A causal perspective on Bell's theorem

WAQ

4 November 2024

Bell's theorem

nonlocality

nonclassicality

indeterminism / randomness

Determinism + locality + free choice $\longrightarrow$ Bell inequalities

Local causality + free choice $\longrightarrow$ Bell inequalities

Determinism + locality + free choice $\longrightarrow$ Bell inequalities

Local causality + free choice $\longrightarrow$ Bell inequalities

"Experimental metaphysics"

Determinism + locality + free choice $\longrightarrow$ Bell inequalities

Local causality + free choice $\longrightarrow$ Bell inequalities

<u>Classical explanation</u> /

<u>causal explanation</u>

Causal perspective

$\downarrow$

(classical) causal modelling

classical causal modelling + causal structure

$$
\begin{tikzcd}
 & A & & B & \\
X \arrow[ur] & & \Lambda \arrow[ul] \arrow[ur] & & Y \arrow[ul]
\end{tikzcd}
$$

$\rightsquigarrow$ Bell inequalities

classical causal modelling + no fine-tuning $\rightsquigarrow$ Bell inequalities

Wood & Spekkens (NJP, 2015): "The lesson of causal discovery algorithms for quantum correlations: causal explanations of Bell-inequality violations require fine-tuning."

Outline

- Classical causal modelling
- Bell inequalities from causal models
- Responses:
  - Reject causal structure
  - Reject causal modelling paradigm

Classical causal modelling

Fundamental physics : causation

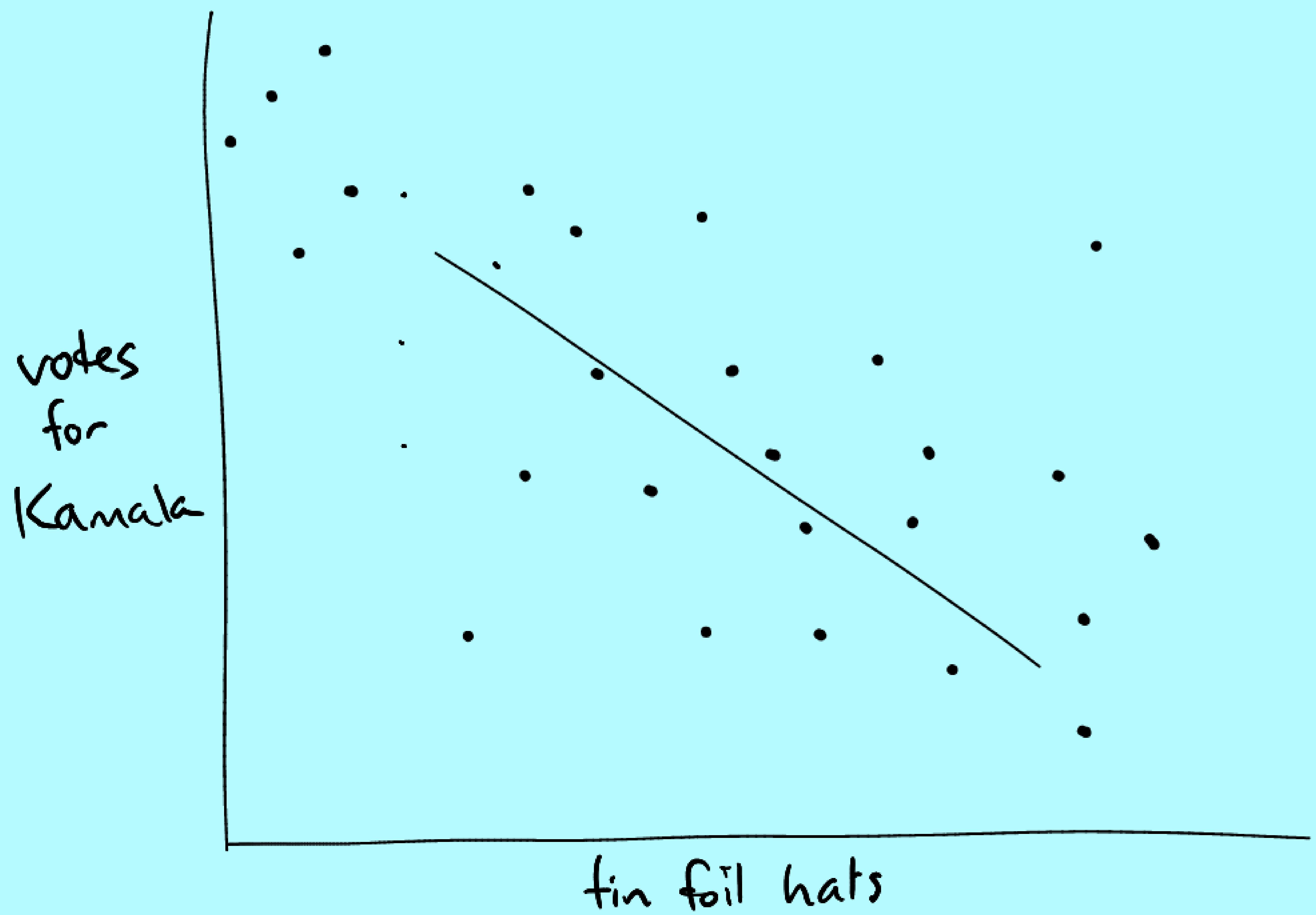
Special sciences : causation

Observation
Big data
Correlation

$\longrightarrow$

Experimentation
Intervention
Counterfactuals

Had Bernie worn a tin foil hat on the way to the polling station, would he still have voted for Kamala?

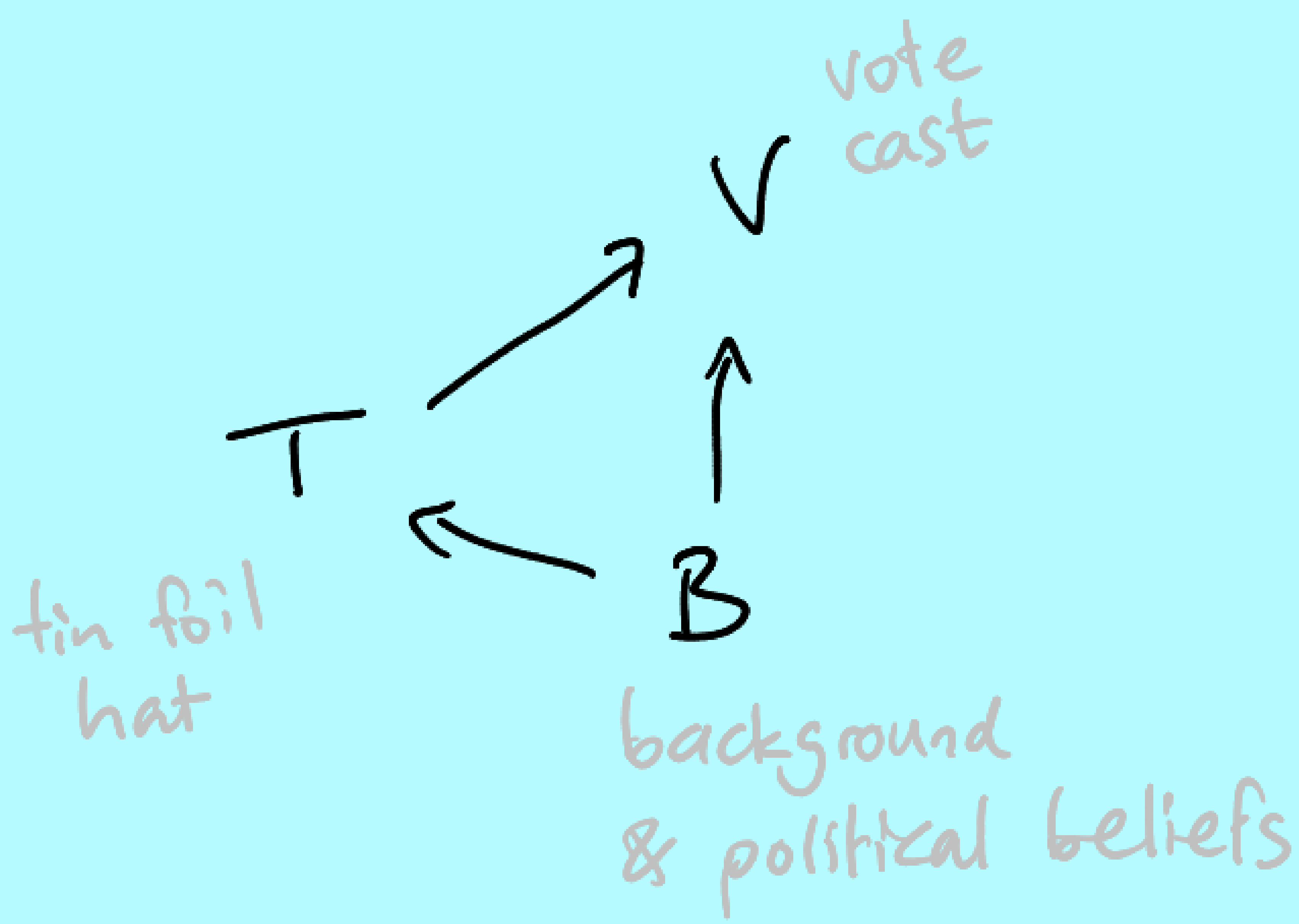

votes
for
Kamala
tin foil hats

Def A <u>causal model</u> for $P(X_1, \dots, X_n)$ is a directed acyclic graph G on $\{X_1, \dots, X_n\}$ and $P_i(X_i \mid Pa(X_i))$ for $i = 1, \dots, n$

such that
$$\boxed{P(X_1 \cdots X_n) = \prod_{i=1,\dots,n} P_i(X_i \mid Pa(X_i))}$$

<u>causal structure</u>

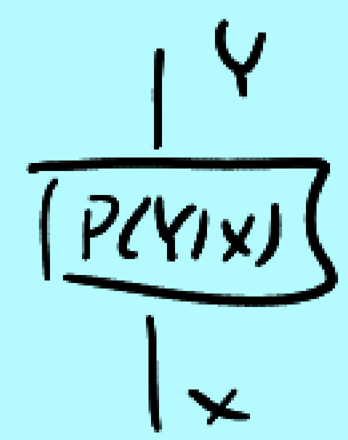
<u>autonomous</u> mechanisms



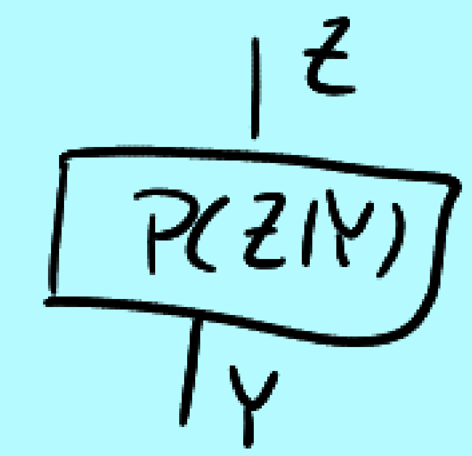
$$P(BTV) = P_B(B)\, P_T(T \mid B)\, P_V(V \mid BT)$$

Interlude: diagrammatic reasoning

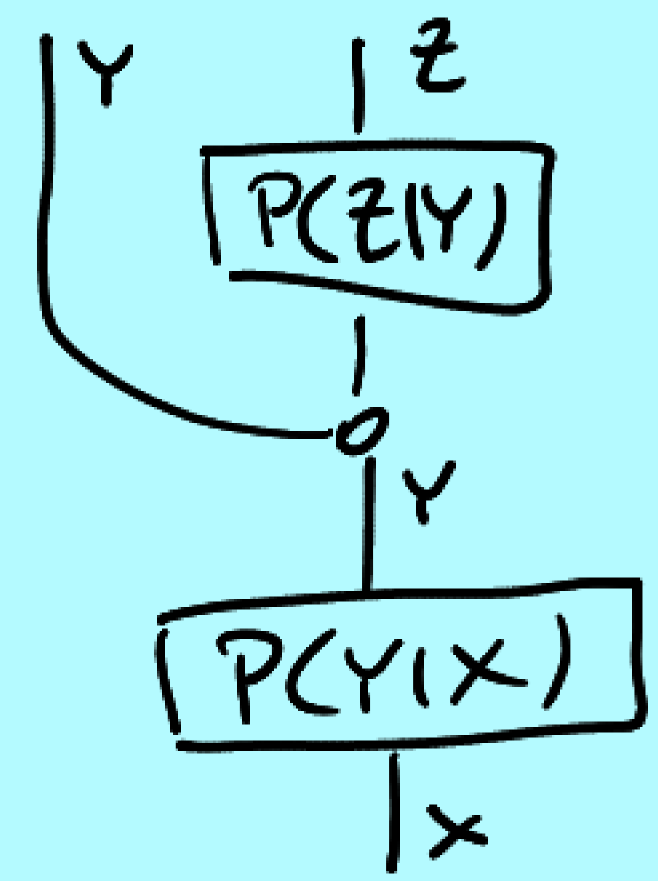
$P(Y|X) \rightsquigarrow$

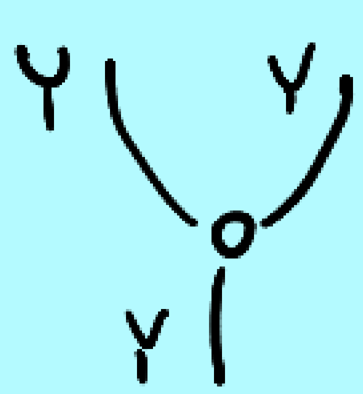


$P(Z|Y) \rightsquigarrow$



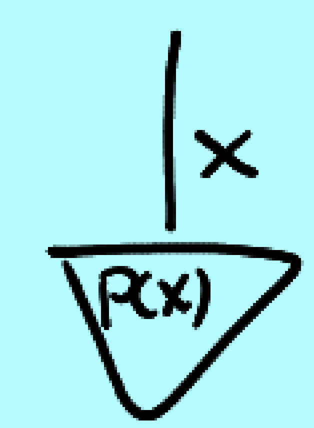
$P(YZ|X) = P(Y|X)\,P(Z|Y) \rightsquigarrow$





