An introduction to quantum nonlocality
Tutorial: Quantum foundations

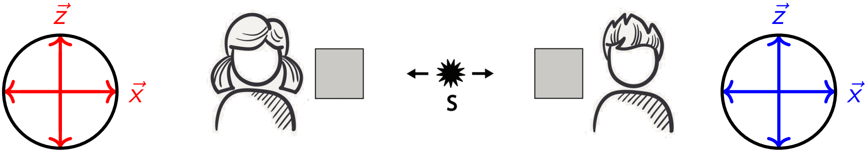
Martin J. Renner, University of Vienna

Paris, 4th November 2024, WAQ

Content of the talk

- Bell's theorem
- Toner-Bacon model
- Random Access Coding
- Hardy's paradox
- Elitzur-Vaidman bomb tester
- Werner's model

EPR argument



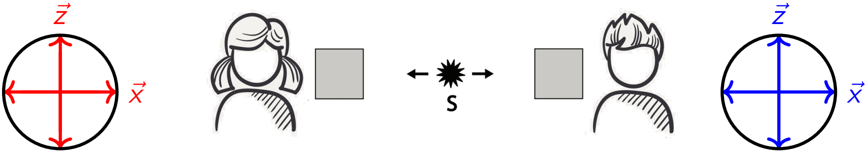
$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

- Uncertainty: Cannot measure $\vec{z}$ and $\vec{x}$ simultaneously
- Anticorrelation when Alice and Bob measure the same direction

"While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, 777 (1935)

EPR argument



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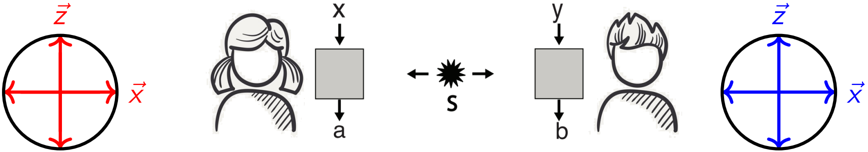
Bell



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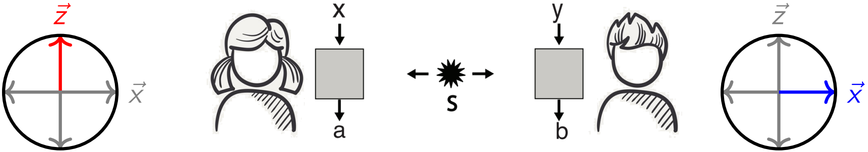
Hidden variable model:

	Alice's measurement	Bob's measurement
λ_1	(+z; +x)	(-z; -x)
λ_2	(+z; -x)	(-z; +x)
λ_3	(-z; +x)	(+z; -x)
λ_4	(-z; -x)	(+z; +x)

$$p(a,b|x,y) = \sum_{i=1}^{4} \frac{1}{4}\, p_A(a|x,\lambda_i) \cdot p_B(b|y,\lambda_i)$$

Bell



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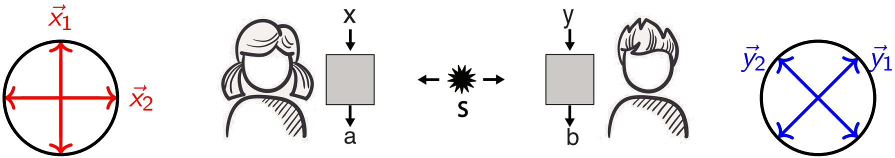
Bell's theorem



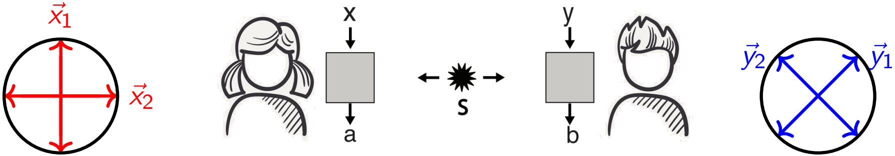
$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Hidden variable model: ???

	Alice's measurement	Bob's measurement
λ_1	(+z; +x)	($-y_1$; $-y_2$)
λ_2	(+z; -x)	($-y_1$; $+y_2$)
λ_3	(-z; +x)	($+y_1$; $-y_2$)
λ_4	(-z; -x)	($+y_1$; $+y_2$)

$$p(a,b|x,y) \neq \sum_i p(\lambda_i)\ p_A(a|x,\lambda_i) \cdot p_B(b|y,\lambda_i)$$

Bell's theorem



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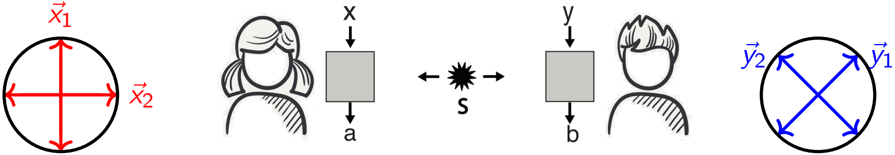
CHSH inequality as a game:

	$+\vec{y}_1$	$-\vec{y}_1$	$+\vec{y}_2$	$-\vec{y}_2$
$+\vec{x}_1$	0.07	0.43	0.07	0.43
$-\vec{x}_1$	0.43	0.07	0.43	0.07
$+\vec{x}_2$	0.07	0.43	0.43	0.07
$-\vec{x}_2$	0.43	0.07	0.07	0.43

