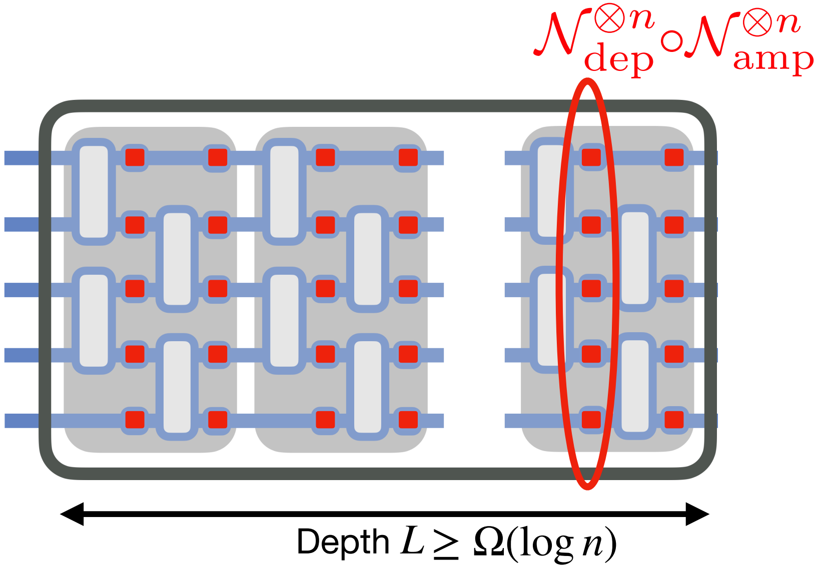

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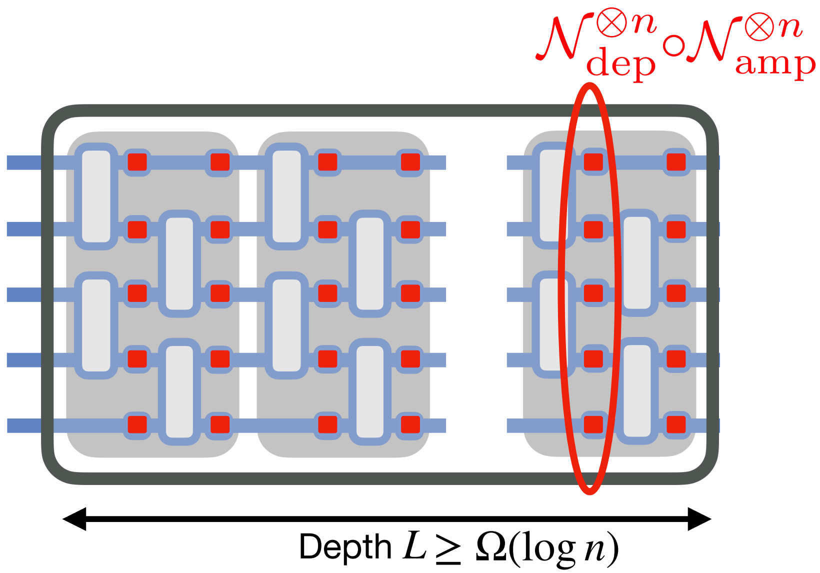

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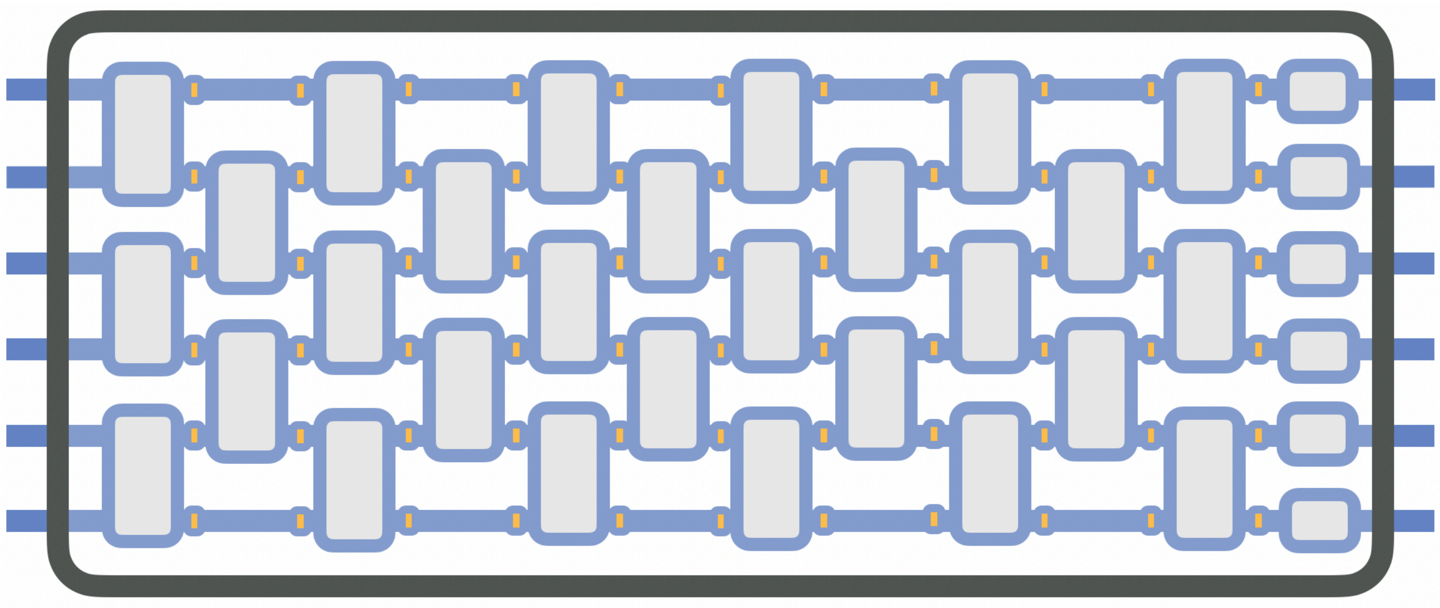
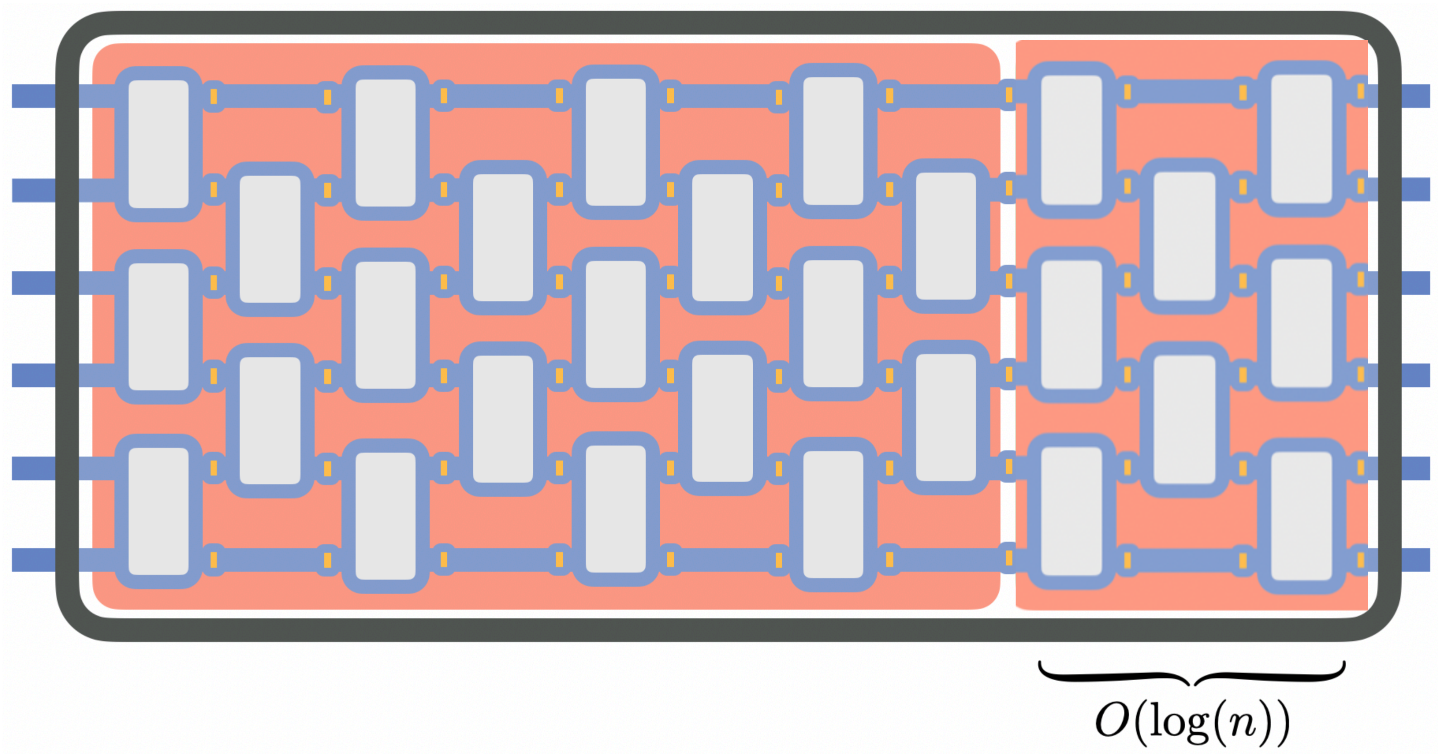
What happens for generic noisy circuits?