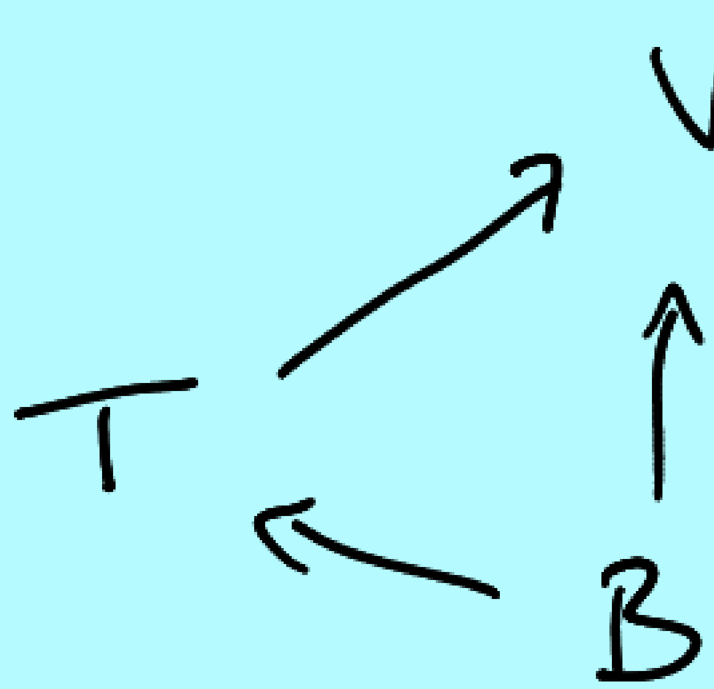
copy map

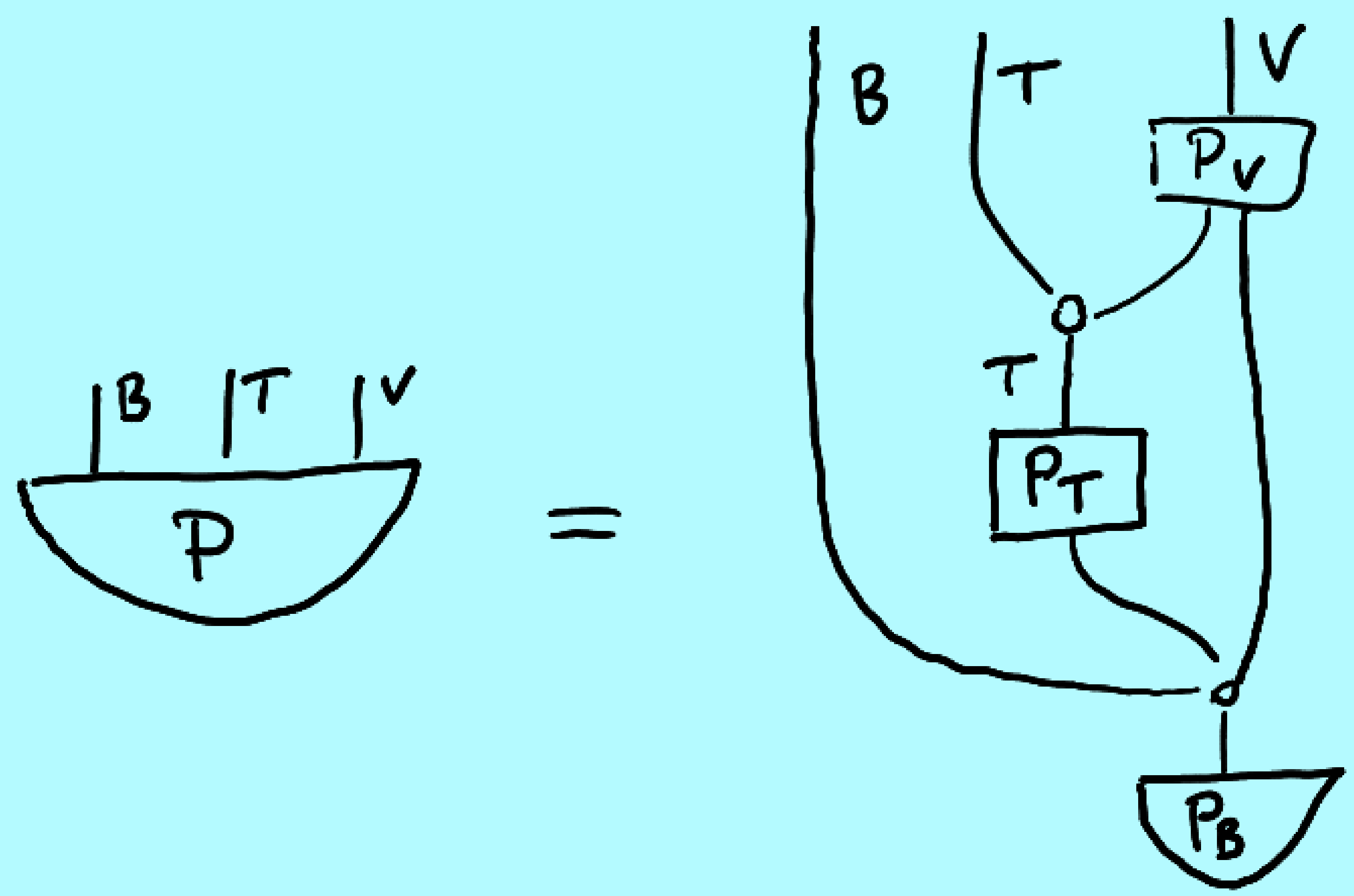
$P(X) \rightsquigarrow$



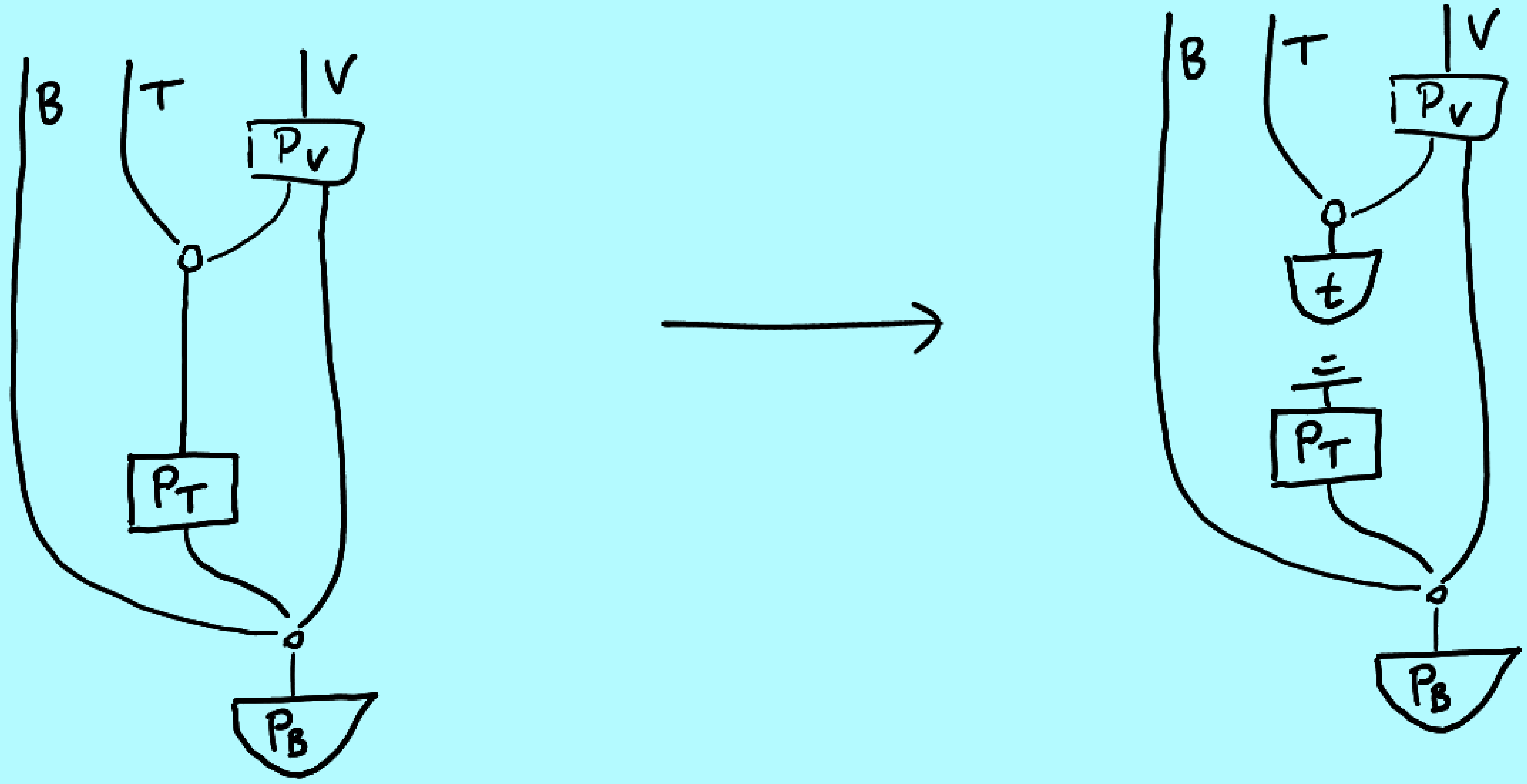
$$P(BTV) = P_B(B)\,P_T(T|B)\,P_V(V|BT)$$

autonomous mechanisms





Intervention



Explaining correlation

$$P(TV) \neq P(T)P(V)$$

$T \quad V$

$T \to V$

$V \to T$

$$\begin{tikzcd} V & & T \\ & B \arrow[ul] \arrow[ur] & \end{tikzcd}$$

$$P(VT) = \Sigma_B P(VTB)$$

$$P(VT|B) = P(V|B)P(T|B)$$

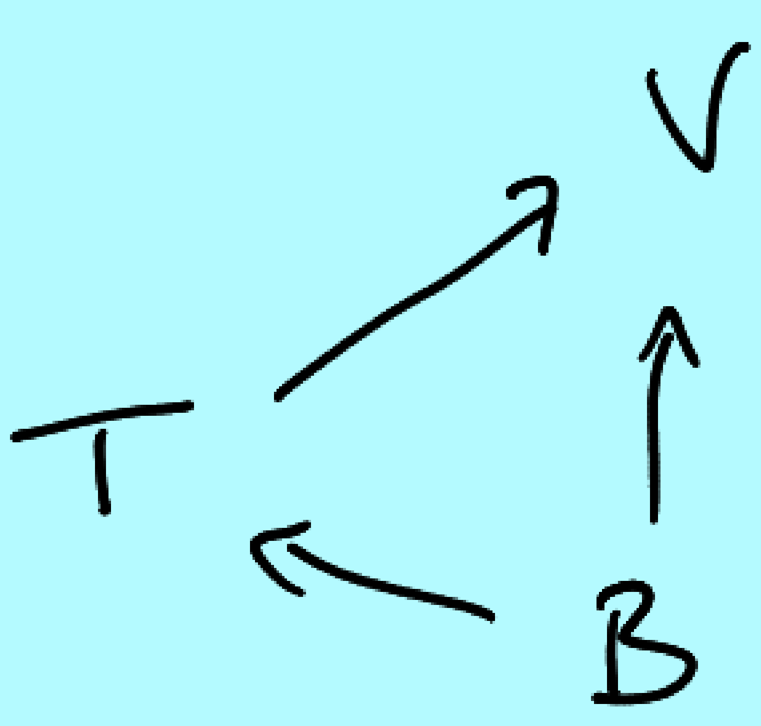
$\boxed{\text{Def}}$ A <u>functional causal model</u> for $P(X_1 \cdots X_n)$ is a DAG G

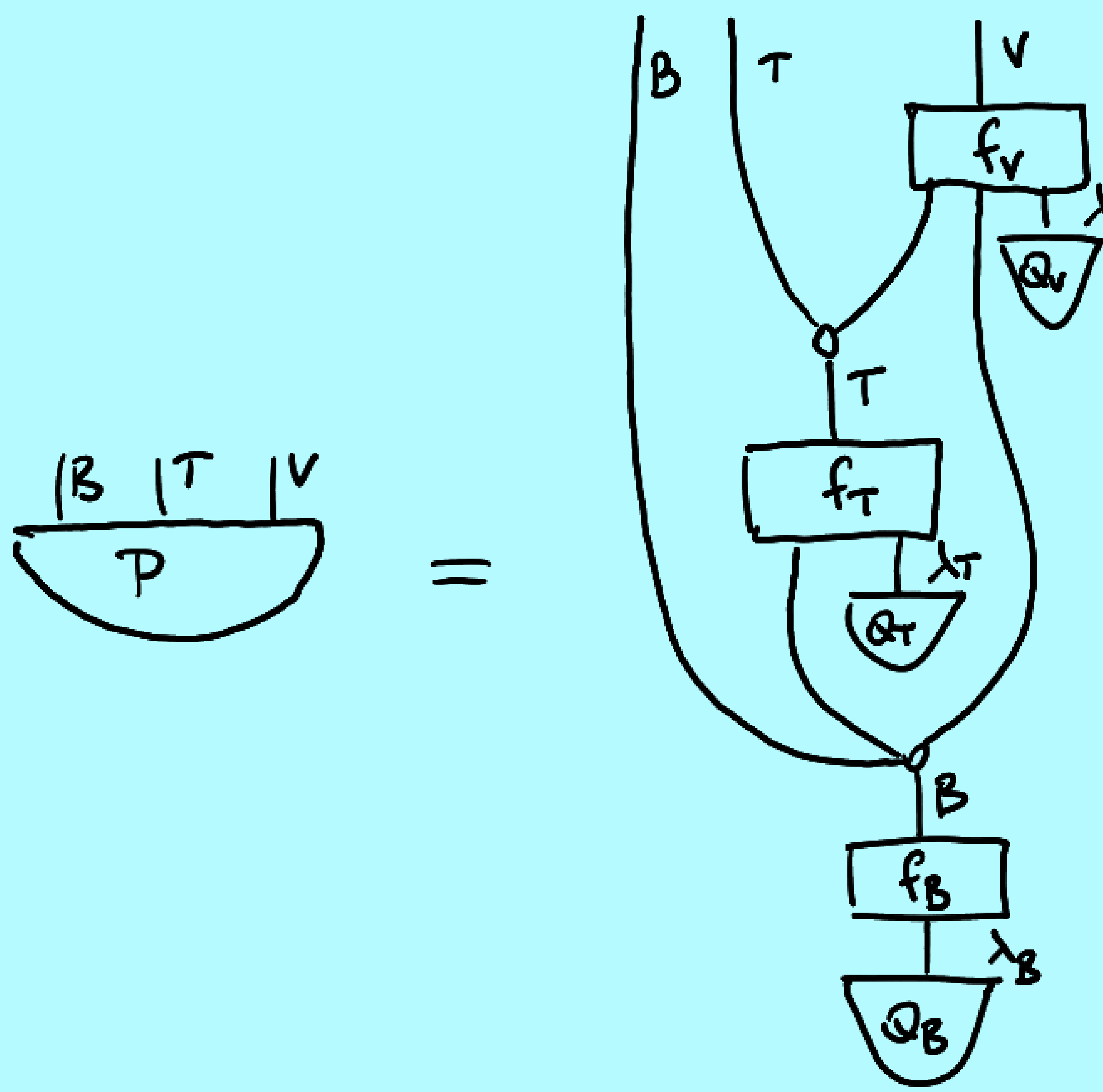
on $\{X_1, \ldots, X_n\}$ and for each $i \in \{1, \ldots, n\}$

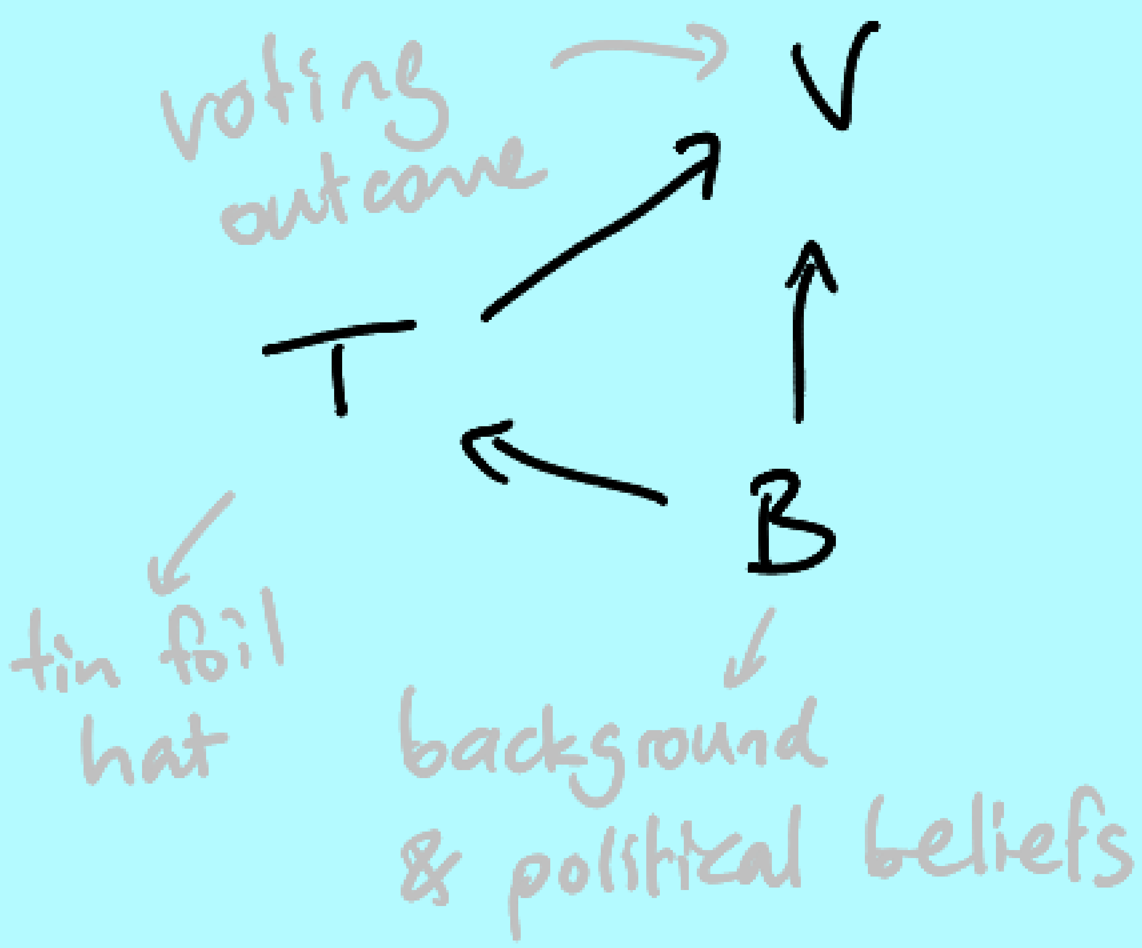
$\lambda_i \sim Q_i(\lambda_i)$ <u>local</u> noise

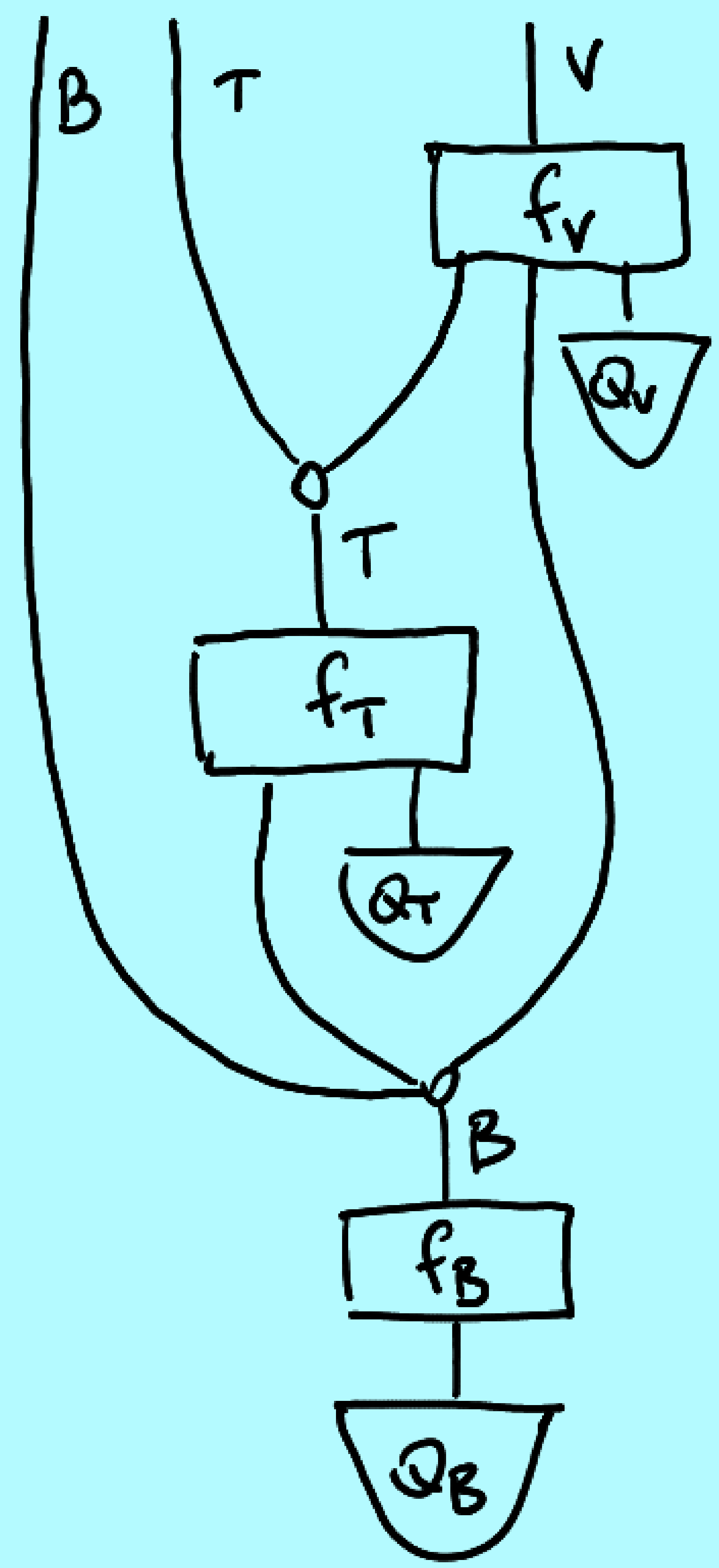
$f_i : \lambda_i \times Pa(X_i) \longrightarrow X_i$ <u>deterministic</u> mechanisms

s.t. $$P(X_1 \cdots X_n) = \sum_{\lambda_1 \cdots \lambda_n} \prod_{i=1,\ldots,n} \delta_{X_i = f(\lambda_i, Pa(X_i))} \, Q_i(\lambda_i)$$