Bell's theorem



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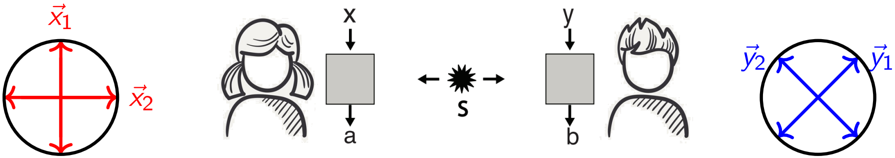
Winning probability: 85%

Bell's theorem



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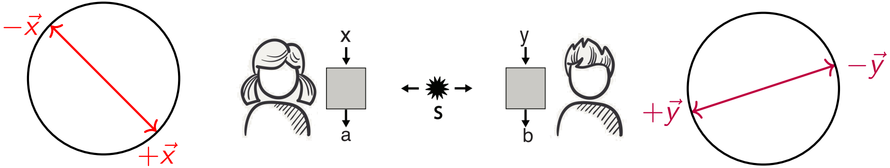
CHSH inequality as a game:

with predetermined strategy: $\lambda_A = (+,+)$, $\lambda_B = (-,-)$

	$+\vec{y}_1$	$-\vec{y}_1$	$+\vec{y}_2$	$-\vec{y}_2$
$+\vec{x}_1$	0	1	0	1
$-\vec{x}_1$	0	0	0	0
$+\vec{x}_2$	0	1	0	1
$-\vec{x}_2$	0	0	0	0

Winning probability: 75%

Quantifying Bell nonlocality

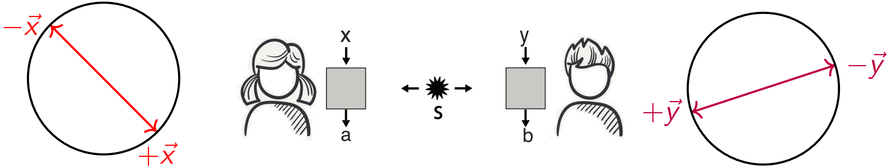


$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

- Correlations: $p(a = \pm 1, b = \pm 1|\vec{x}, \vec{y}) = (1 - (ab)\ \vec{x} \cdot \vec{y})/4$

Bell's theorem:
Correlations cannot be reproduced with local hidden variables!

Quantifying Bell nonlocality



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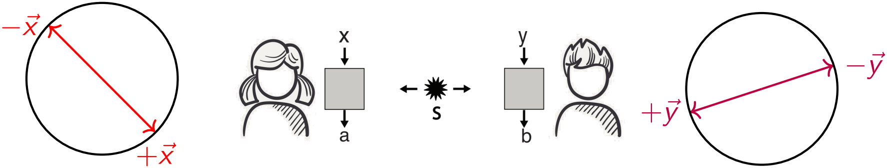

Correlations: $p(a, b|\vec{x}, \vec{y}) = (1 - (ab)\ \vec{x} \cdot \vec{y})/4$

Question:

Can we simulate these correlations with local hidden variables plus some classical communication?

- Maudlin (1992): LHV + infinite communication
- Brassard, Cleve, Tapp (1999): LHV + 8 bits
- Toner and Bacon (2003): LHV + 1 bit

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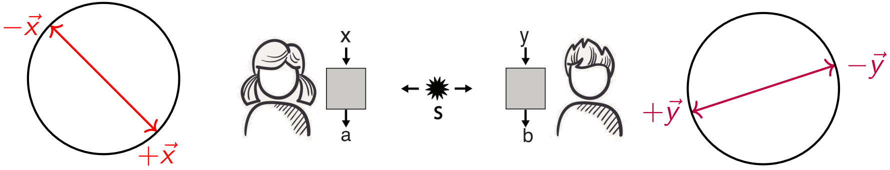

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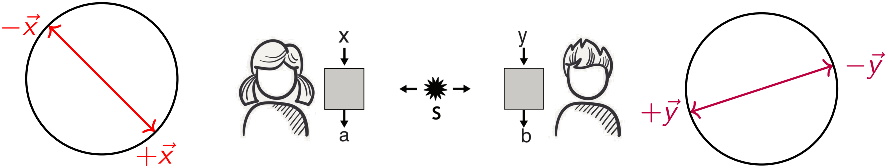

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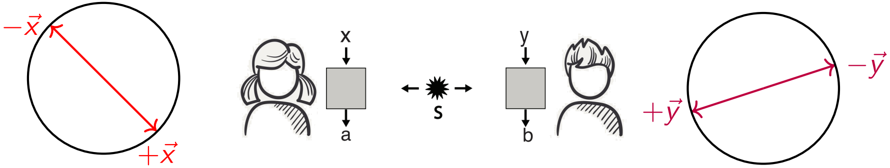

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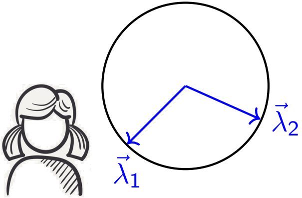
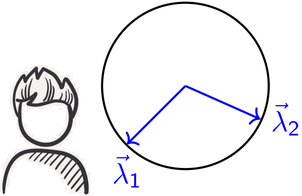
Toner-Bacon model

0. Alice and Bob share two random vectors $\vec{\lambda}_1, \vec{\lambda}_2 \in S_2$





Alice:

1. choose measurement $\vec{x}$
2. select $\vec{\lambda}_i$ closest to $\pm\vec{x}$
3. output $a = -sgn(\vec{x} \cdot \vec{\lambda}_i)$

Bob:

4. choose measurement $\vec{y}$
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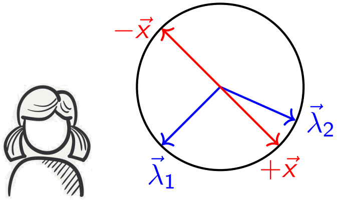