If the noise is **depolarizing**, no layers matter.



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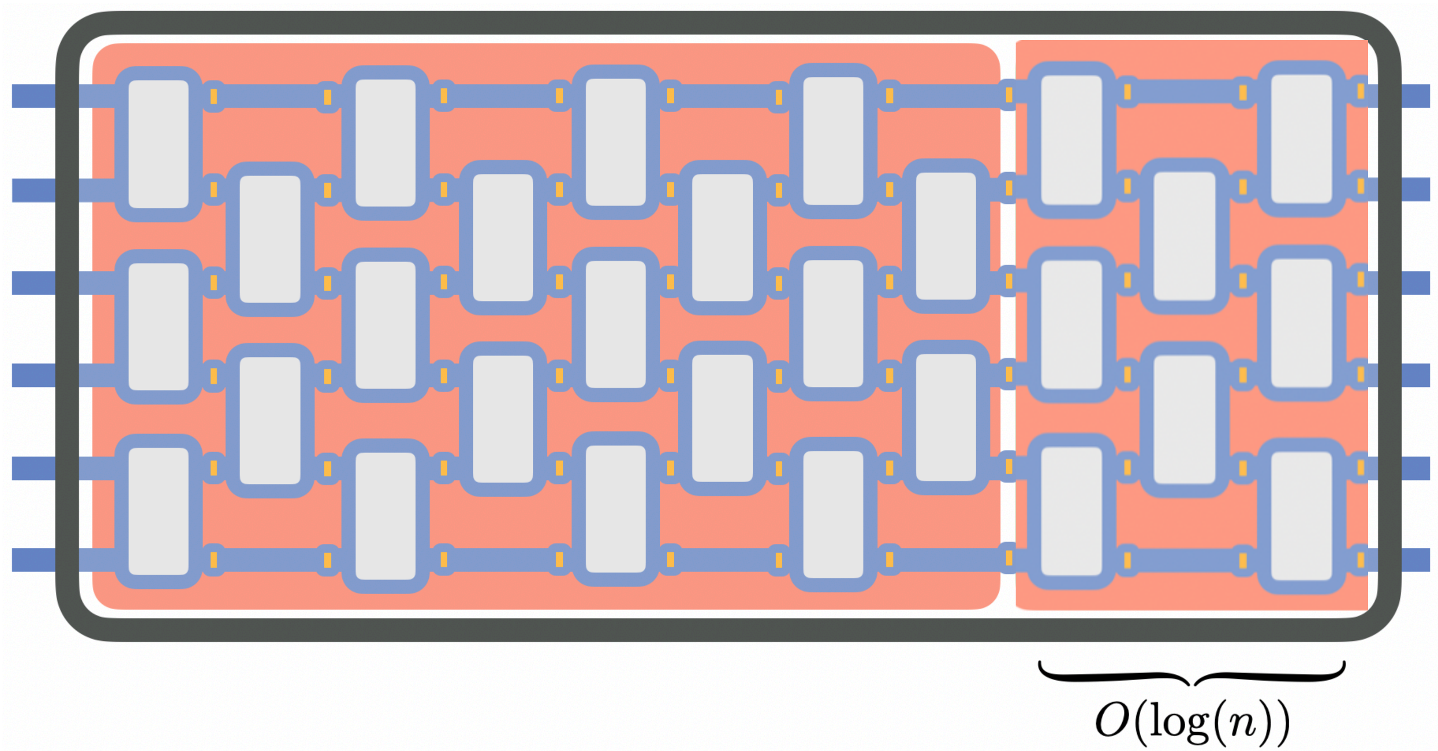

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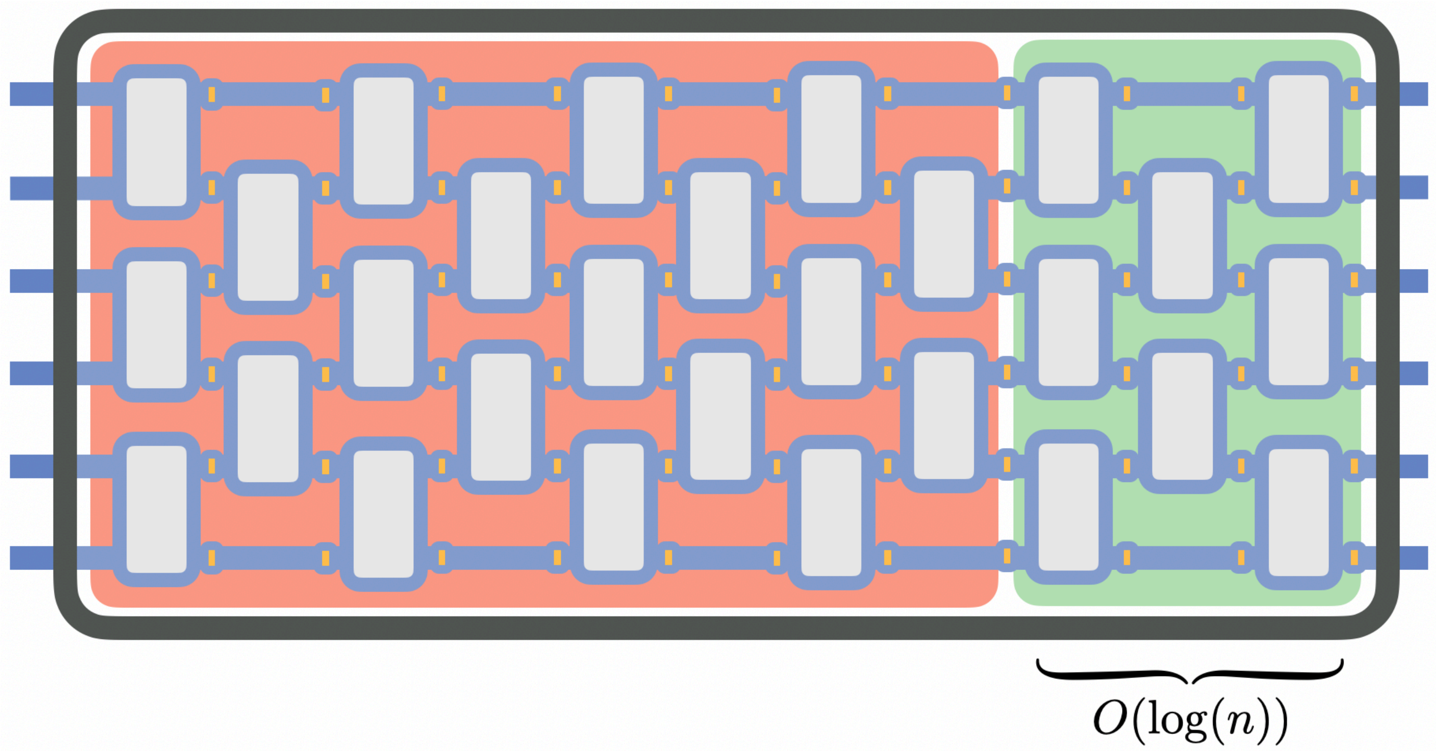