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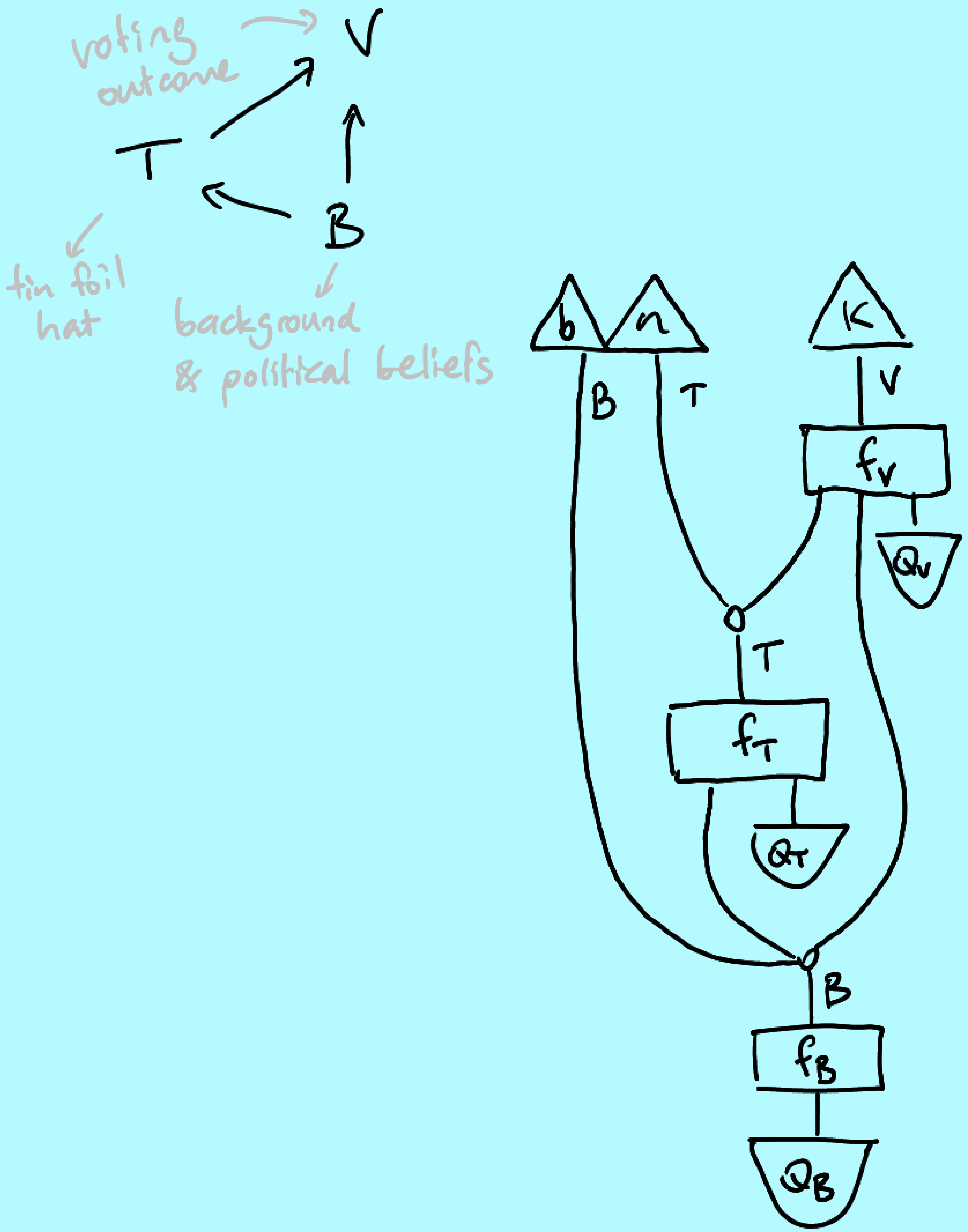

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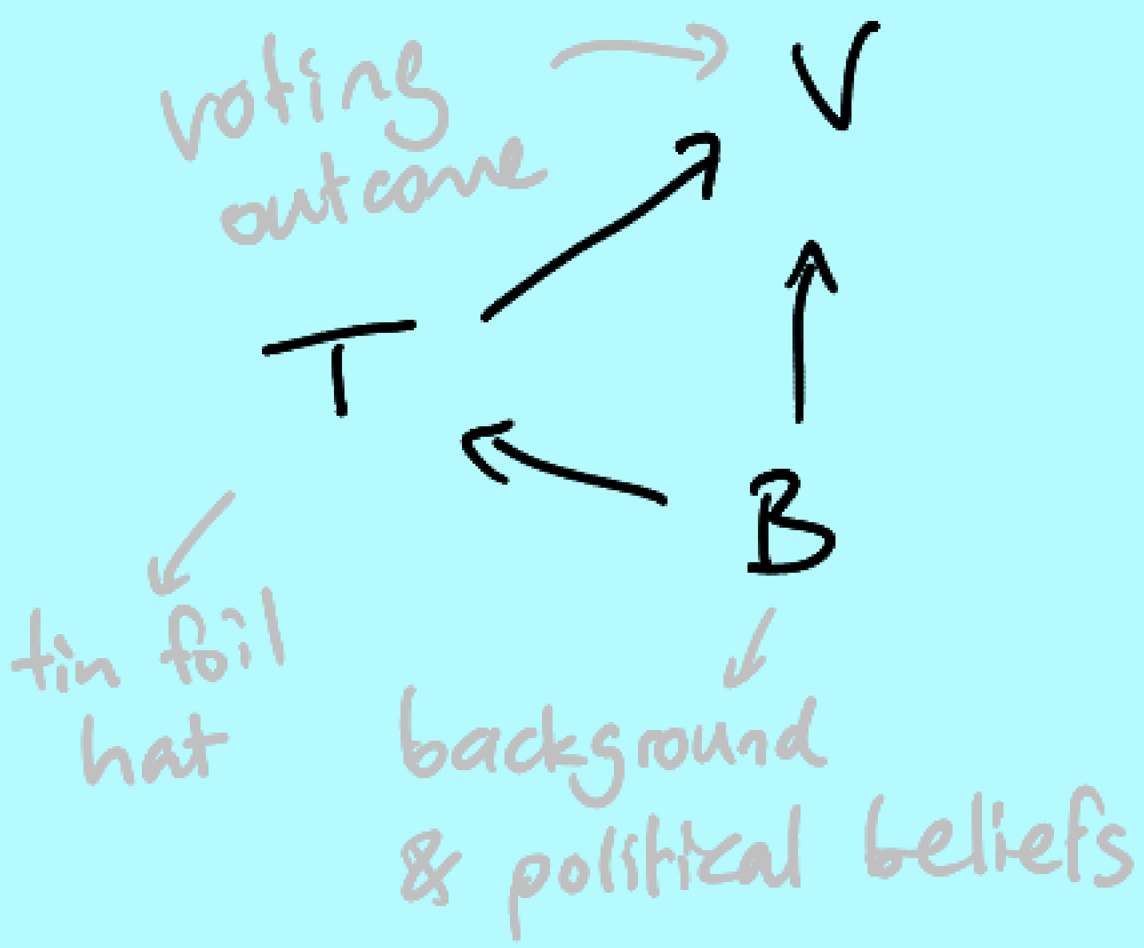

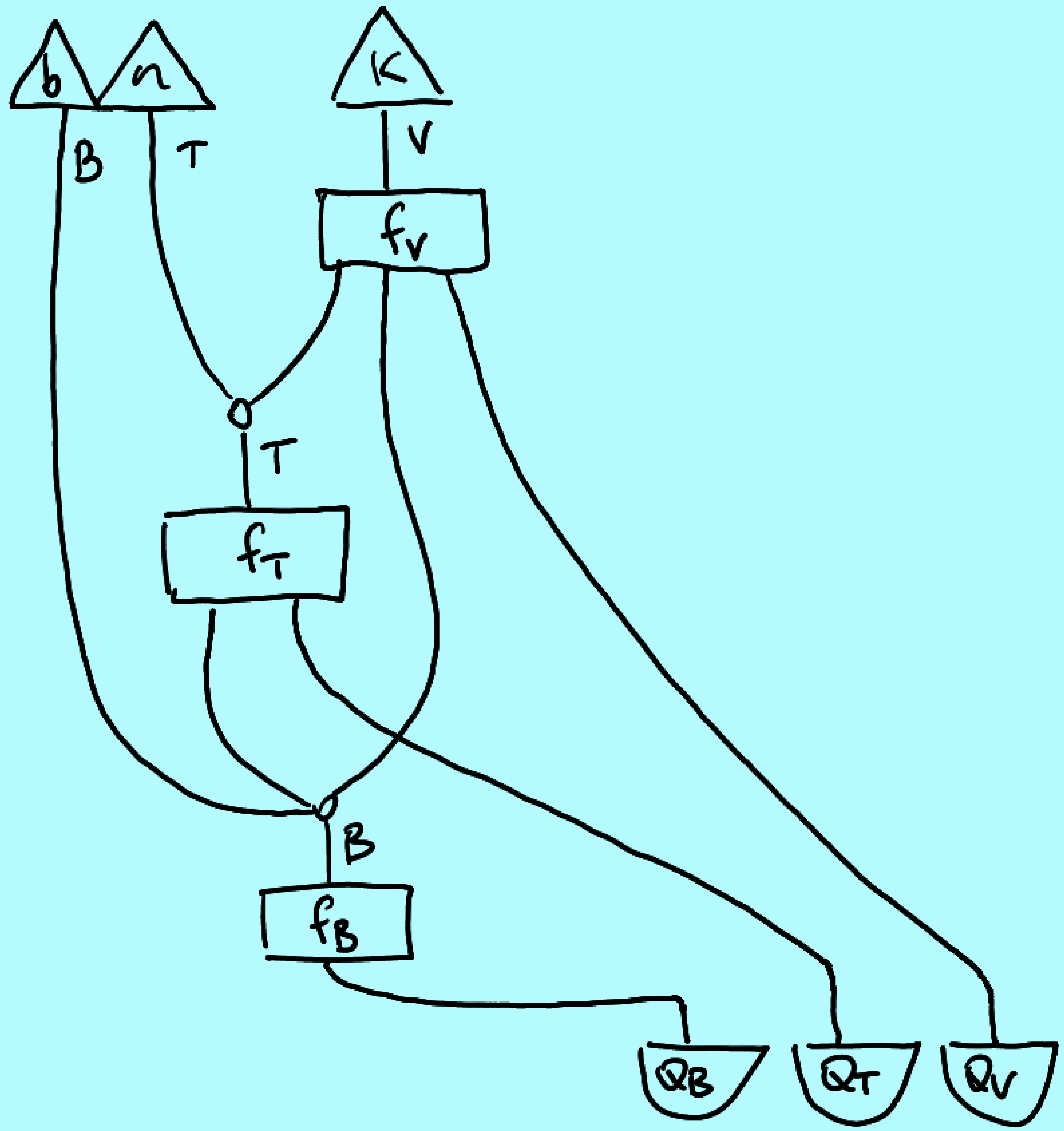

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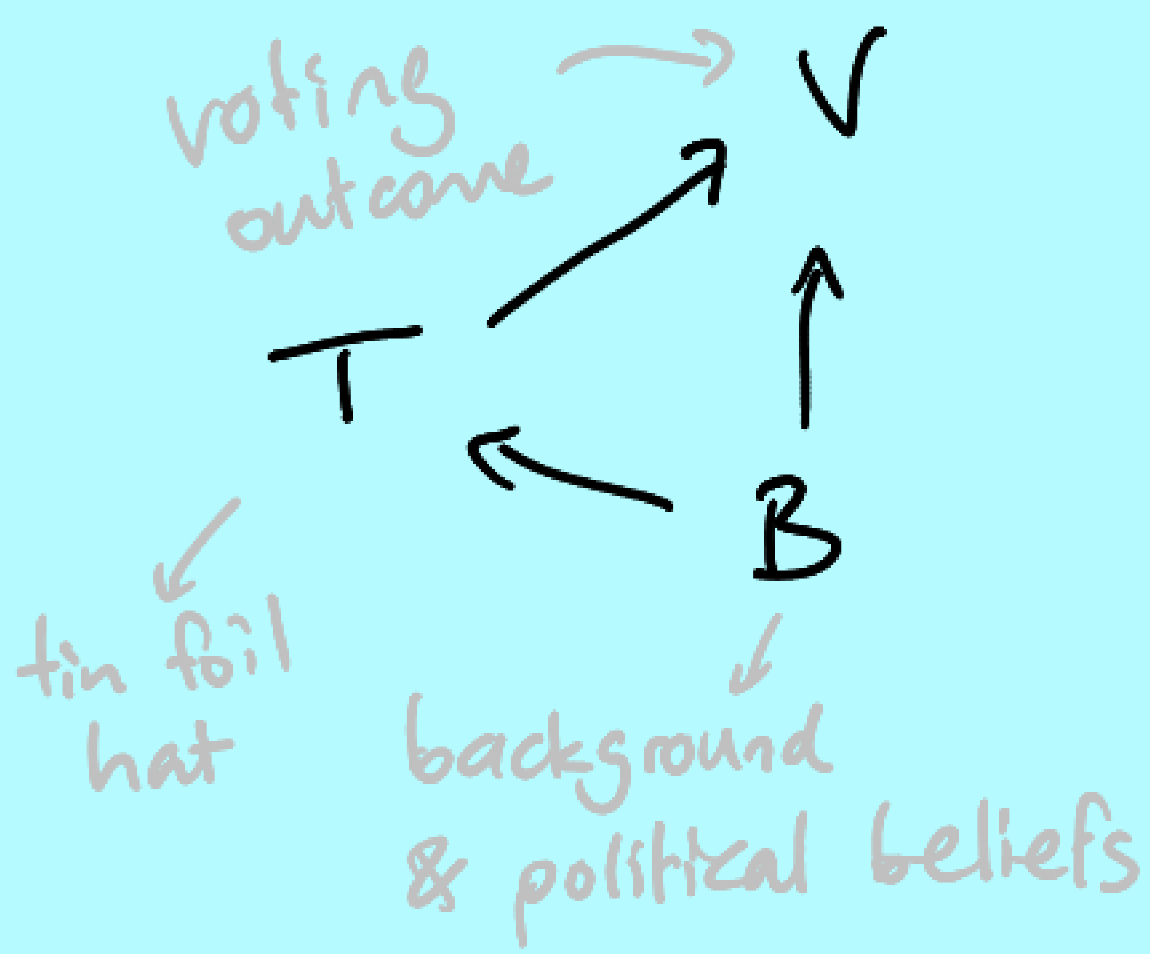