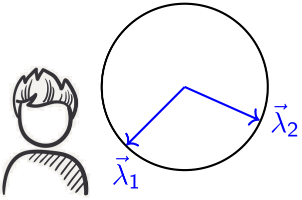

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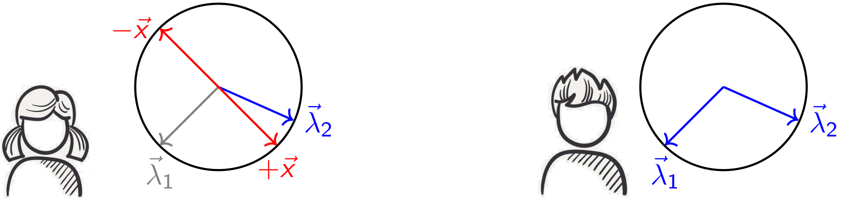

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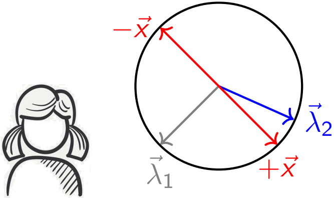

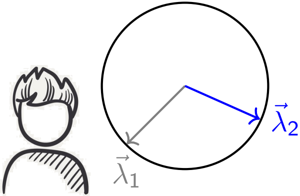

Output: "$a = -1$"

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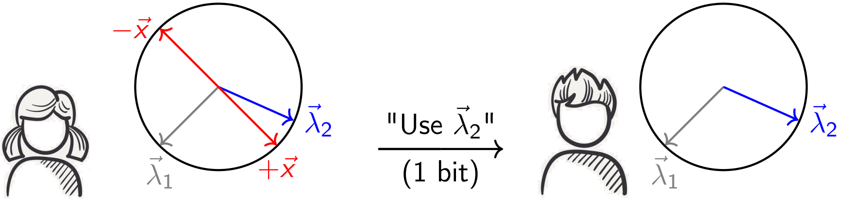

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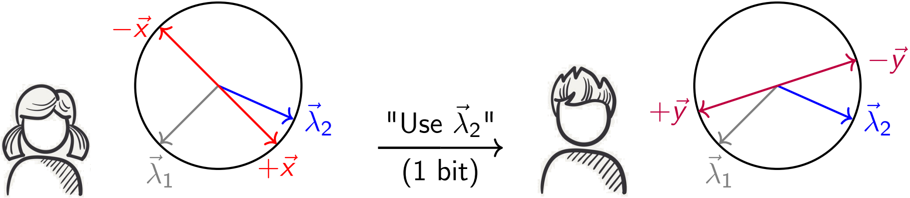

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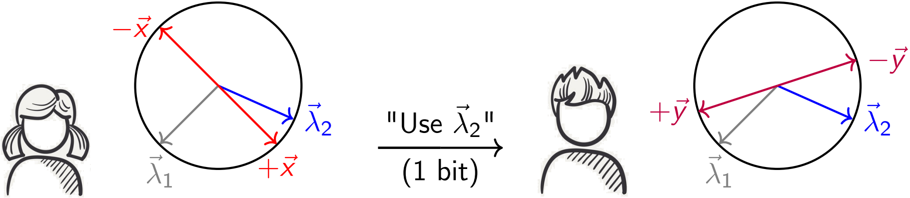

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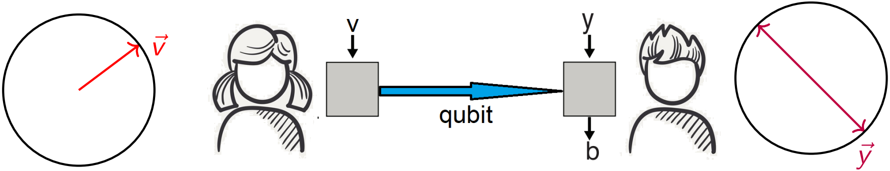
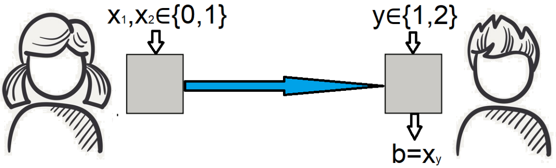
Qubit Prepare-and-Measure scenario



Task: Random Access Coding



Winning probabilities for different resources:

- 1 Bit: 75 %
- 2 Bits: 100 %
- 1 Qubit: 85 %

Quantum advantage: 1 Bit < 1 Qubit

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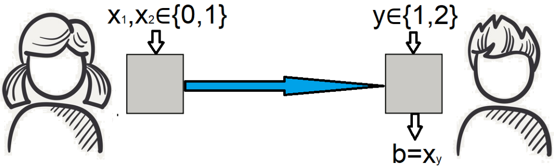

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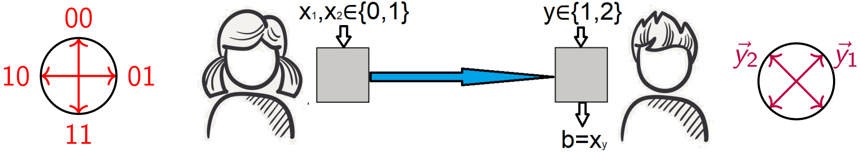


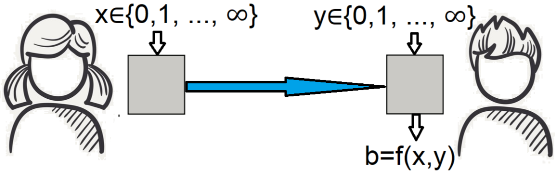
Qubit strategy:

- Alice prepares one of four states (according to her input)
- Bob measures either in $\vec{y}_1$ or $\vec{y}_2$ basis, depending on which bit he wants to read out

Winning probability: 85 %

S. Wiesner, Conjugate Coding, SIGACT News. 15 (1): 78–88 (1983)

What about a general task?