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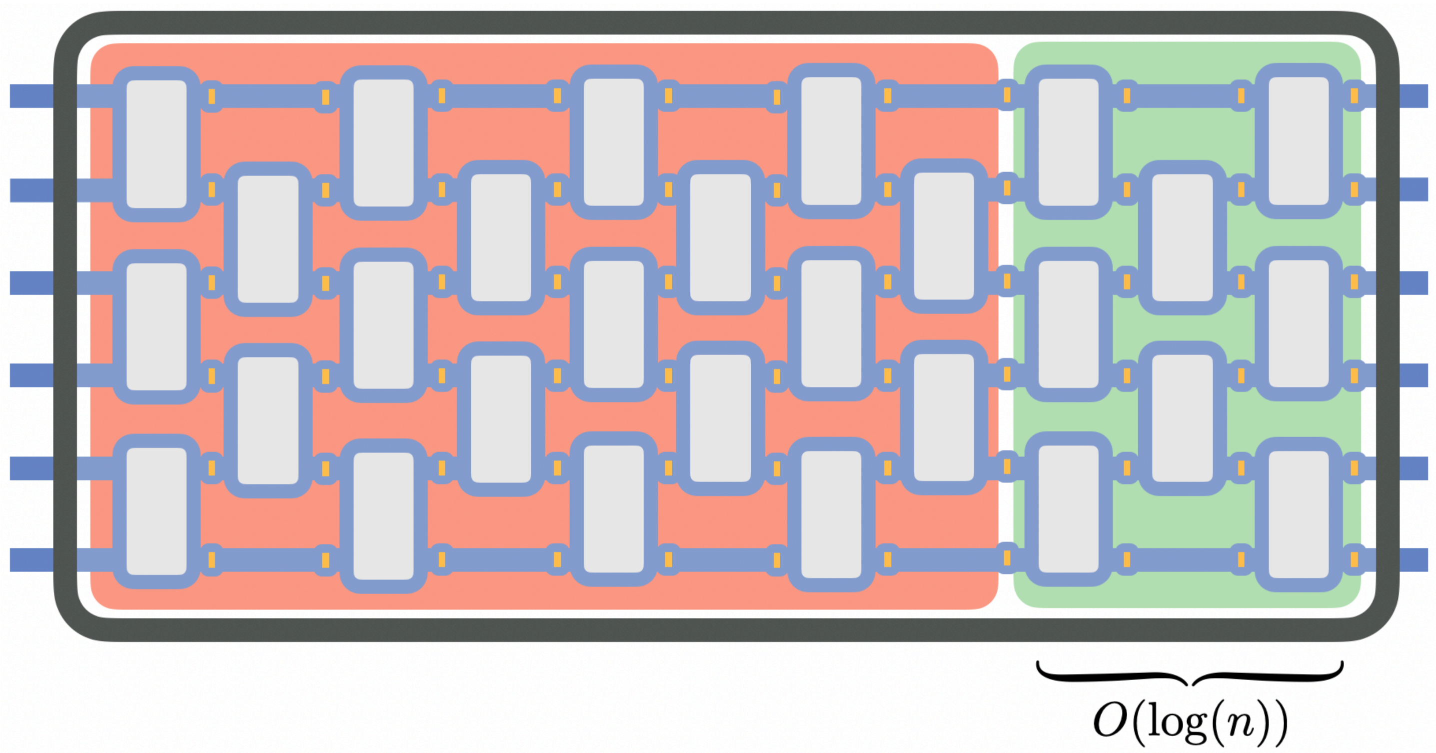

Effective shallow circuits

Proposition.

For any initial state ρ_0 (possibly complex) and observable O,

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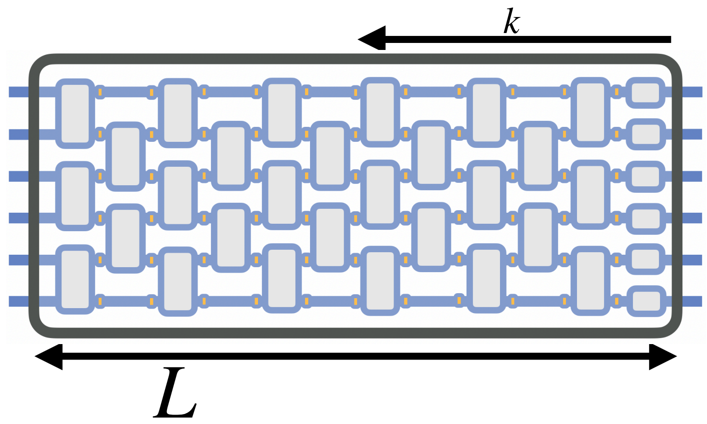

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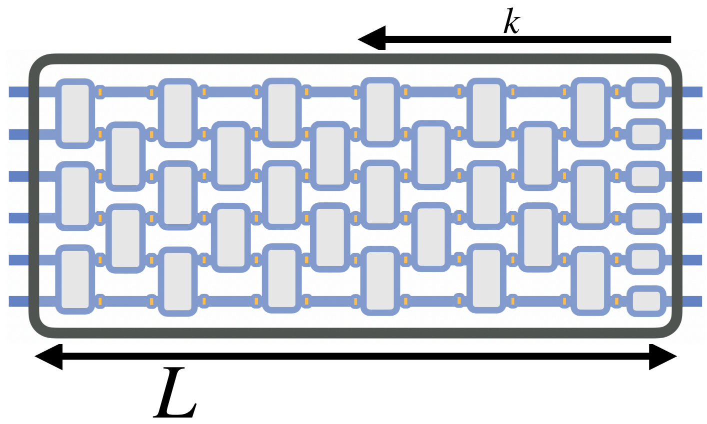