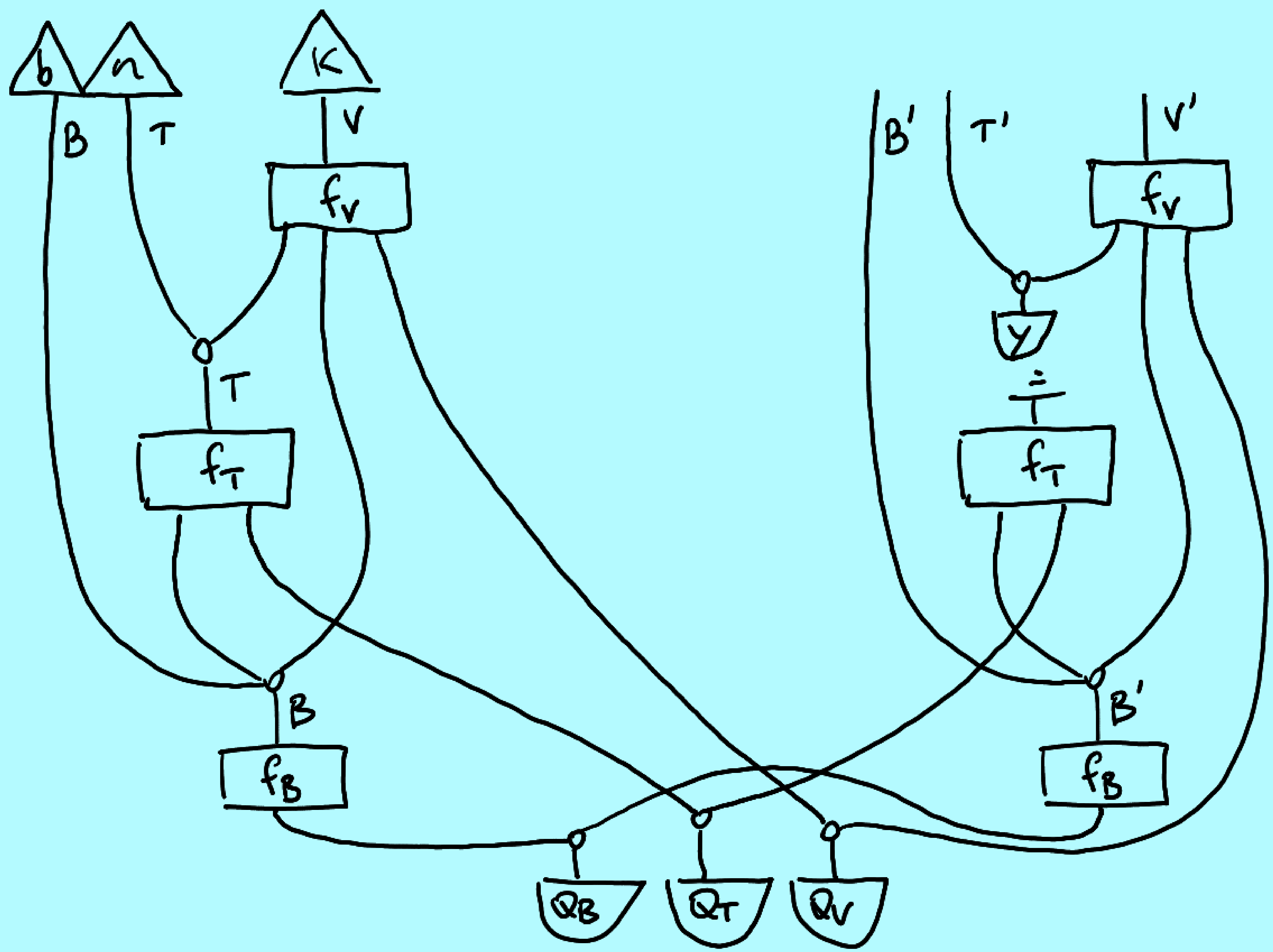

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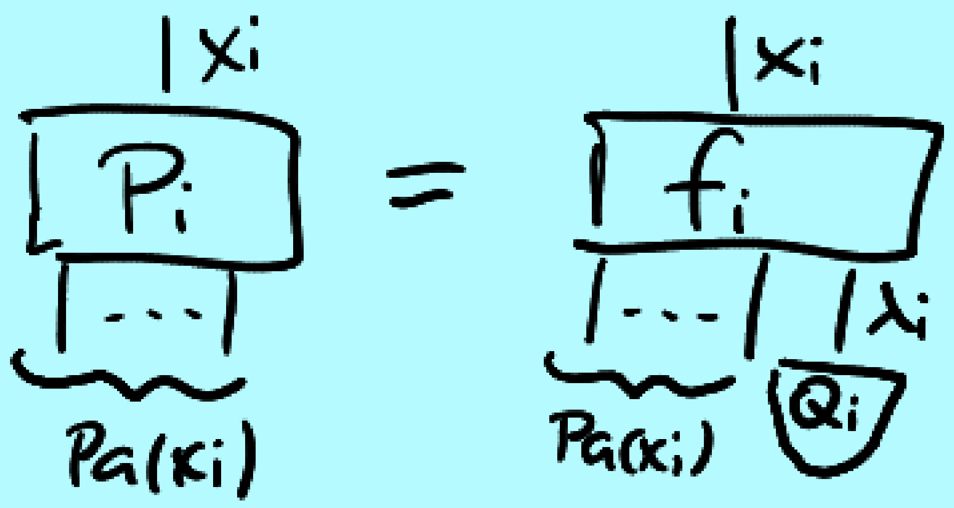
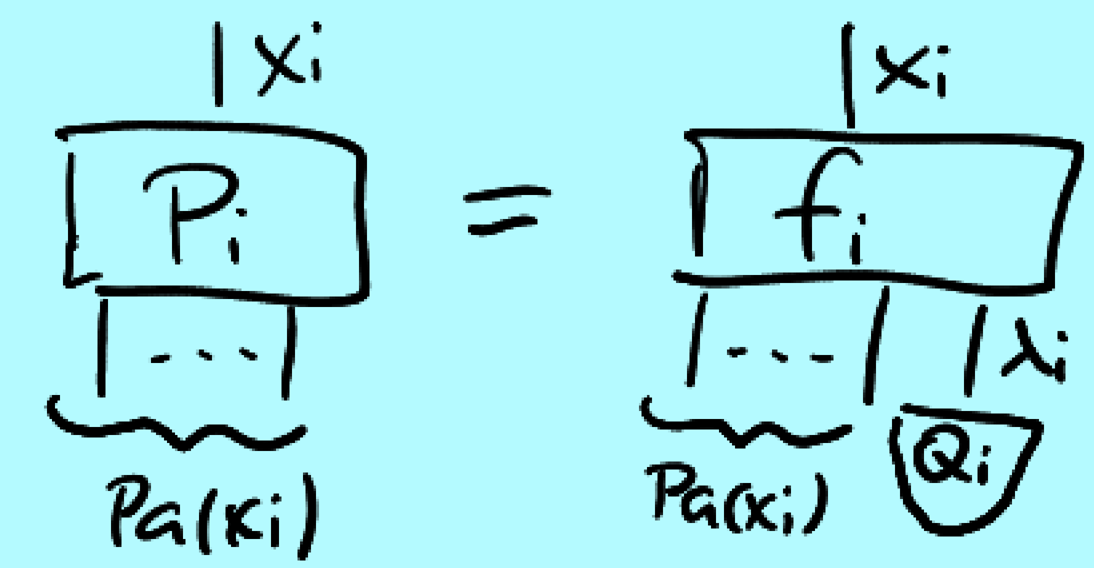


3 Functional causal model

$G, \{\lambda_i, f_i\}$

Determinism; counterfactuals

functional dilation:



2 Causal model

$G, \{P_i\}$

Intervention; causation

$P = \prod_i P_i$

$A \to B$

$B \to A$

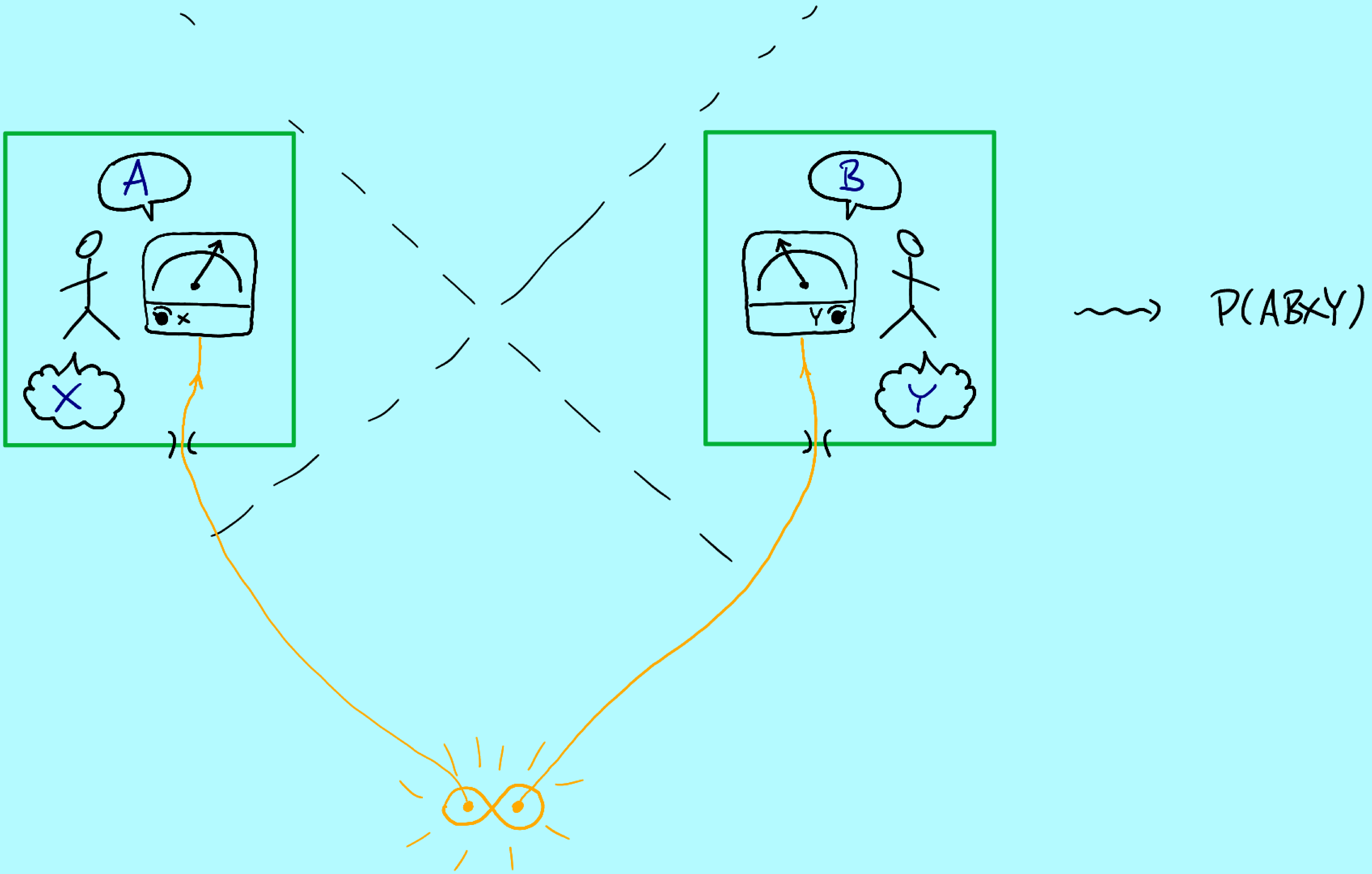
$A \leftarrow \Lambda \to B$

1 Data

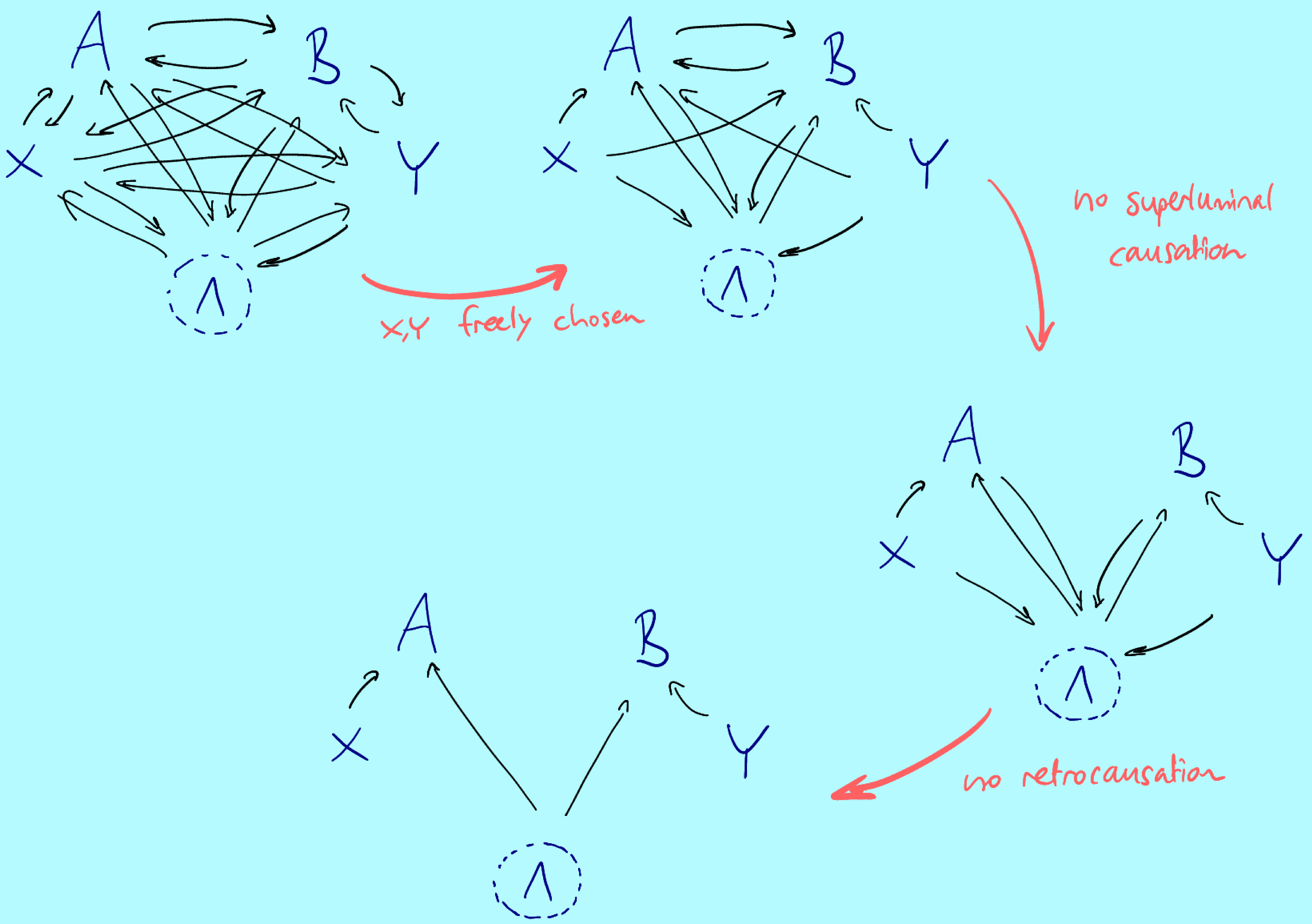
$P(x_1 \cdots x_n)$

Observation; correlation

Bell's theorem vs causal modelling



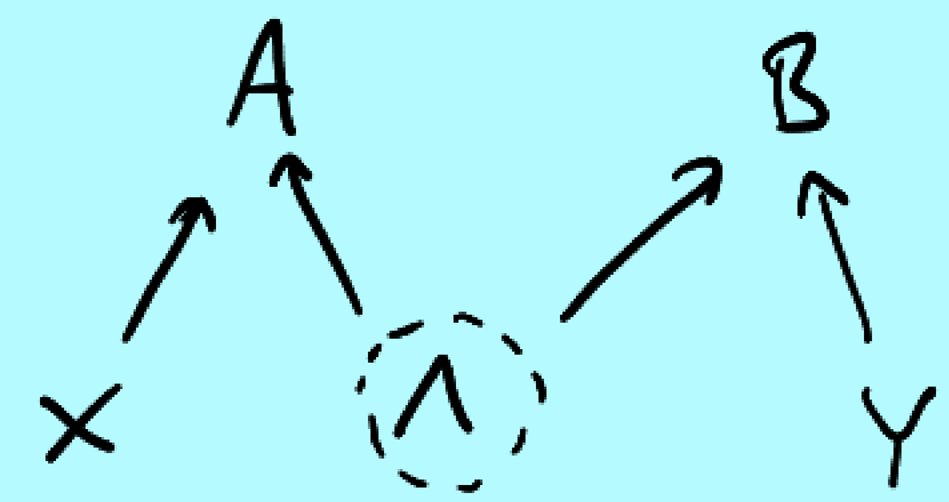
A
x
X
B
Y
Y
P(ABXY)



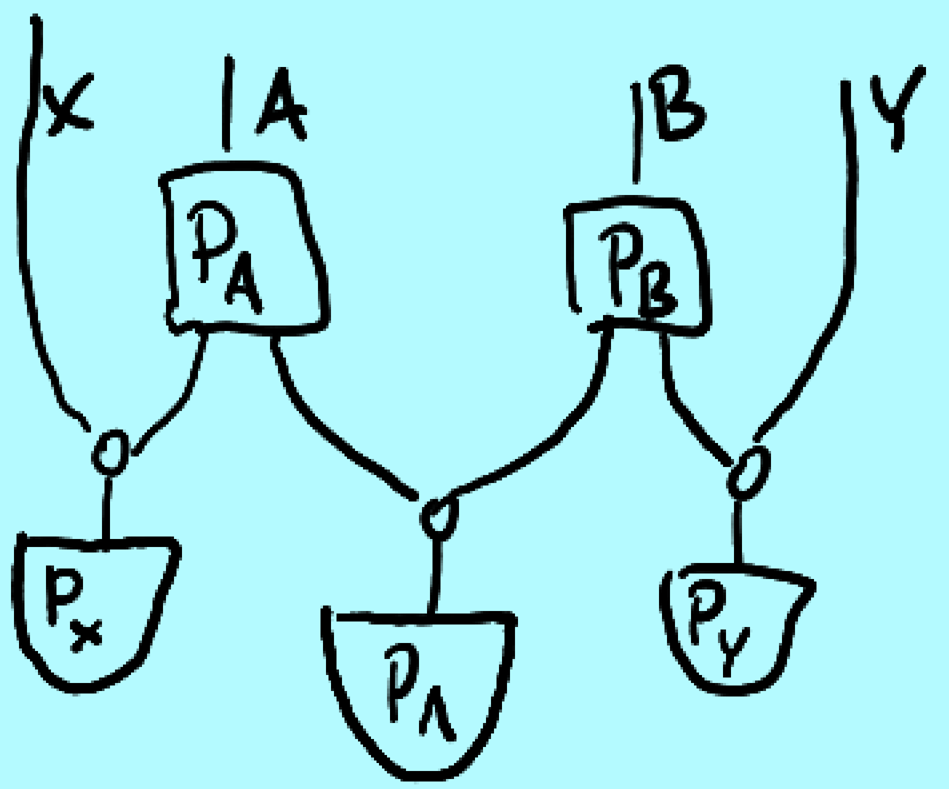
A
B
X
Y
Λ
X,Y freely chosen
no superluminal causation
no retrocausation