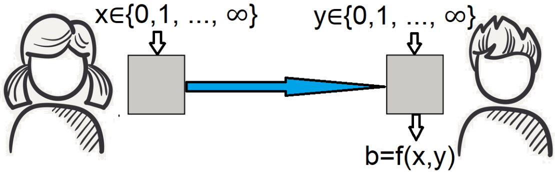


Winning probabilities for different resources?

Main message: 1 Bit $<$ 1 Qubit $\leq$ 2 Bits

Why? Two bits can simulate all qubit correlations!

What about a general task?



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Toner-Bacon model

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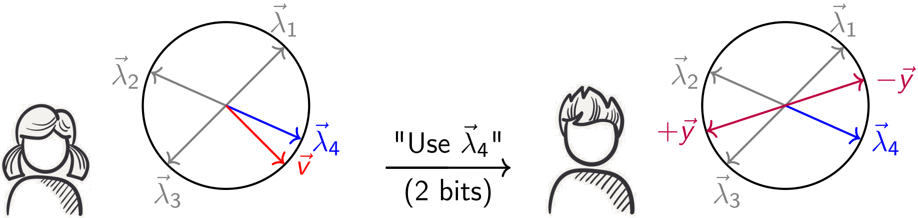

Output: "$b = -1$"

Alice:
1. select state $\vec{v}$
2. select $\vec{\lambda}_i$ closest to $\vec{v}$
3. send i

Bob:

4. select measurement $\vec{y}$
5. output $b = sgn(\vec{y} \cdot \vec{\lambda}_i)$

$p(b = \pm 1) = (1 \pm \vec{v} \cdot \vec{y})/2$

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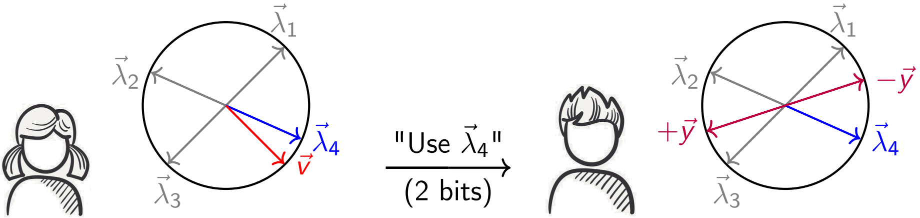

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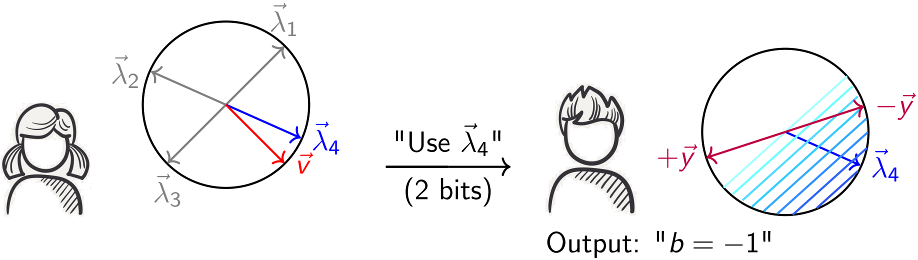
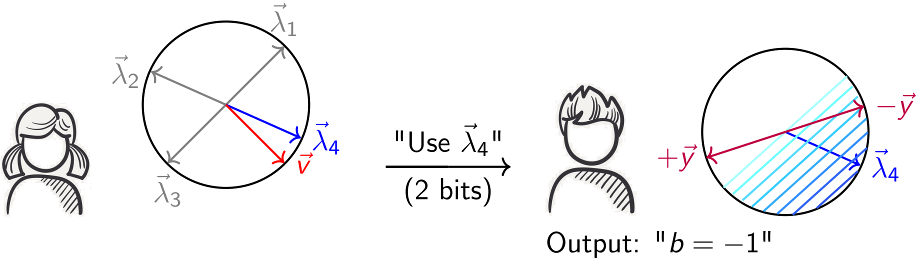


$$p(b = \pm 1 | \vec{v}, \vec{y}) = (1 \pm \vec{v} \cdot \vec{y})/2$$

Idea: Selected $\vec{\lambda}$ forms a LHV model of the qubit state $\vec{v}$

B. F. Toner and D. Bacon, PRL 91, 187904 (2003)
M. J. Renner, A. Tavakoli, and M. T. Quintino, PRL 130, 120801 (2023)

Toner-Bacon model



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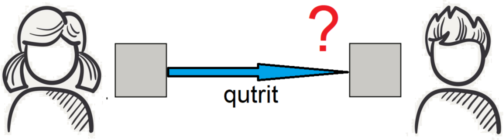
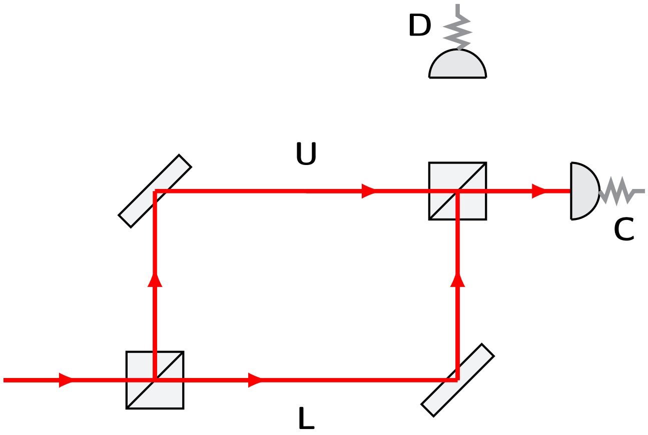
Interesting open problems