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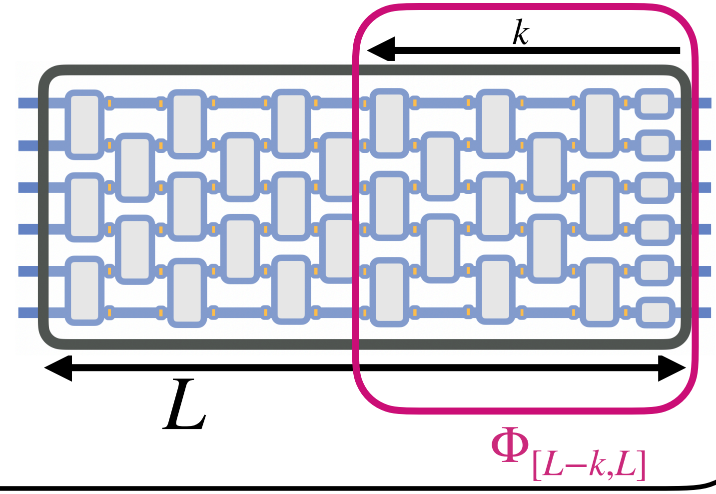
$\Phi =$ 

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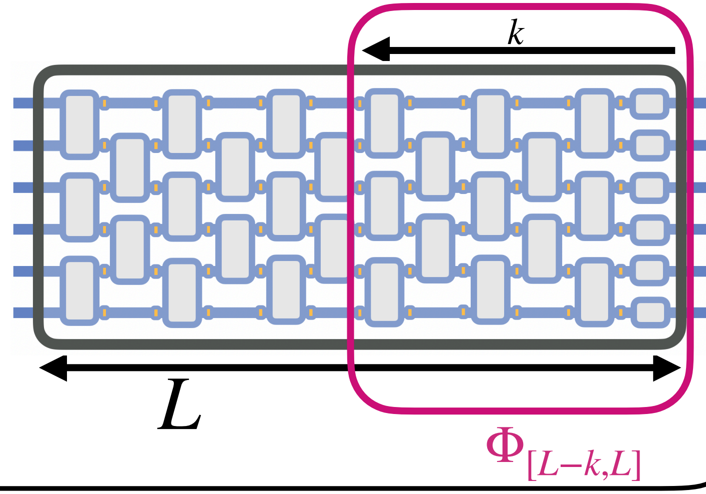
$\Phi =$ 

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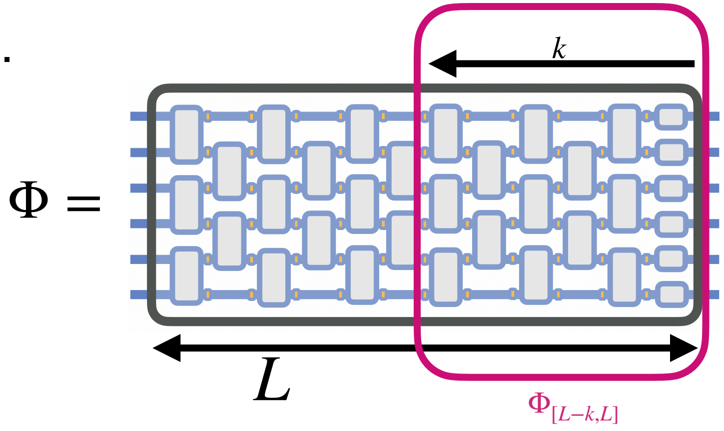

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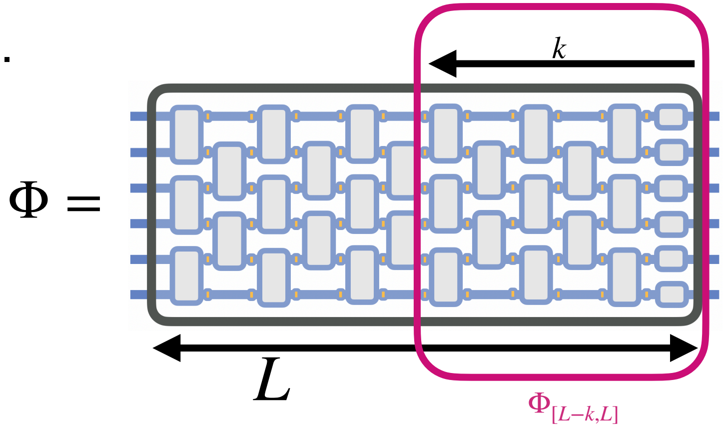

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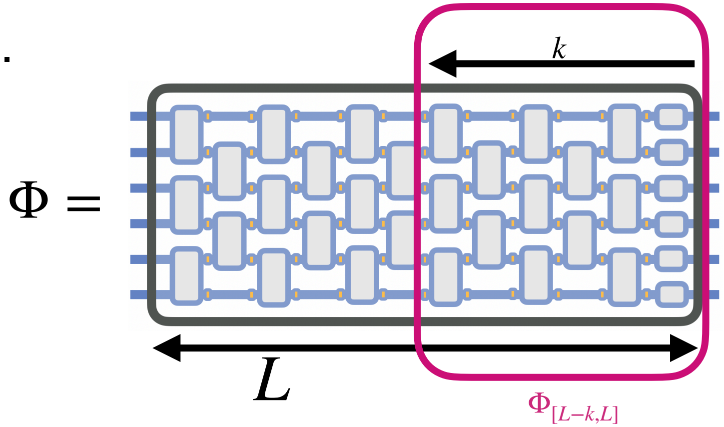

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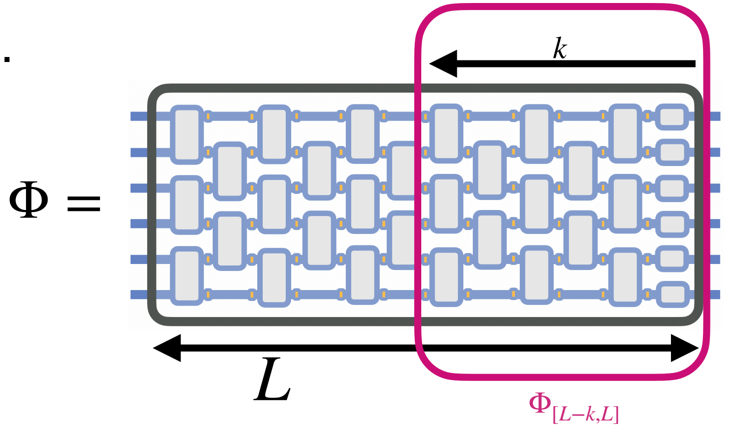

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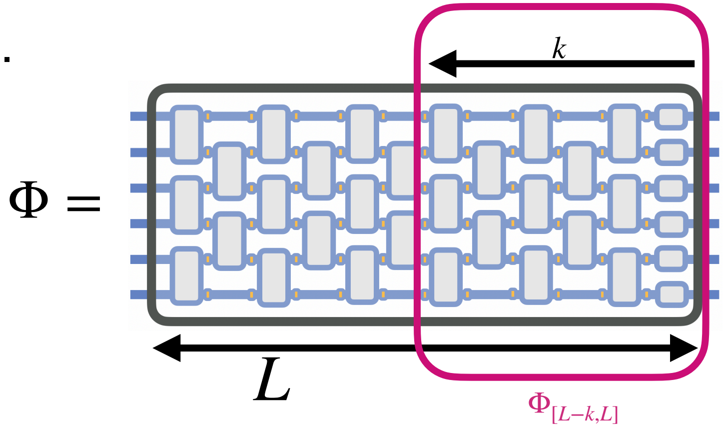