Let $P(ABXY)$ be the marginal of a causal model with causal structure

$P(ABXY\Lambda)$,



i.e. $P(ABXY) = \sum_\Lambda P_\Lambda(\Lambda)\, P_X(X)\, P_Y(Y)\, P_A(A|X\Lambda)\, P_B(B|Y\Lambda) =$



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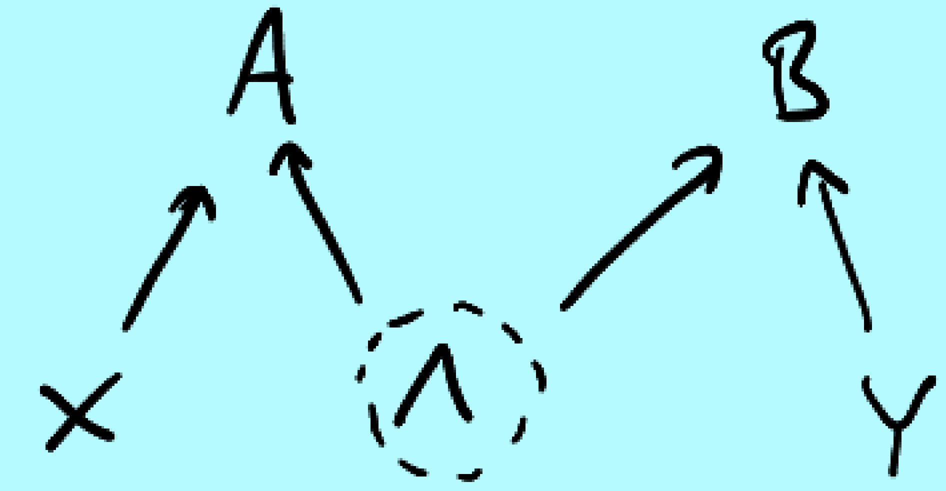
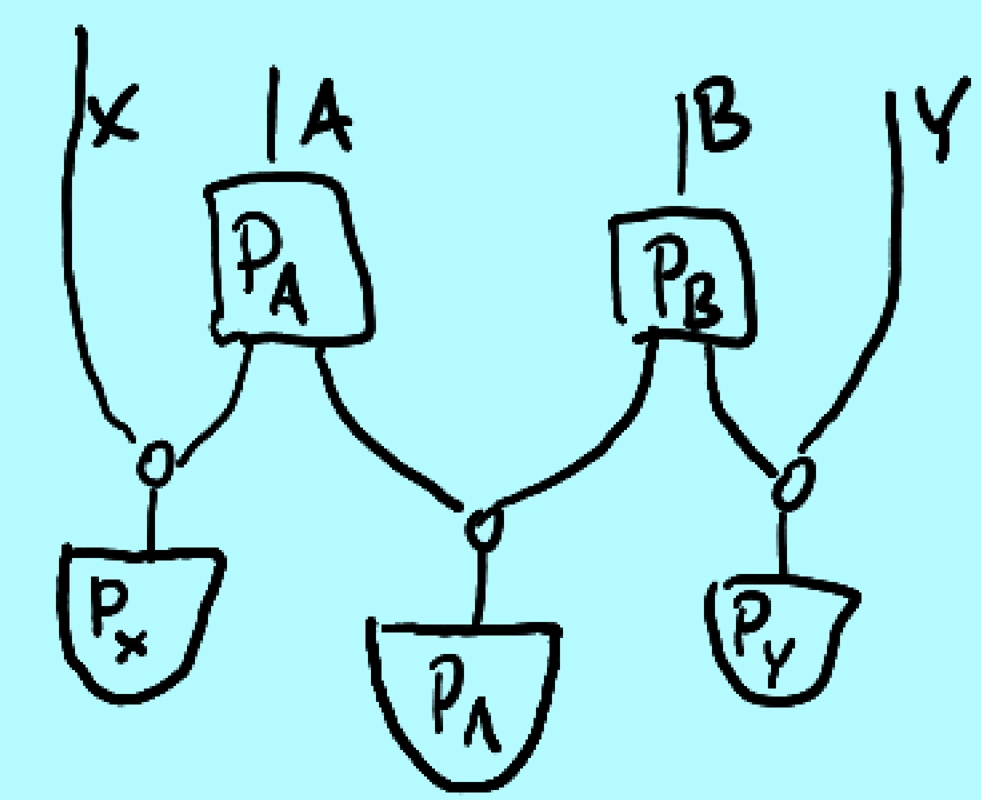
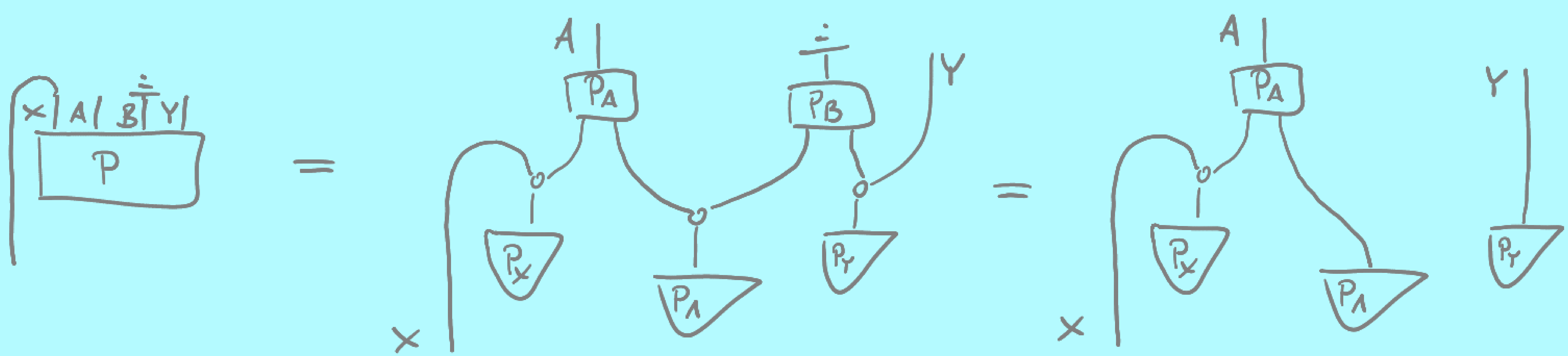
$P(ABXY\Lambda)$, 

i.e. $P(ABXY) = \sum_{\Lambda} P_{\Lambda}(\Lambda)\, P_X(X)\, P_Y(Y)\, P_A(A|X\Lambda)\, P_B(B|Y\Lambda)$ = 

Then $(A \perp\!\!\!\perp Y \,|\, X)_P$; $(B \perp\!\!\!\perp X \,|\, Y)_P$ ("no superluminal signalling")

$P(AY|X) = P(A|X)\,P(Y|X)$



Interlude: d-separation

$\boxed{\text{Def}}$ In a DAG G with vertices $V = \{X_1, \dots, X_n\}$, and disjoint $S, T, U \subseteq V$

S is <u>d-separated</u> from T by U, notated $(S \perp\!\!\!\perp T \mid U)_G$, iff

...

$X \to Y \to Z$

$(X \perp\!\!\!\perp Z \mid Y)_G$

$\Downarrow$

$(X \perp\!\!\!\perp Z \mid Y)_P$

$$\begin{tikzcd} X & & Z \\ & Y \arrow[ul] \arrow[ur] & \end{tikzcd}$$

$(X \perp\!\!\!\perp Z \mid Y)_G$

$\Downarrow$

$(X \perp\!\!\!\perp Z \mid Y)_P$

$$\begin{tikzcd} & Y & \\ X \arrow[ur] & & Z \arrow[ul] \end{tikzcd}$$

$(X \perp\!\!\!\perp Z)_G$

$\Downarrow$

$(X \perp\!\!\!\perp Z)_P$

$$\begin{tikzcd} & Y & \\ X \arrow[ur] & & Z \arrow[ul] \end{tikzcd}$$

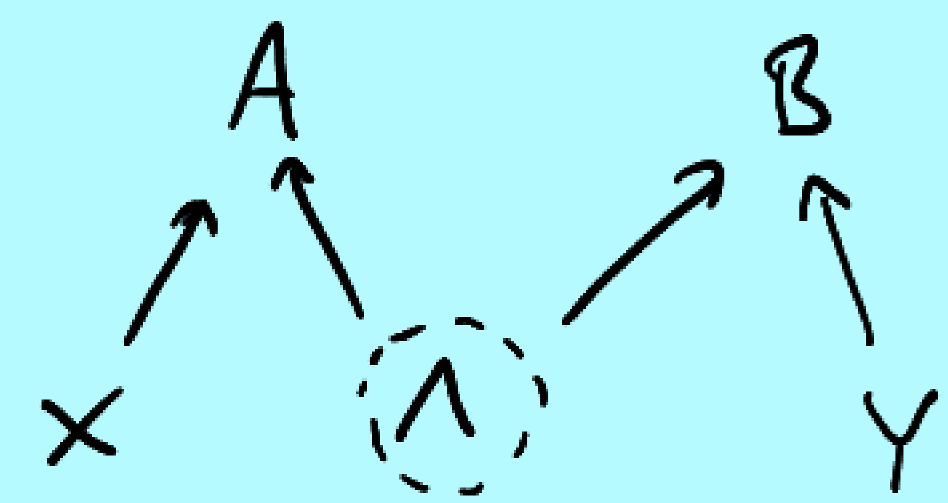
$\neg (X \perp\!\!\!\perp Z \mid Y)_G$

$\boxed{\text{Thm}}$ d-separation is <u>sound</u> for statistical independence: if P admits a G-model

then $$(S \perp\!\!\!\perp T \mid U)_G \implies (S \perp\!\!\!\perp T \mid U)_P .$$

Let $P(ABXY)$ be the marginal of a causal model with causal structure

$P(AB X Y \Lambda)$,

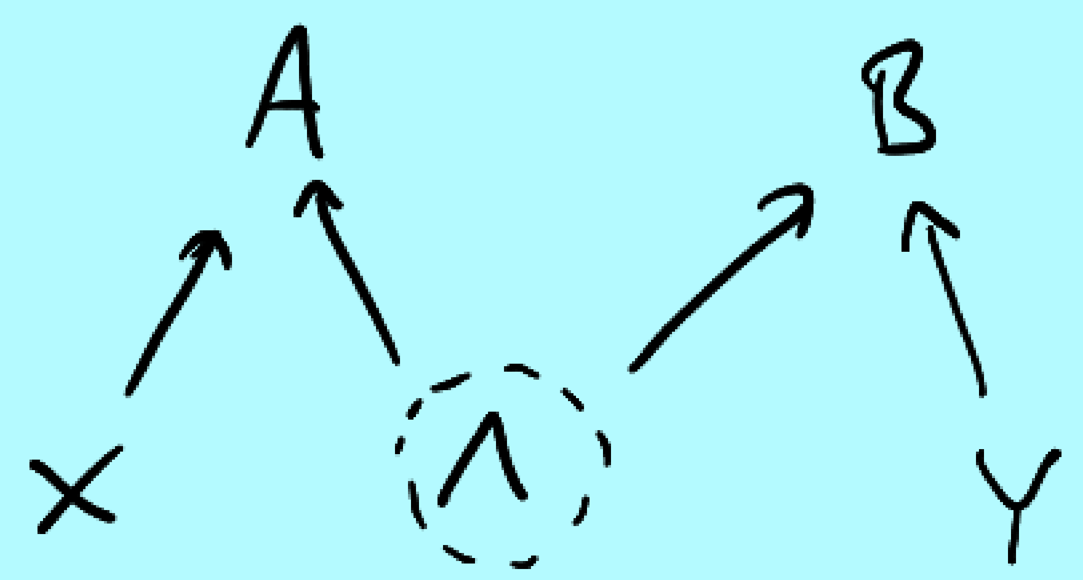


Then $(A \perp\!\!\!\perp Y \mid X)_P$; $(B \perp\!\!\!\perp X \mid Y)_P$ "no superluminal signalling"

$(X \perp\!\!\!\perp \Lambda \mid Y)_P$; $(Y \perp\!\!\!\perp \Lambda \mid X)_P$ "free choice"

$(A \perp\!\!\!\perp Y \mid X\Lambda)_P$; $(B \perp\!\!\!\perp X \mid Y\Lambda)_P$ "no superluminal influences"/ parameter independence

$(A \perp\!\!\!\perp B \mid XY\Lambda)_P$ outcome independence



Parameter independence and outcome independence together imply "local causality":

$$P(AB|XY\Lambda) = P(A|X\Lambda)\,P(B|Y\Lambda)$$

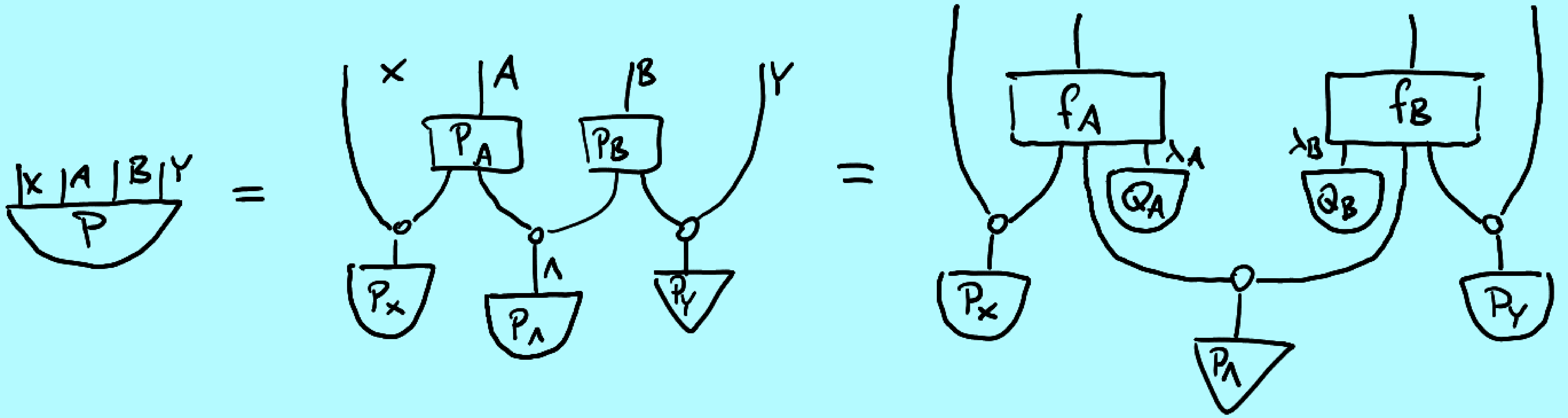
Correlation between A and B is explained through the variance of their complete common cause Λ:

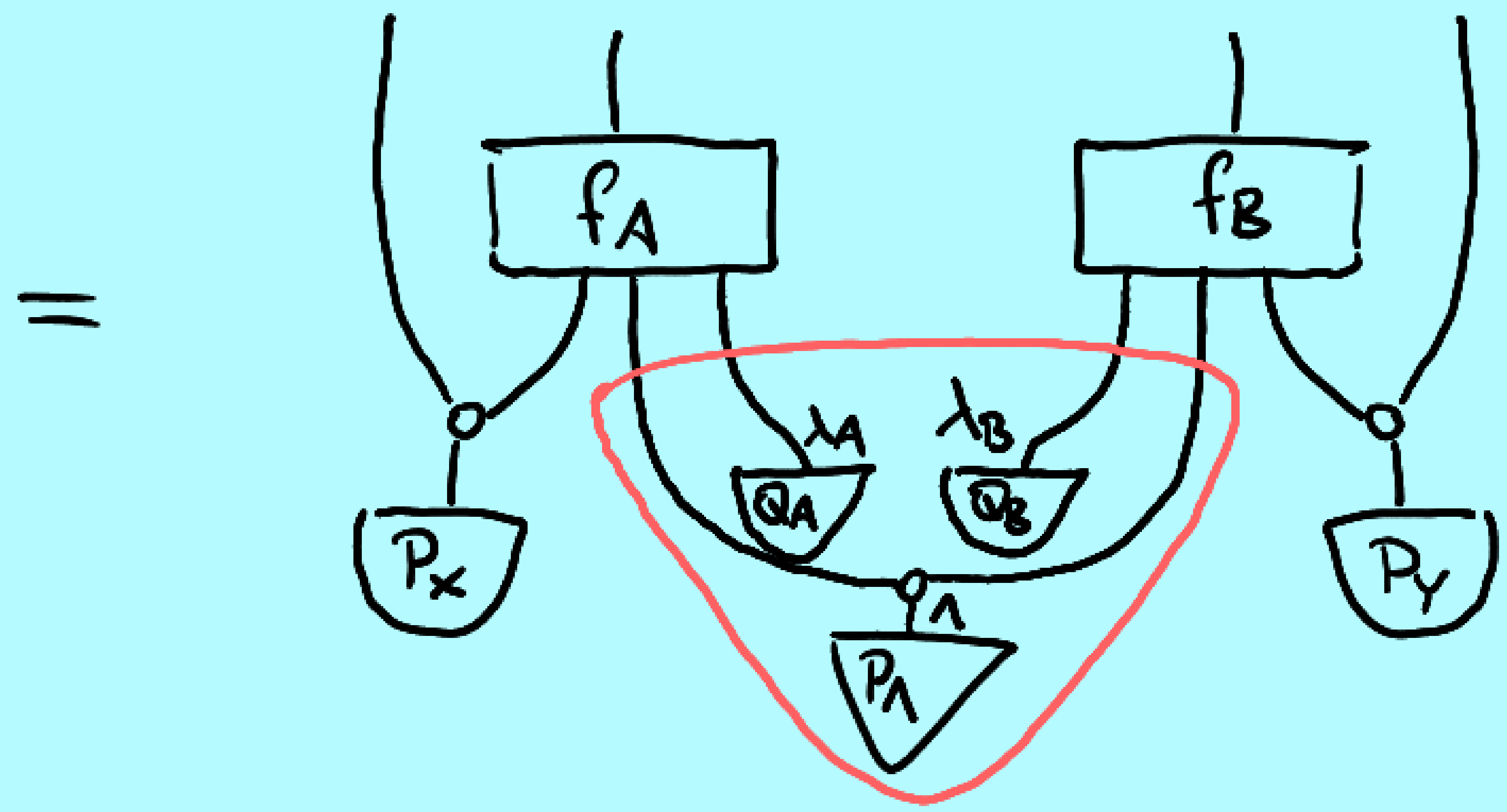
Had Λ been fixed, i.e. known to attain just one value $\Lambda = \lambda$, then A and B would not have been correlated