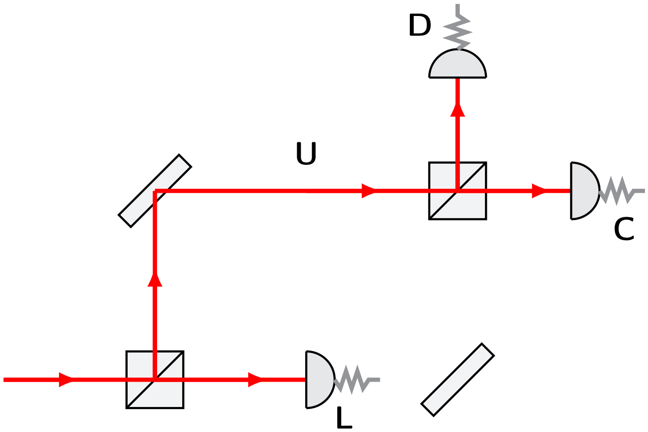
- One bit for every entangled qubit pair?
- Simulating higher dimensional systems
- Simulating multipartite systems, sequential scenarios, entanglement assisted scenarios, ...

Hardy's paradox



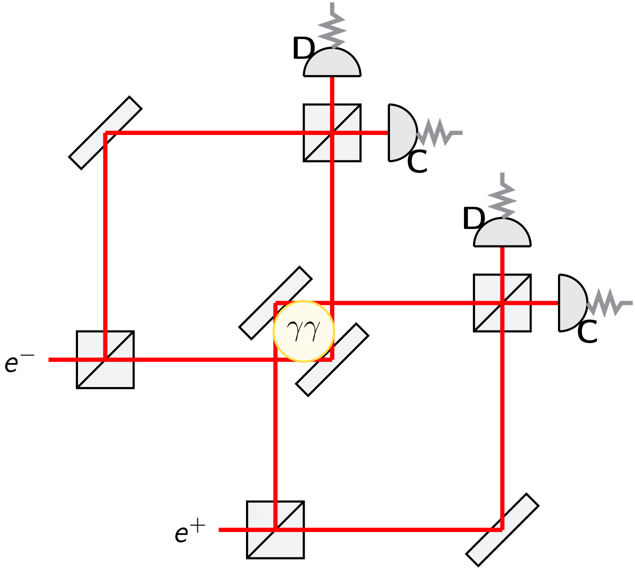
$$p(C) = 1 \quad p(D) = 0$$

Hardy's paradox



$p(L) = 1/2 \quad p(C) = 1/4 \quad p(D) = 1/4$

Hardy's paradox



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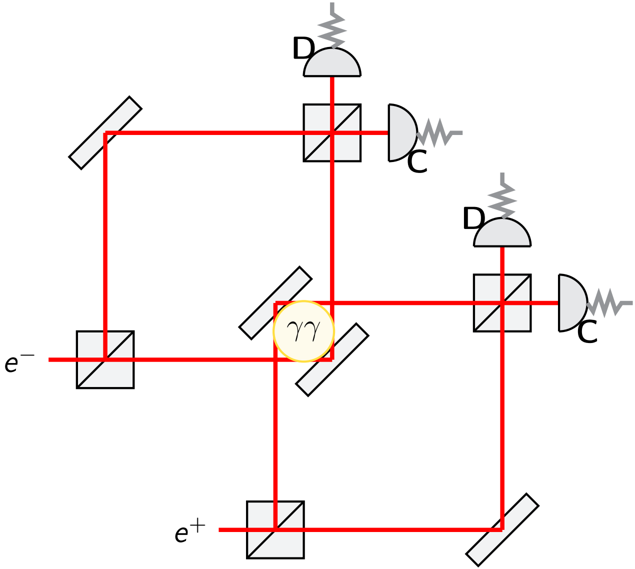

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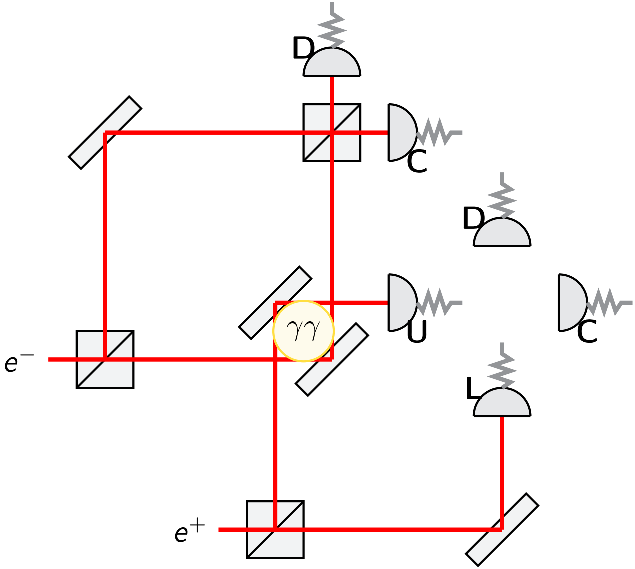