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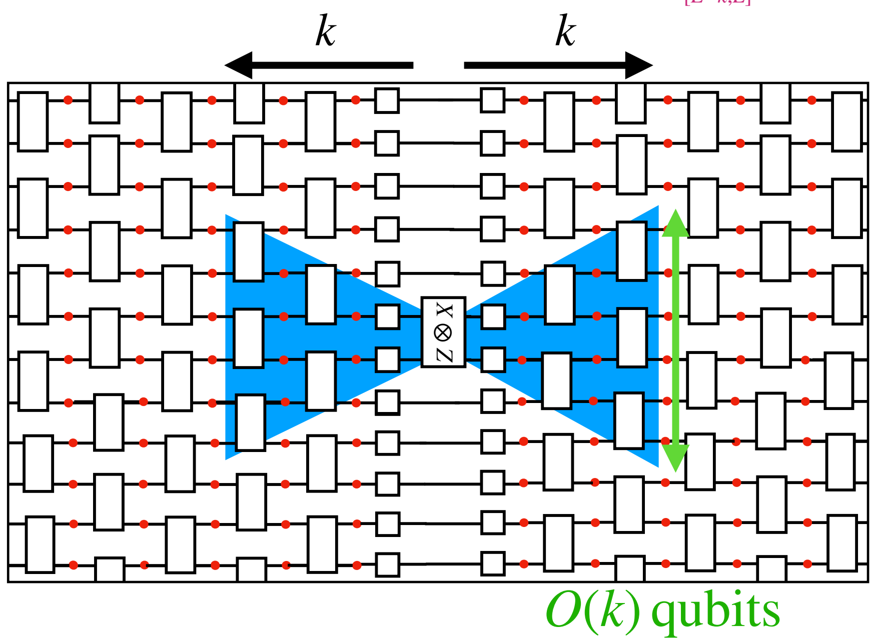
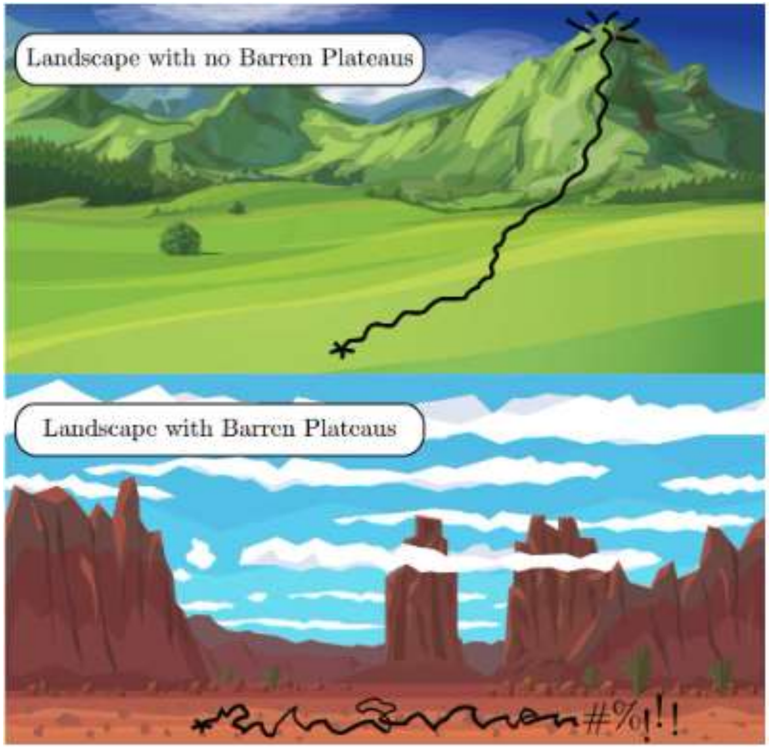
  If P is local, then this is also local

- Thus, this can be computed efficiently for light-cone arguments.



Computational time (for 1D): $\exp(O(k)) = \mathrm{poly}(\varepsilon^{-1})$

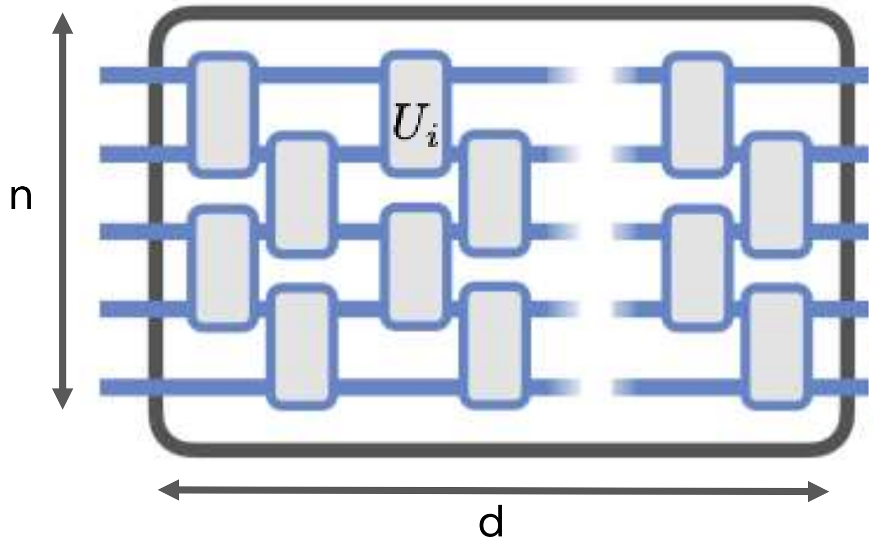
Barren plateaus are bad for variational algorithms



https://www.eurekalert.org/multimedia/739167

Gradient vanishes in all directions; can't figure out where to go!

Barren plateaus make optimization hard



C = gradient of cost function

Math translation of ‘vanishing gradient’:

$$\mathrm{Var}_{U_1,\ldots U_m}[C] = O(\exp(-n))$$

Recall: We are optimizing over

$$U_1, \ldots U_m$$

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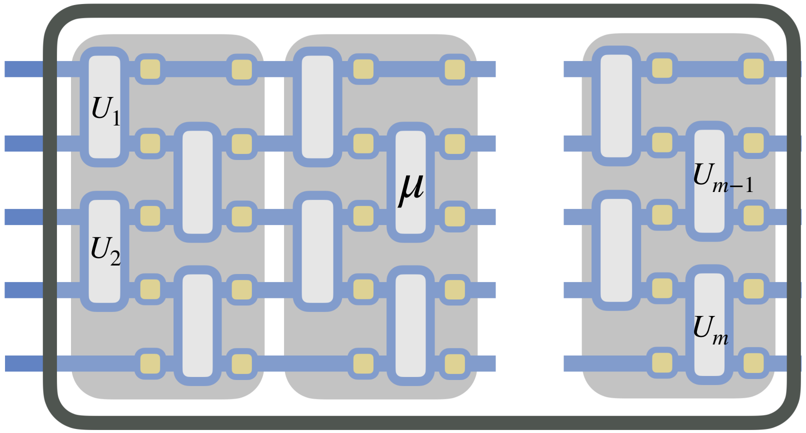