Local causality + free choice $\longrightarrow$ Bell inequalities

Determinism + locality + free choice $\longrightarrow$ Bell inequalities

Dilate causal model to a functional causal model:

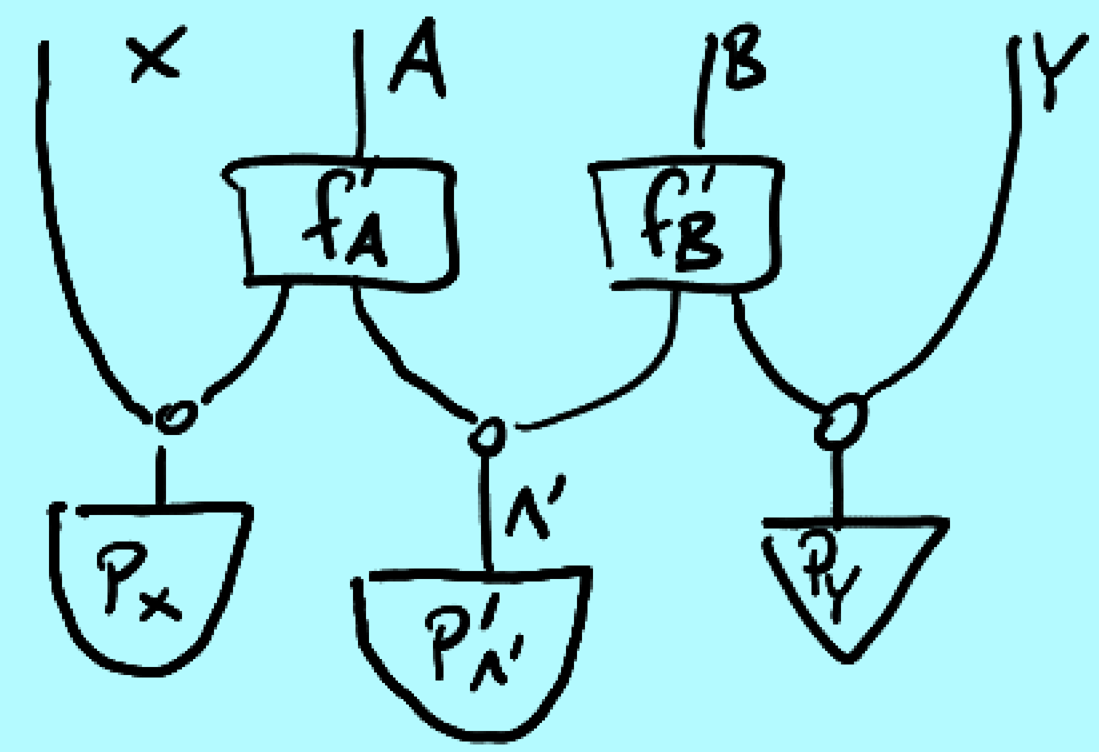




=

$\Downarrow$

$\Lambda' := \Lambda \lambda_A \lambda_B$

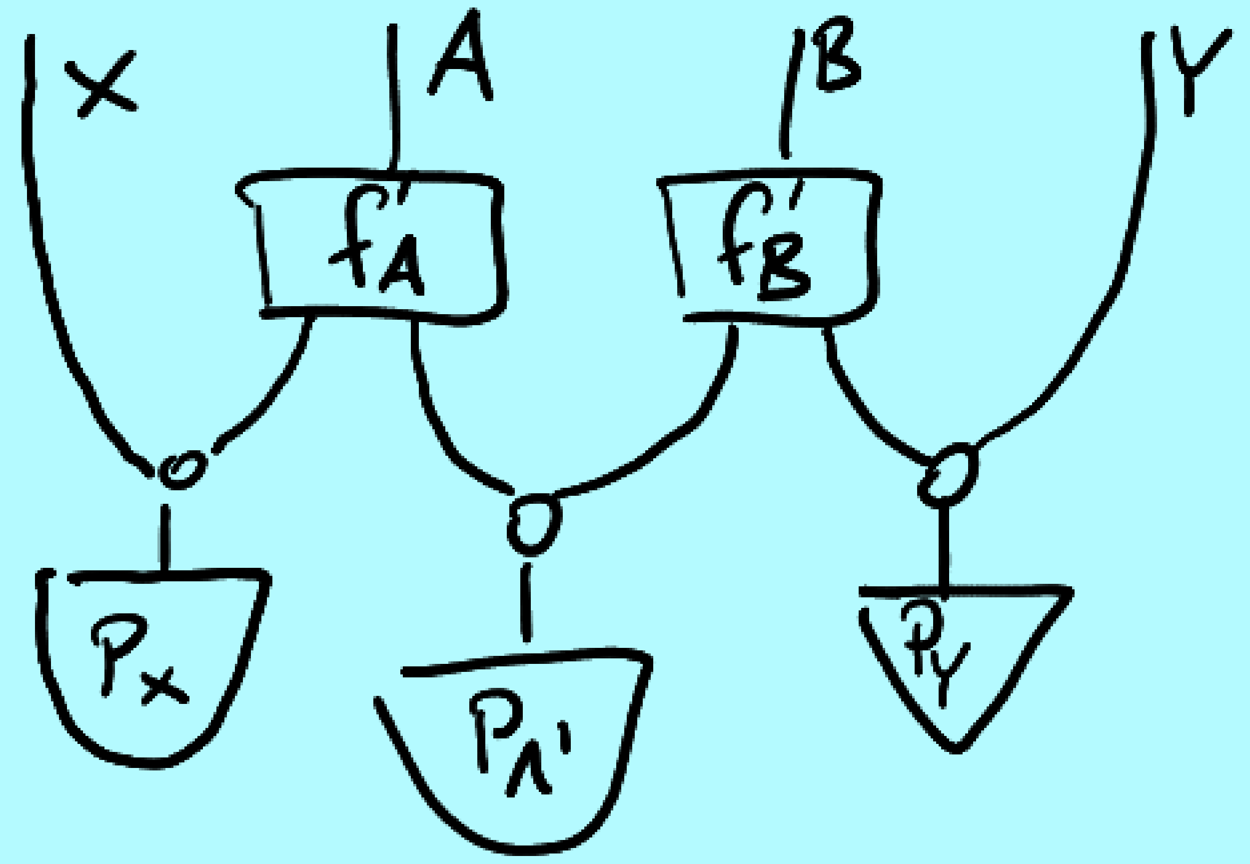


$$P(A \mid X \Lambda') \in \{0,1\}$$

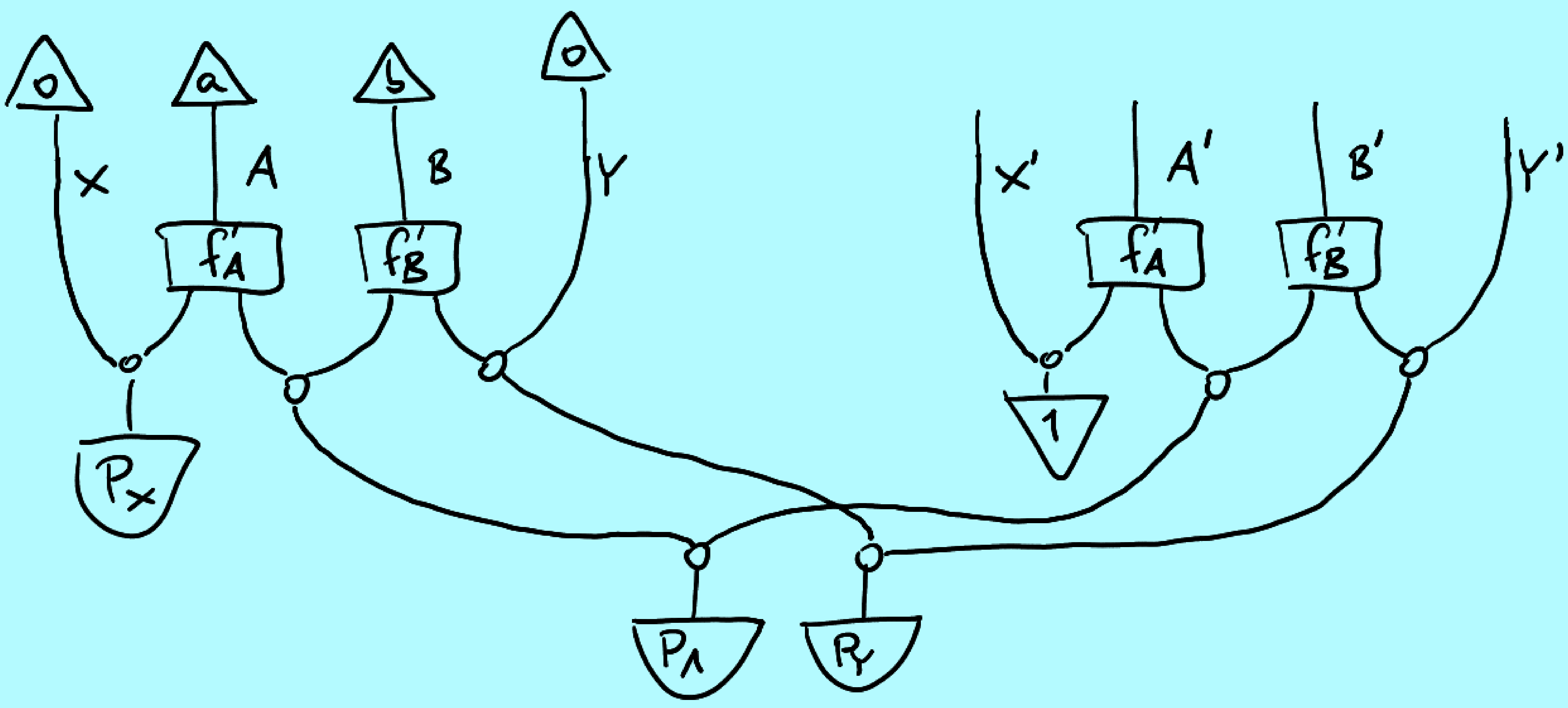
$$P(B \mid Y \Lambda') \in \{0,1\}$$

Determinism

Counterfactuals



Counterfactuals

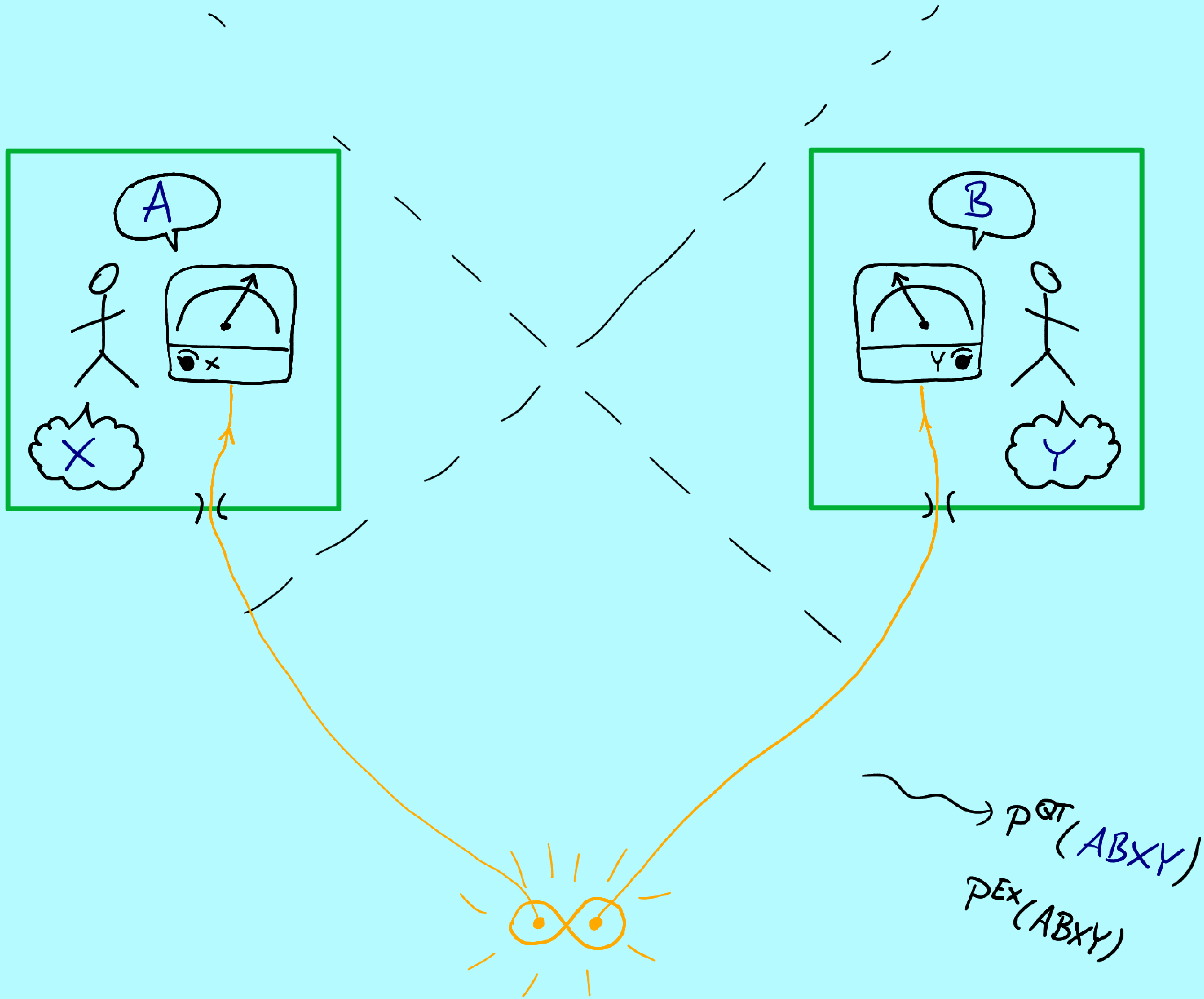


Bell's theorem, phrased in causal models:

If $P(ABXY)$ has a classical causal model with causal structure

$$\begin{tikzcd}
& A & & B & \\
X \arrow[ur] & & \Lambda \arrow[ul] \arrow[ur] & & Y \arrow[ul]
\end{tikzcd},$$

then $\sum_{x,y} \frac{1}{4} P(A \oplus B = XY \mid X=x, Y=y) \leq \frac{3}{4}$.