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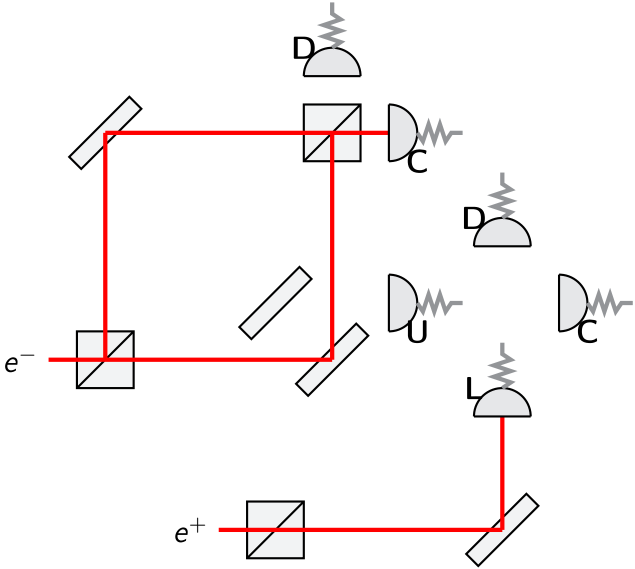

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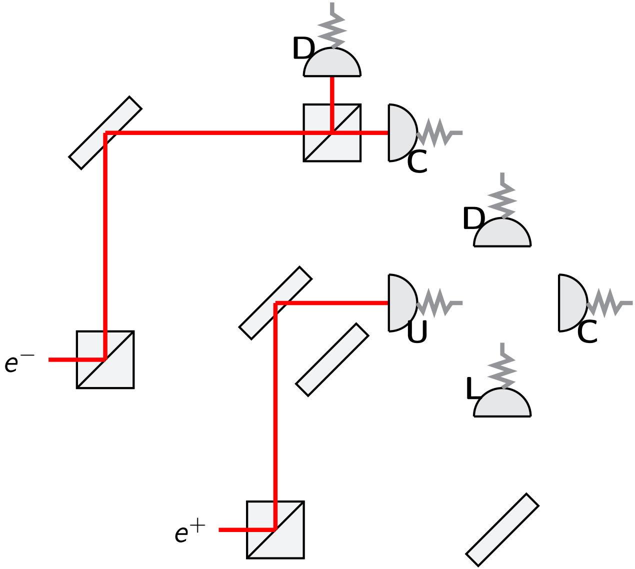

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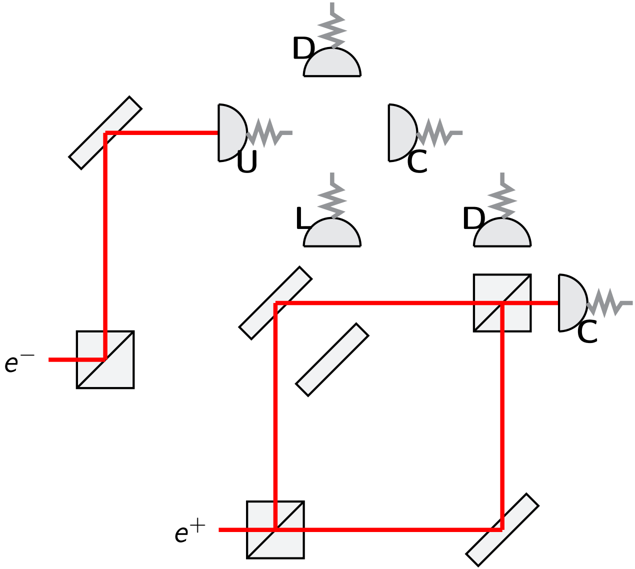

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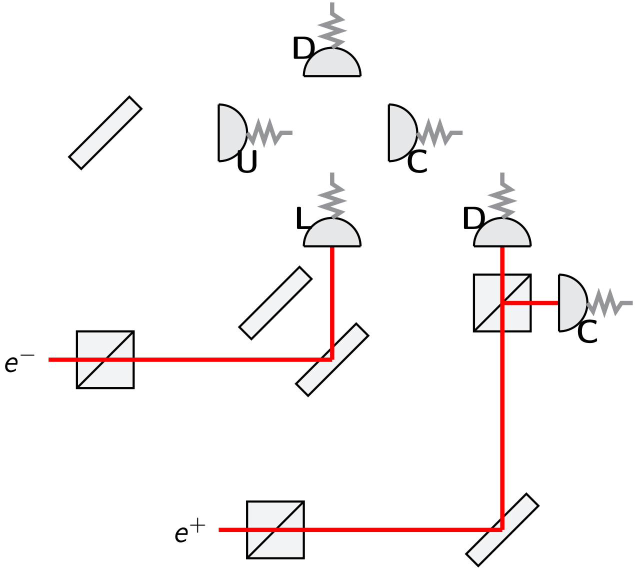

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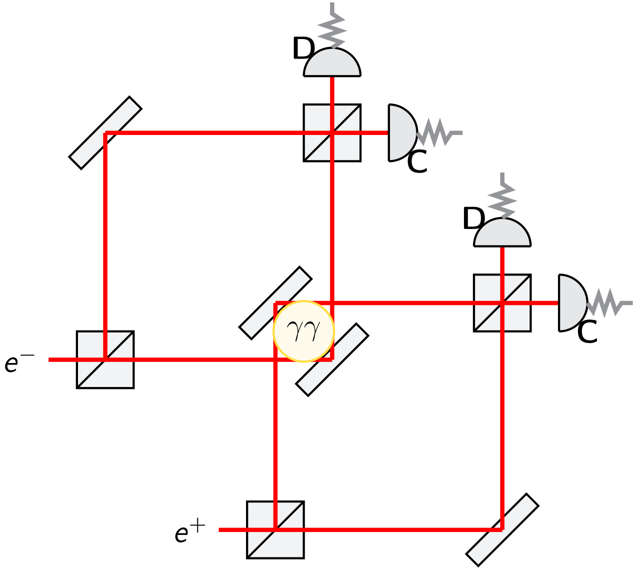
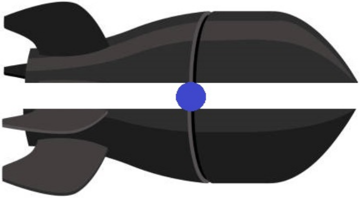