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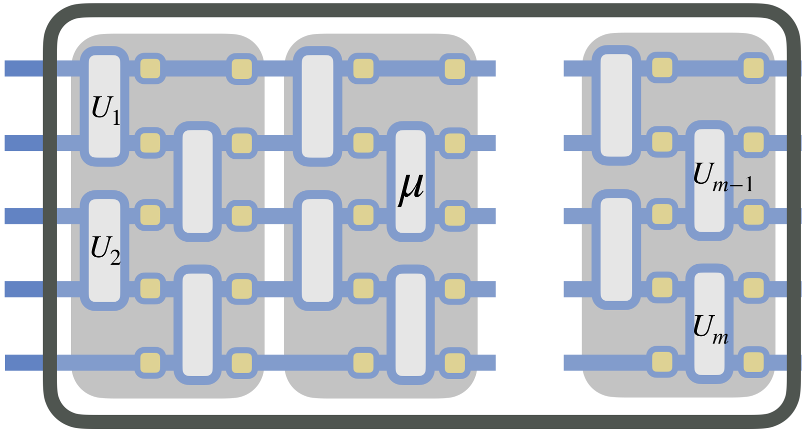

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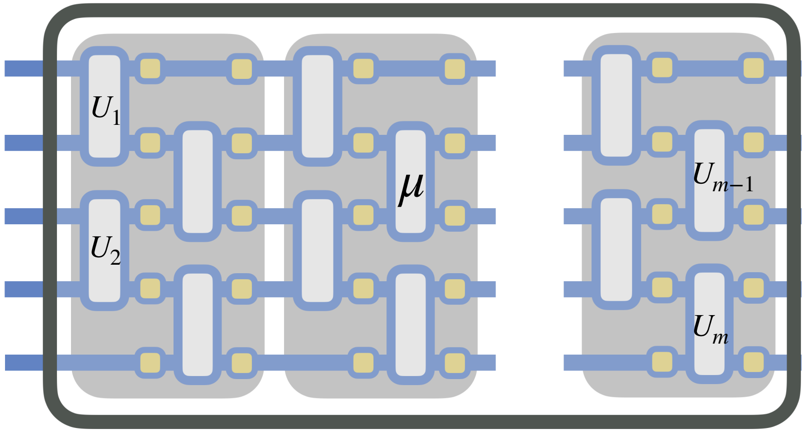

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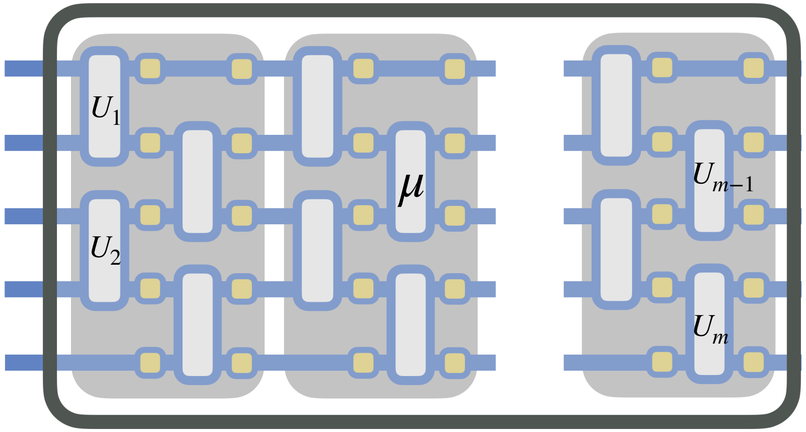

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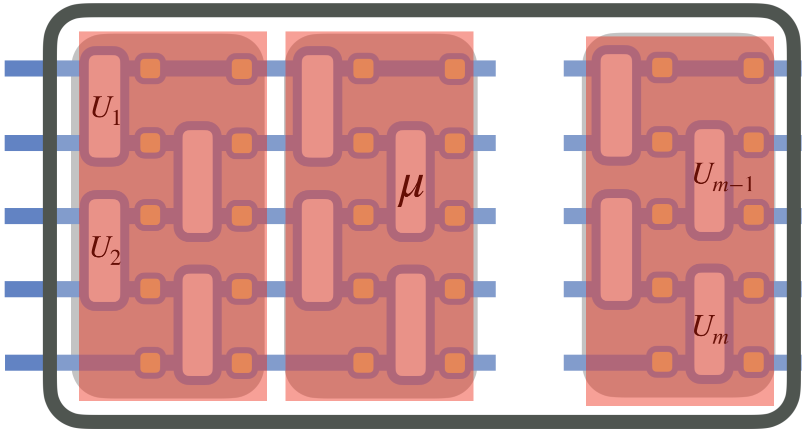

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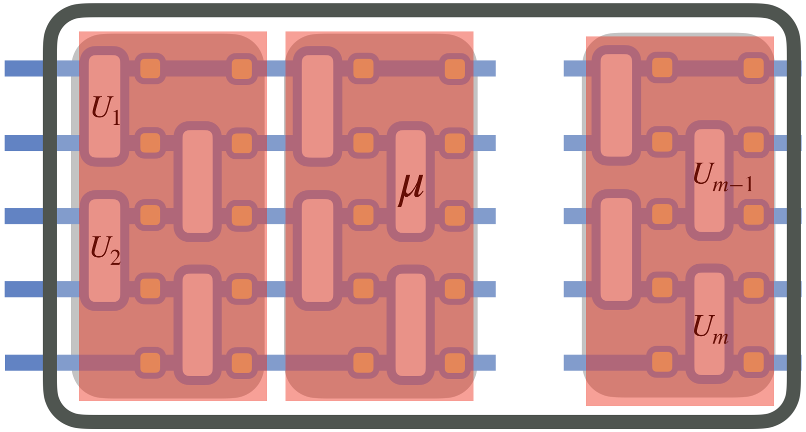
$\Phi =$ 

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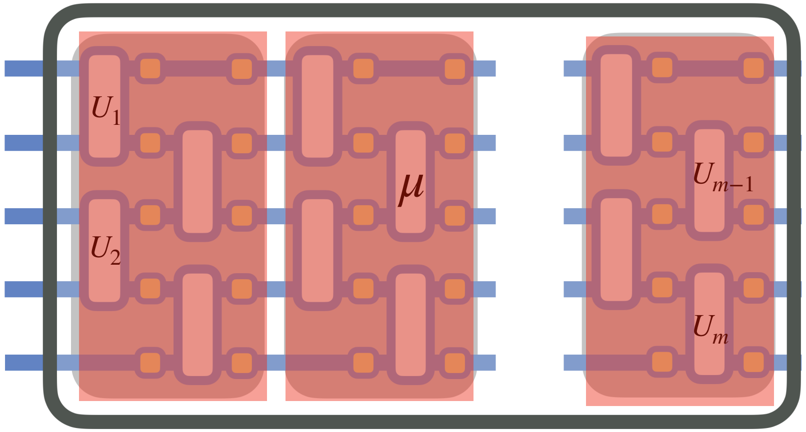
$\Phi =$ 

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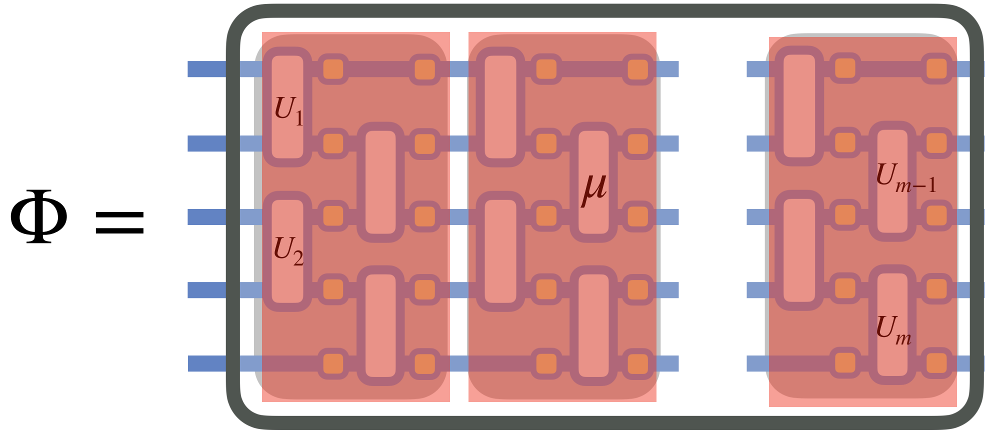