A
X
x
B
Y
Y
P^QT(ABXY)
P^Ex(ABXY)

Bell's theorem, phrased in causal models:

If $P(ABXY)$ has a classical causal model with causal structure

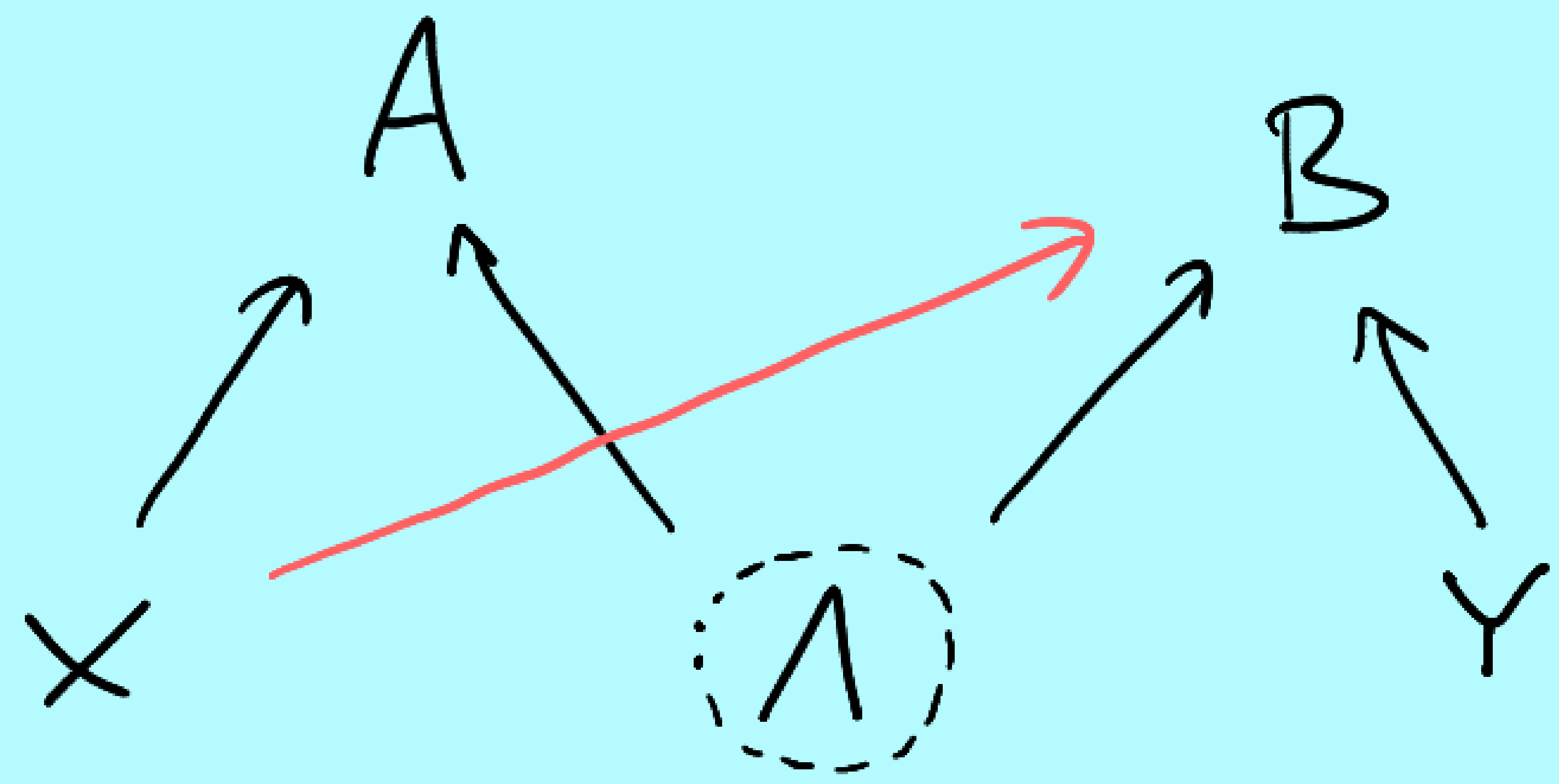
$$\begin{tikzcd}
 & A & & B & \\
X \arrow[ur] & & \Lambda \arrow[ul] \arrow[ur] & & Y \arrow[ul]
\end{tikzcd},$$

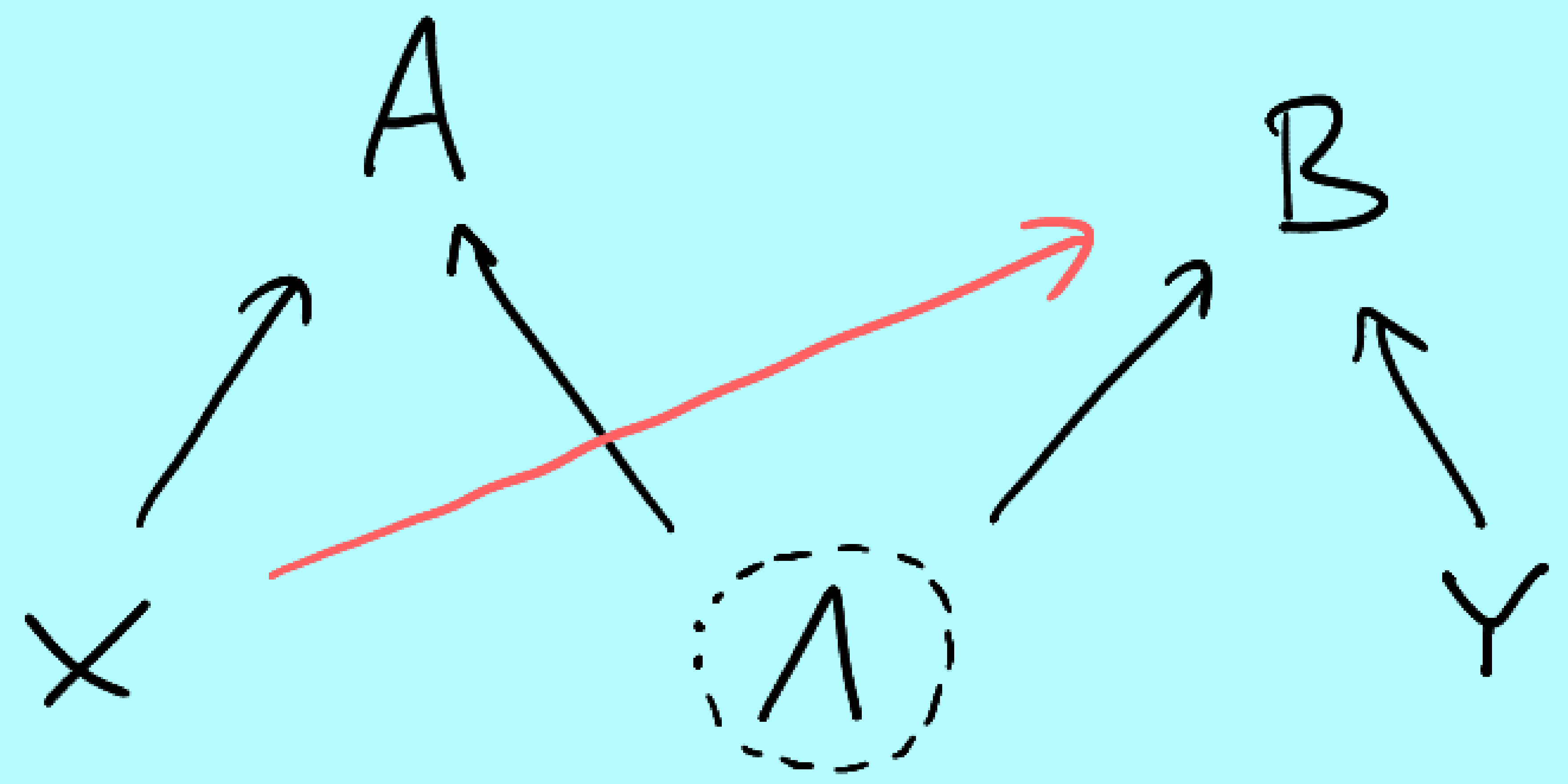
then $$\sum_{x,y} \frac{1}{4} P(A \oplus B = XY \mid X=x, Y=y) \leq \frac{3}{4}.$$

However, $$\sum_{x,y} \frac{1}{4} P^{QT}(A \oplus B = XY \mid X=x, Y=y) > \frac{3}{4}.$$

$$\sum_{x,y} \frac{1}{4} P^{EX}(A \oplus B = XY \mid X=x, Y=y) > \frac{3}{4}.$$

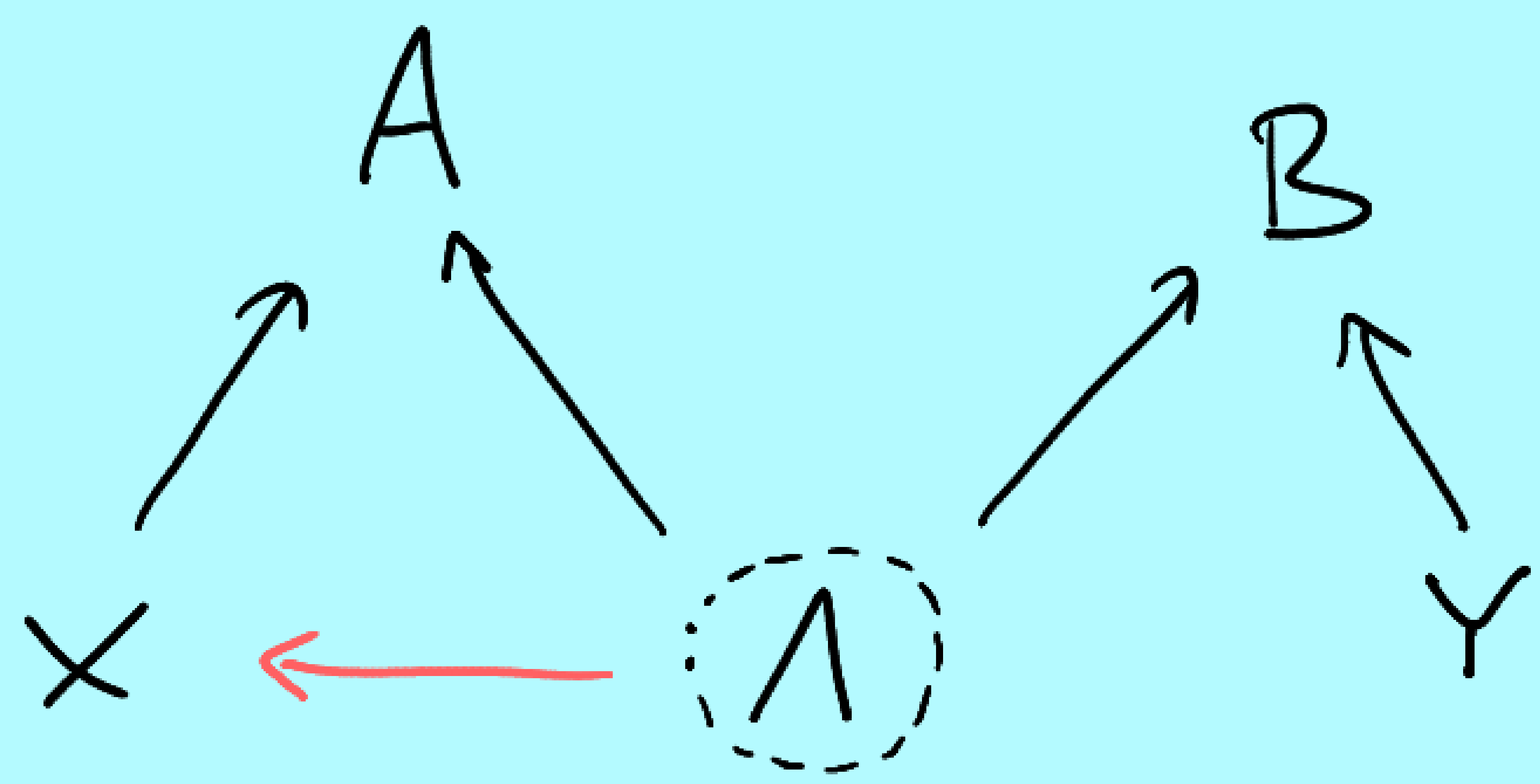
Rejecting the causal structure


A
B
X
Λ
Y

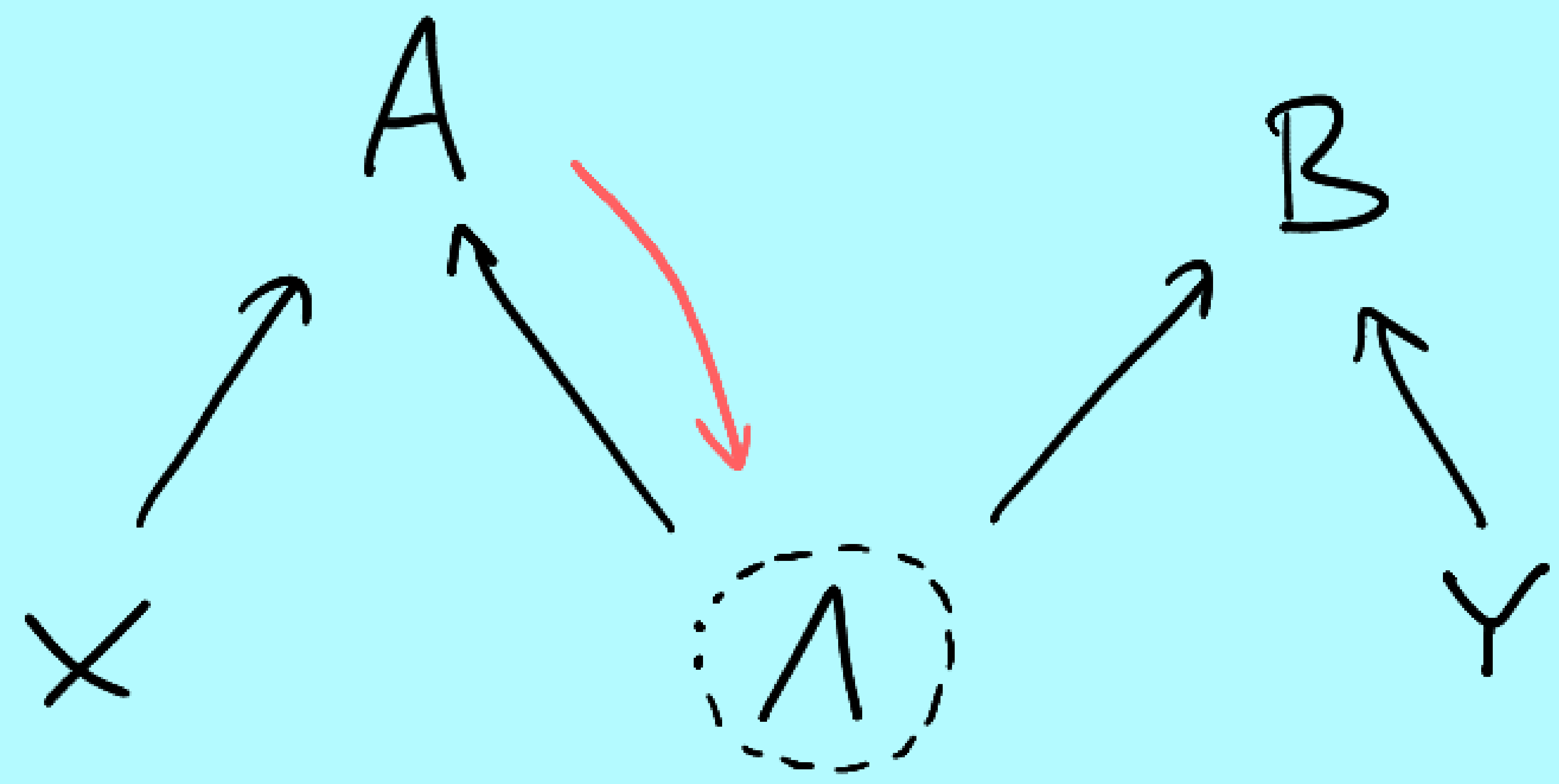


faster-than-light influence

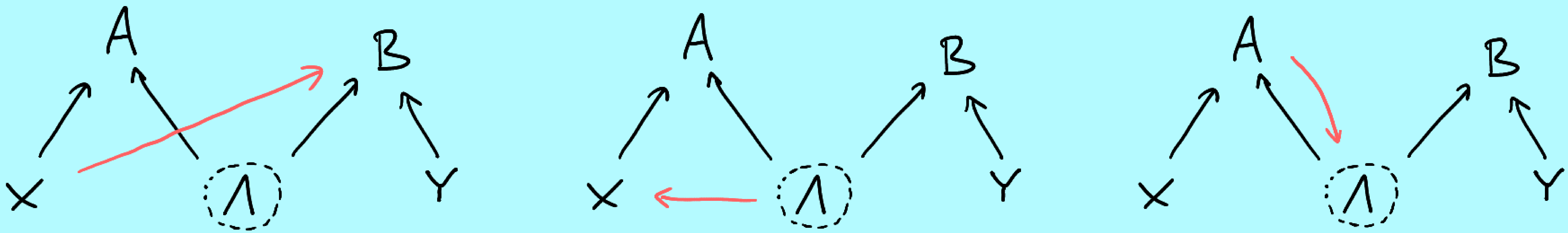
e.g. pilot wave theory

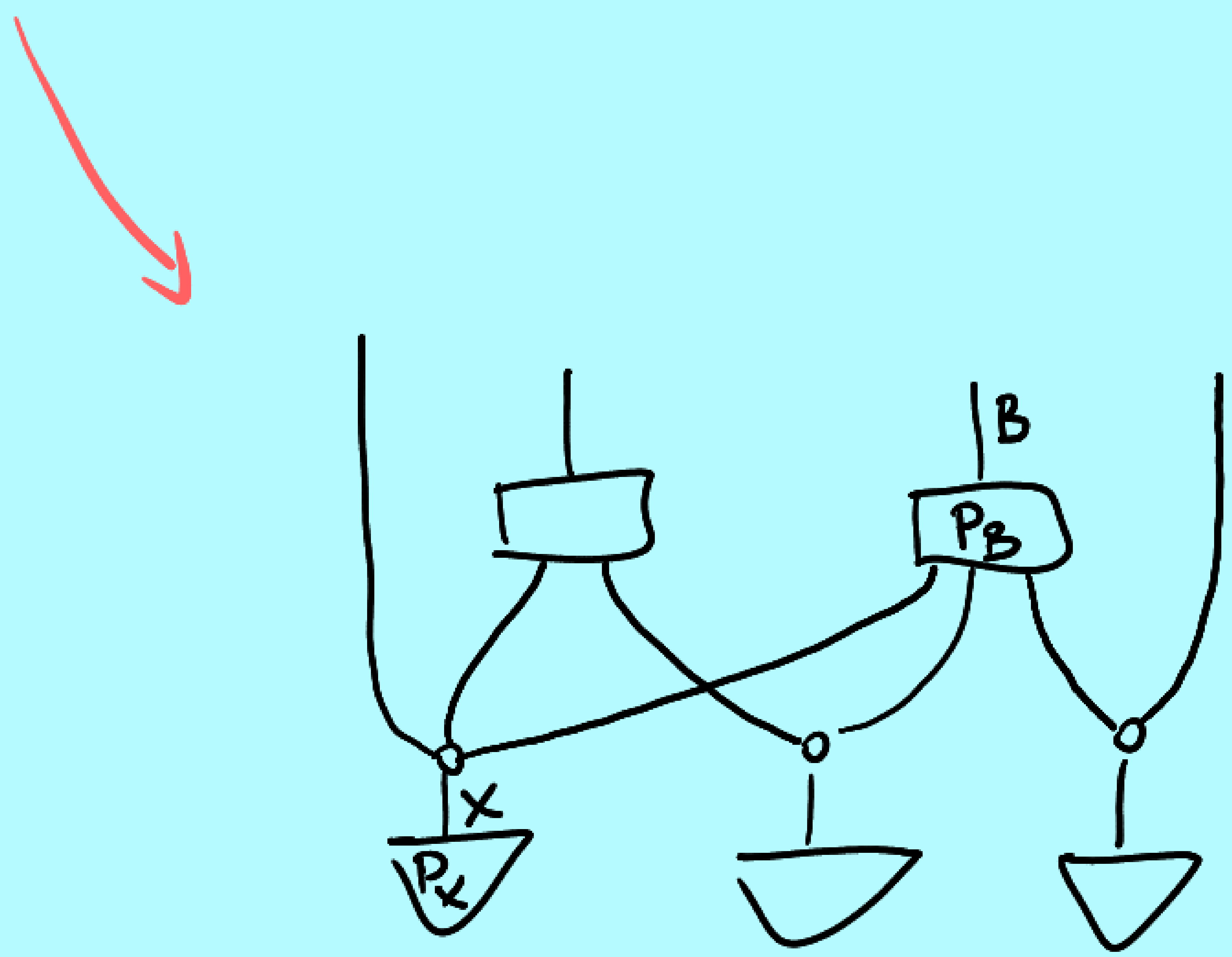


superdeterminism



retrocausation





$(X \perp\!\!\!\perp B)_P$ $\longrightarrow$ fine-tuning

Interlude: d-separation

$\boxed{\text{Def}}$ In a DAG G with vertices $V = \{X_1, \ldots, X_n\}$, and disjoint $S, T, U \subseteq V$,