Hardy's paradox

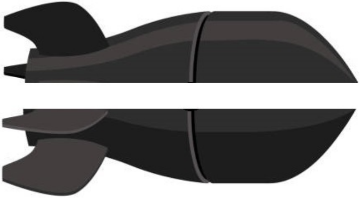
	U	L	C	D
U	1/3	1/3	2/3	0
L	0	1/3	1/6	1/6
C	1/6	2/3	9/12	1/12
D	1/6	0	1/12	1/12

Winning probability: 79% > 75 %

Elitzur-Vaidman bomb tester



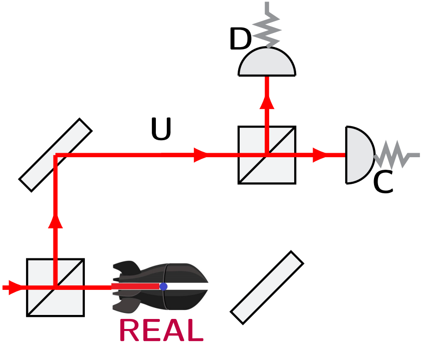
REAL



FAKE

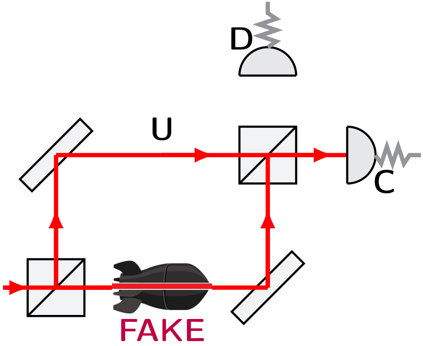
- A single photon can trigger the bomb
- You don't know if the bomb is real or fake

Elitzur-Vaidman bomb tester



$p(L) = 1/2$
(Bomb explodes)

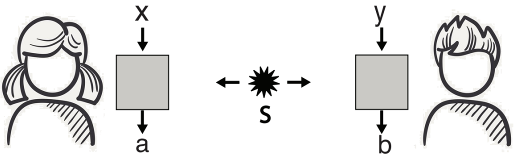
$p(C) = 1/4$
$p(D) = 1/4$



$p(C) = 1$
$p(D) = 0$

If "D" clicks: Bomb is real but did not explode!

Which states violate a Bell inequality?



Alice and Bob share a product state $\rho_{AB} = \rho_A \otimes \rho_B$

$$p(a, b|x, y) = tr[(A_{a|x} \otimes B_{b|y})\ \rho_{AB}] = tr[A_{a|x}\ \rho_A] \cdot tr[B_{b|y}\ \rho_B]$$

$\Longrightarrow$ **p(a,b|x,y)=p(a|x)**$\cdot p(b|y)$ **(local model)**

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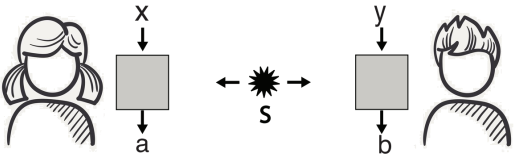

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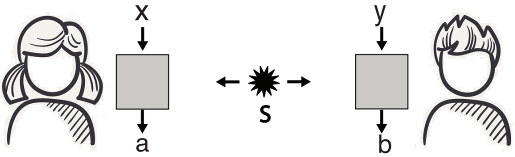

Alice and Bob share a separable state $\rho_{AB} = \sum_\lambda p(\lambda)\ \rho_A^\lambda \otimes \rho_B^\lambda$

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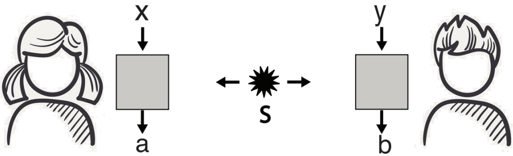

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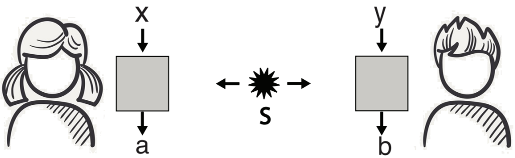


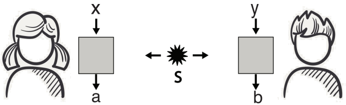
Alice and Bob need to share an entangled state $\rho_{AB} \neq \sum_\lambda p(\lambda)\ \rho_A^\lambda \otimes \rho_B^\lambda$

Do all entangled states violate a Bell inequality?

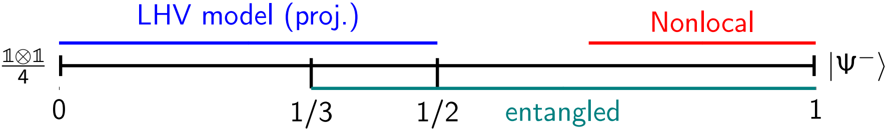
- pure states: yes (Gisin's theorem)
- mixed states: no (Werner's model)

N. Gisin, Bell's inequality holds for all non-product states, Physics Letters A 154, 201-202 (1991)
R. F. Werner, Quantum states with EPR correlations admitting a hidden-variable model, Phys. Rev. A 40, 4277–4281 (1989)

Non-locality of two-qubit Werner states



$$\rho_W^\eta = \eta \, |\Psi^-\rangle \langle\Psi^-| + (1-\eta)\, \mathbb{1}/4$$