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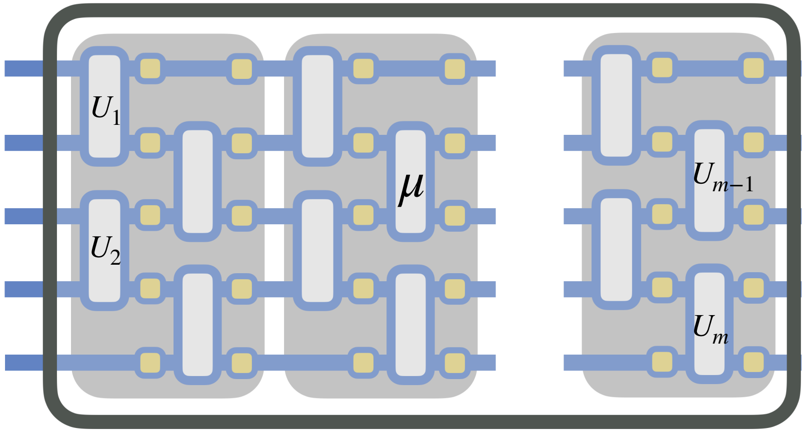
$\Phi =$ 

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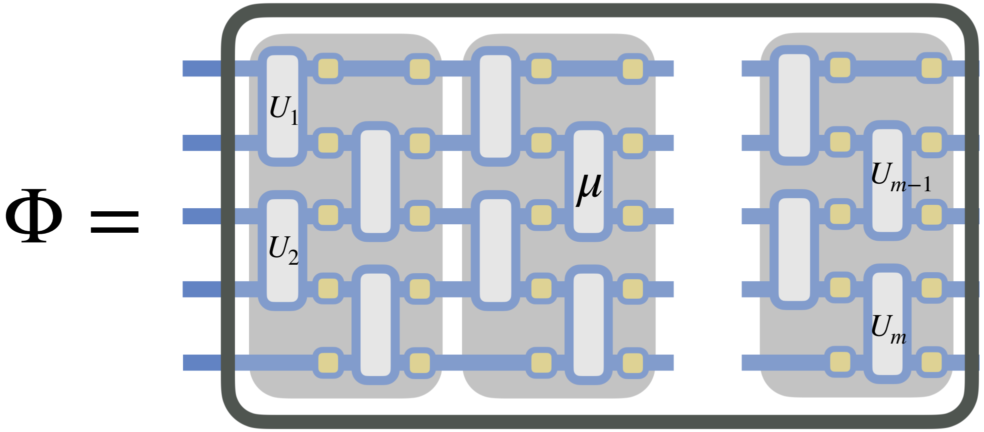

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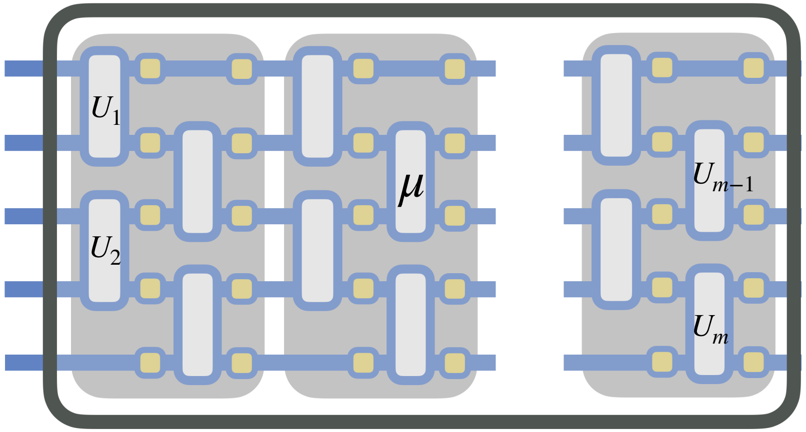

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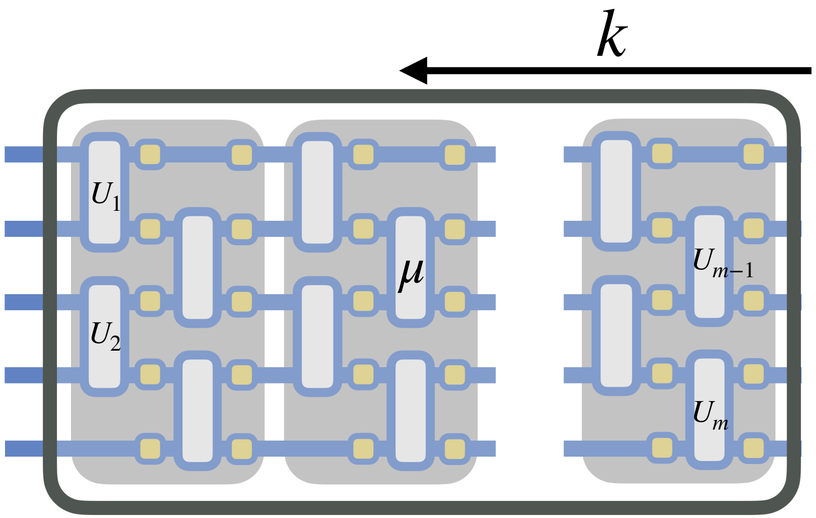

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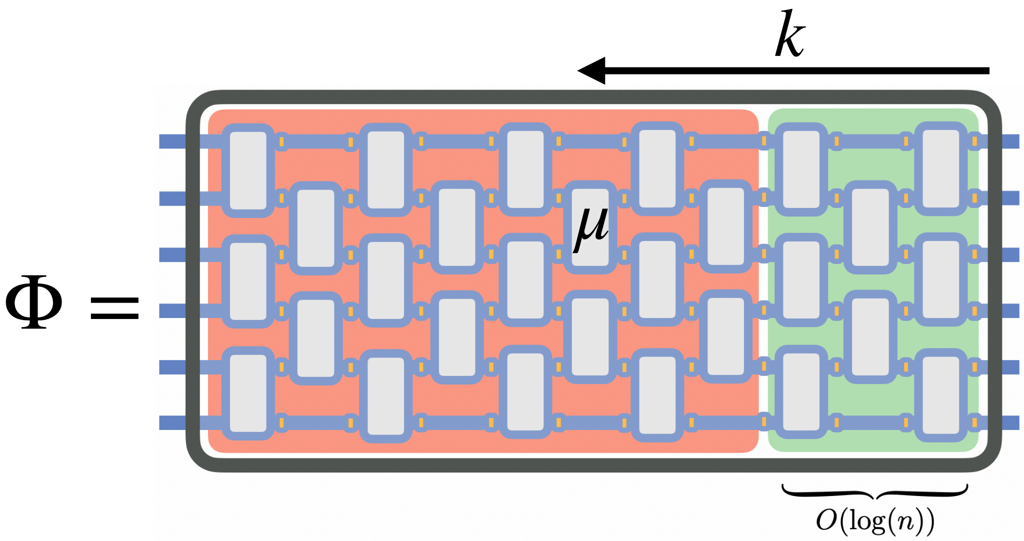