S is <u>d-separated</u> from T by U, notated $(S \perp\!\!\!\perp T \mid U)_G$, iff...

$$X \to Y \to Z \qquad \begin{tikzcd} X & & Z \\ & Y \arrow[ul] \arrow[ur] & \end{tikzcd} \qquad \begin{tikzcd} & Y \arrow[dl] \arrow[dr] & \\ X & & Z \end{tikzcd} \qquad \begin{tikzcd} & Y \arrow[dl] \arrow[dr] & \\ X & & Z \end{tikzcd}$$

$$(X \perp\!\!\!\perp Z \mid Y)_G \qquad (X \perp\!\!\!\perp Z \mid Y)_G \qquad (X \perp\!\!\!\perp Z)_G \qquad \neg(X \perp\!\!\!\perp Z \mid Y)_G$$

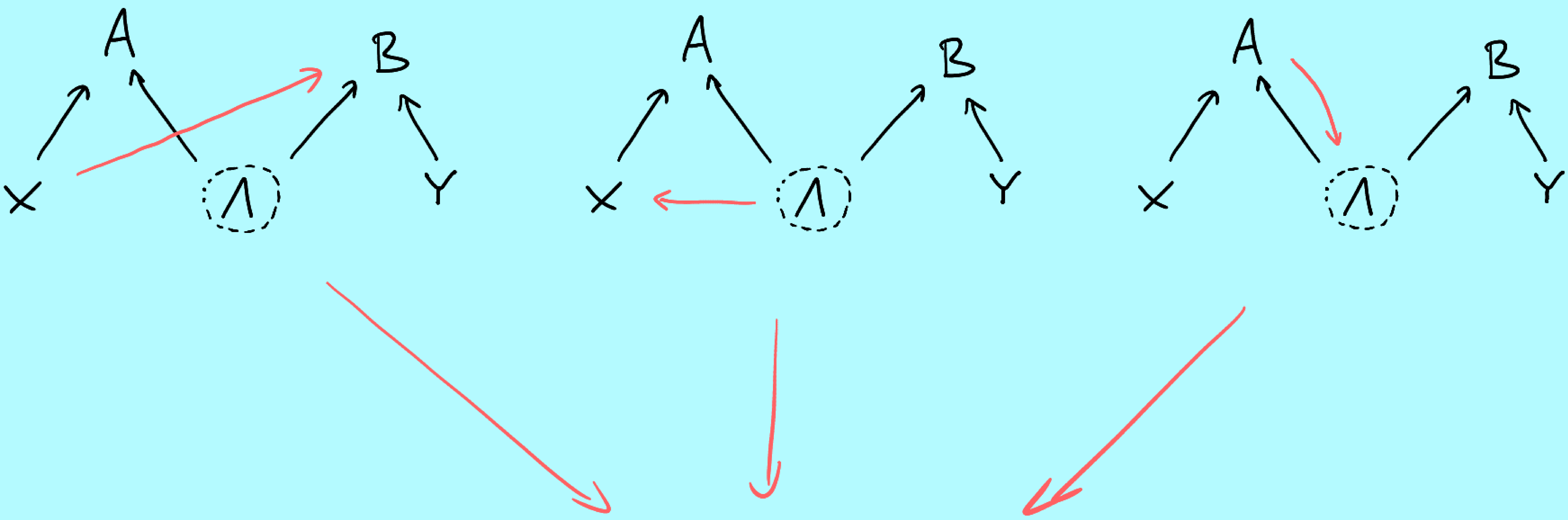
$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$(X \perp\!\!\!\perp Z \mid Y)_P \qquad (X \perp\!\!\!\perp Z \mid Y)_P \qquad (X \perp\!\!\!\perp Z)_P$$

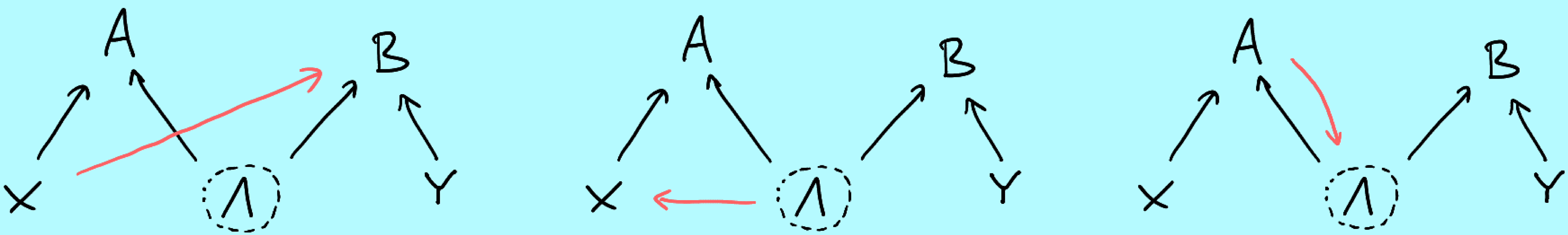
$\boxed{\text{Thm}}$ (i) d-separation is <u>sound</u> for statistical independence: if P has a model with structure G then $(S \perp\!\!\!\perp T \mid U)_G \implies (S \perp\!\!\!\perp T \mid U)_P$.

(ii) d-separation is also <u>complete</u> for statistical independence:

$$\neg(S \perp\!\!\!\perp T \mid U)_G \implies \neg(S \perp\!\!\!\perp T \mid U)_P \text{ almost always}$$

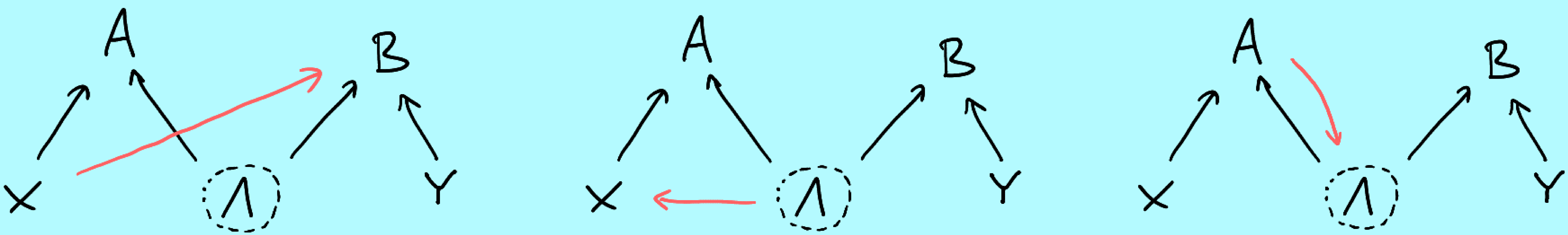


$(X \perp\!\!\!\perp B)_p$ requires fine-tuning in these models



These models

- explain the correlation between A and B through variation of an unobserved common cause
- do not explain the absence of correlation between X and B



These models

- <u>explain</u> the correlation between A and B through variation of an unobserved common cause
- <u>do not explain</u> the absence of correlation between X and B

and they can only be reconciled with that absence through <u>fine-tuning</u>.

Bell's theorem

If $P(ABXY)$ has a <u>classical causal model</u> with <u>causal structure</u>

$$\begin{tikzcd}
 & A & & B & \\
X \arrow[ur] & & \Lambda \arrow[ul] \arrow[ur] & & Y \arrow[ul]
\end{tikzcd},$$

then $\displaystyle \sum_{x,y} \frac{1}{4} P(A \oplus B = XY \mid X=x, Y=y) \leq \frac{3}{4}.$

Bell's theorem

If $P(ABXY)$ has a classical causal model with no fine-tuning

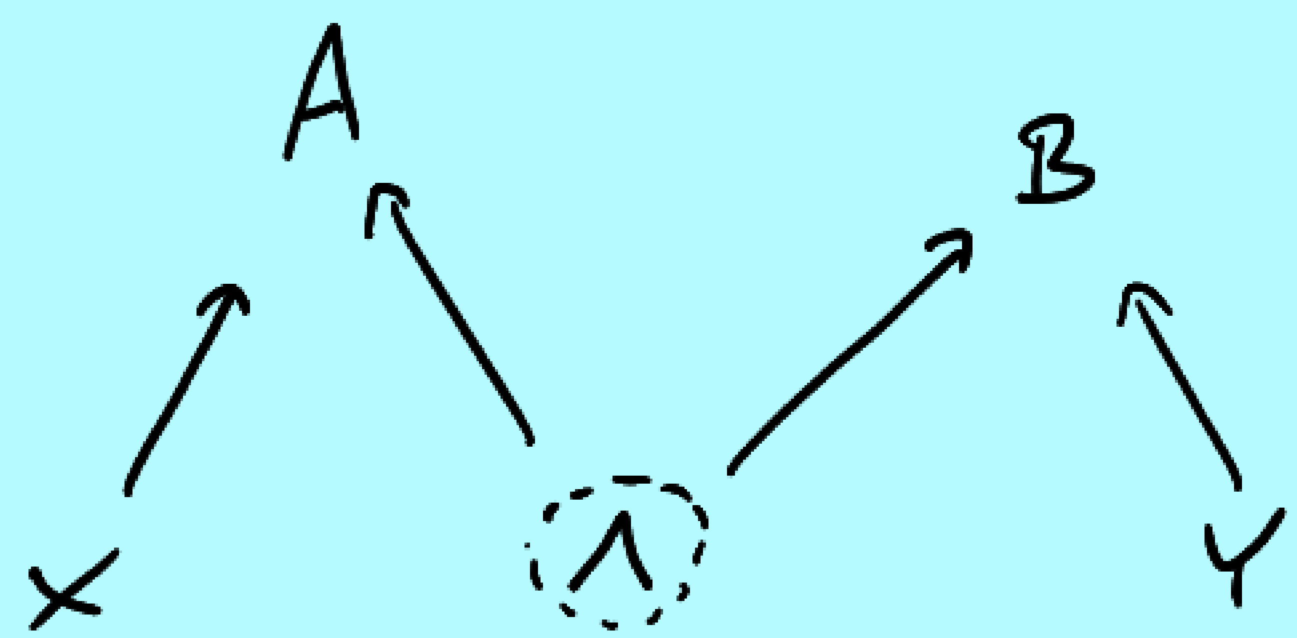
and if $(X \perp\!\!\!\perp B | Y)_P$, $(Y \perp\!\!\!\perp A | X)_P$, and $(X \perp\!\!\!\perp Y)_P$,

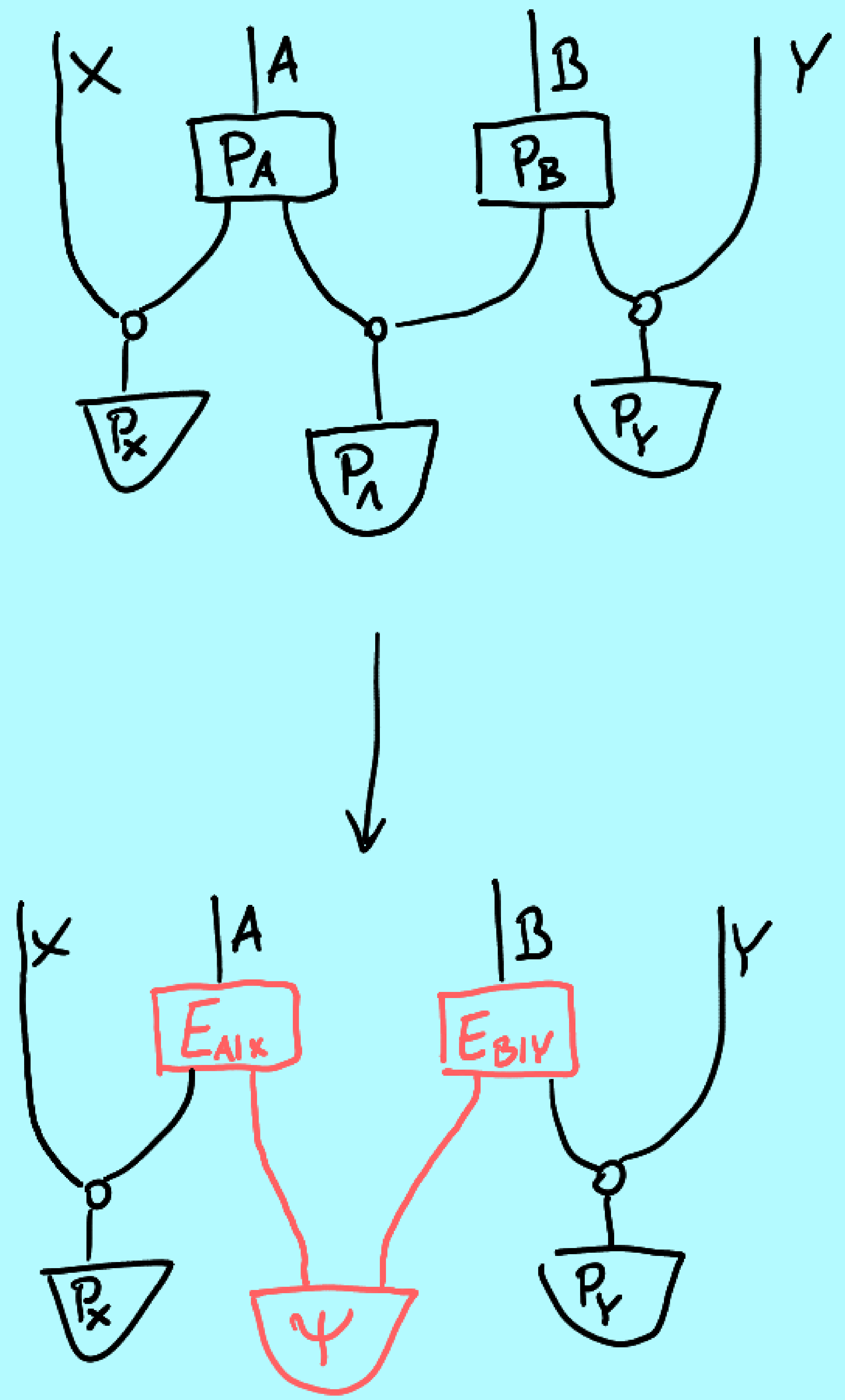
then $$\sum_{x,y} \frac{1}{4} P(A \oplus B = XY \mid X=x, Y=y) \leq \frac{3}{4}.$$

Rejecting classical causal models

Quantum causal models

+





Henson, Lal, Pusey (2014);

Fritz (2014)

Fundamental determinism

+

local noise

↓

Fundamental unitarity

+

local noise

Allen et al. (2017);

Barrett, Lorenz, Oreshkov (2019)

Suggested reading

Causal modelling:

Judea Pearl, "Book of Why" / "Causality: models, reasoning, inference"

Jacobs, Kissinger, Zanasi: "Causal Inference by String Diagram Surgery"

... and Bell's theorem:

Wood & Spekkens (NJP 2015) "Lessons of causal discovery (...) for quantum correlations"

Wiseman & Cavalcanti (2017) "Causarum Investigatio and the two Bell's theorems"

Quantum causal models:

Allen et al. (2017) "Quantum Common Causes and Quantum Causal Models"

Henson, Lal, Pusey (2014) "Theory-independent limits on correlations from generalised Bayesian networks"