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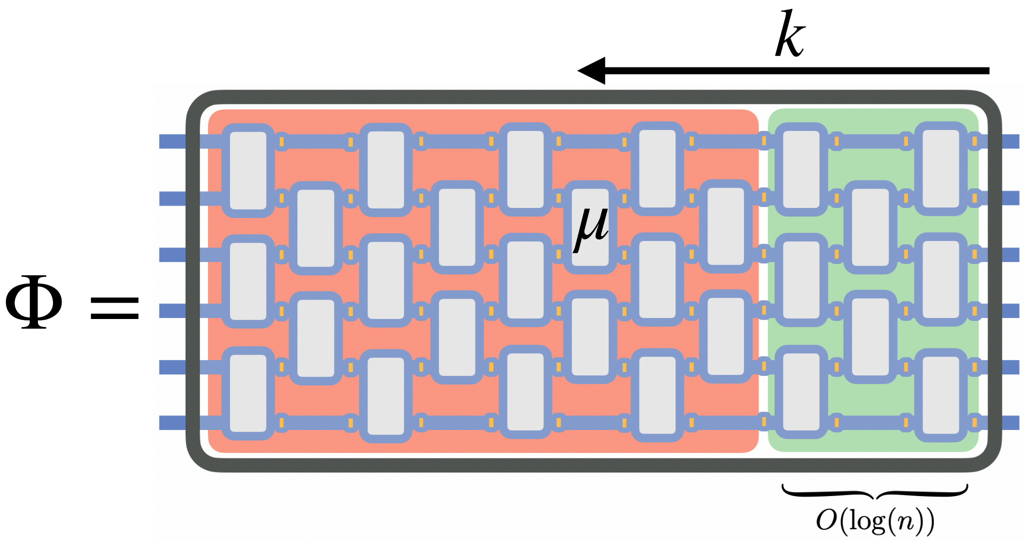

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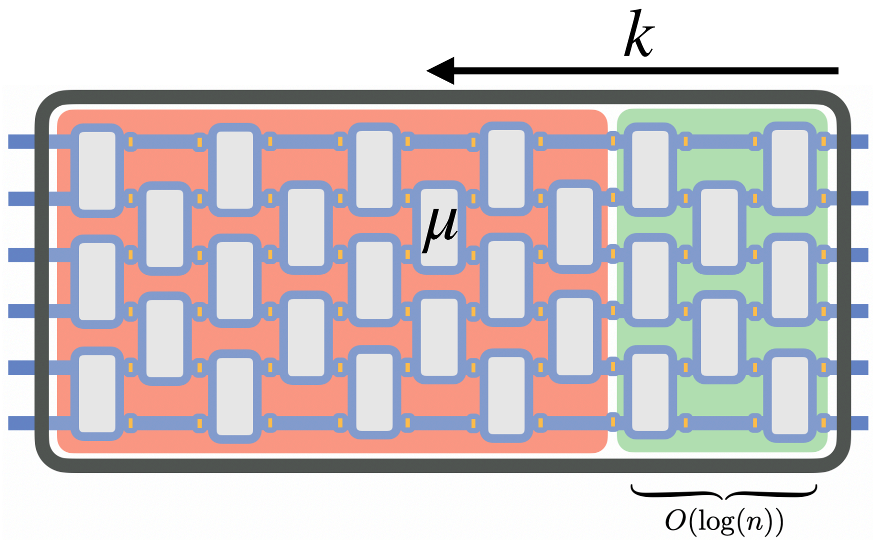

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<u>**Compare with**</u>: [3] Beyond unital noise in variational quantum algorithms: noise-induced barren plateaus and fixed points. Singkanipa *et al.*, ArXiv. (2024).

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$$\mathcal{N}(I) = I + t_X X + t_Y Y + t_Z Z$$

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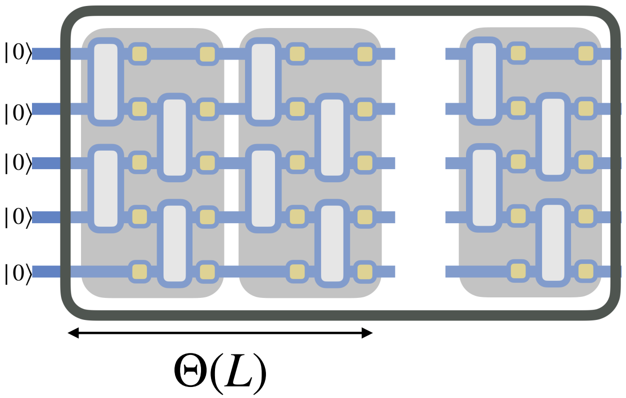

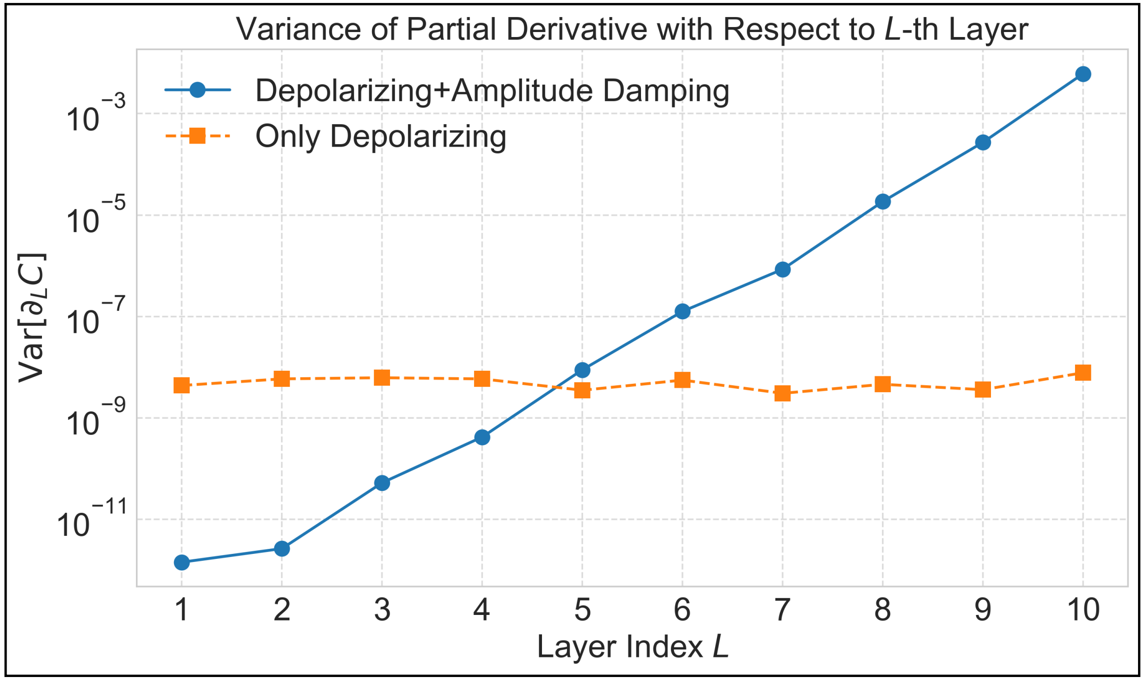

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Thanks a lot for your attention!

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