- entangled for $\eta \geq 1/3$: $\rho_W^\eta \neq \sum_\lambda p(\lambda)\, \rho_A^\lambda \otimes \rho_B^\lambda$
- local for $\eta \leq 1/2$: $p(a,b|x,y) = \int \mathrm{d}\lambda\; p(\lambda) p_A(a|x,\lambda)\; p_B(b|y,\lambda)$

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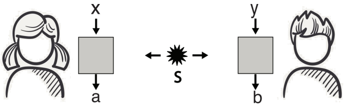

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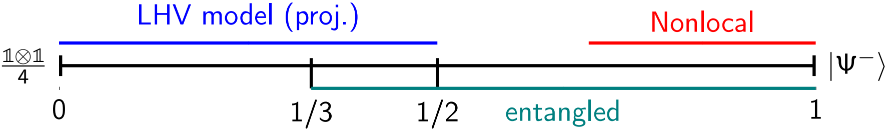

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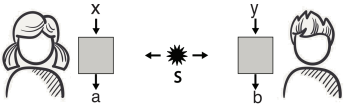

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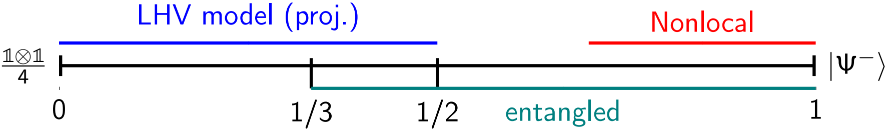

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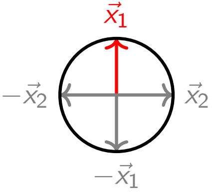
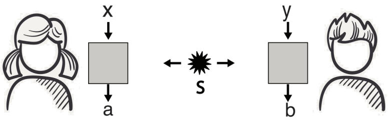
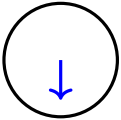
Example

$$\rho_W^\eta = \frac{1}{\sqrt{2}} \left|\Psi^-\right\rangle \left\langle\Psi^-\right| + \left(1 - \frac{1}{\sqrt{2}}\right) \frac{\mathbb{1}}{4}$$







Alice performs measurement $\Longrightarrow$ Bob's qubit is in a noisy state

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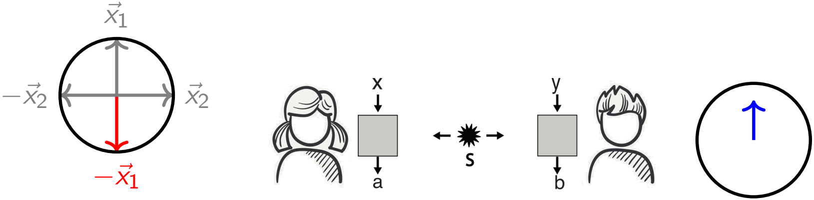

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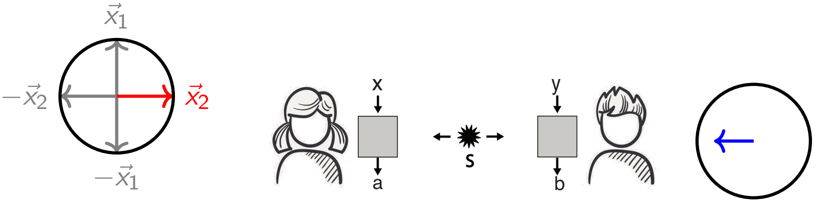

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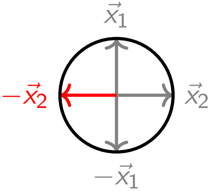

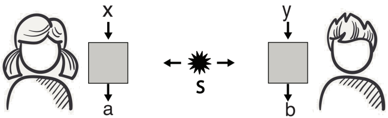

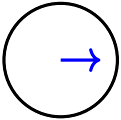

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Local Hidden State model

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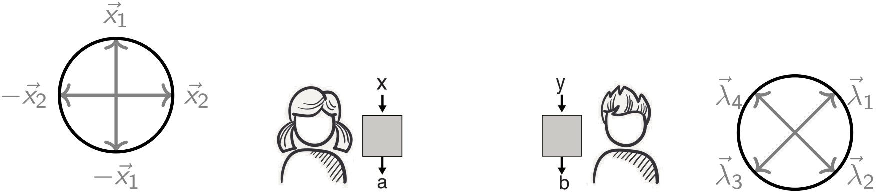

Local Hidden State model:

- Bob's qubit is in one of the four states $\vec{\lambda}_i$ (prob. 1/4 each)

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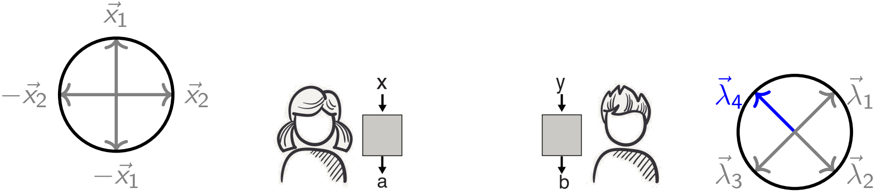

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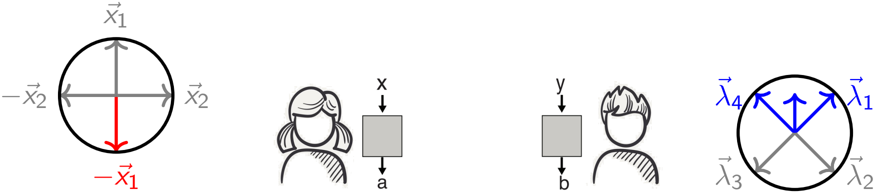

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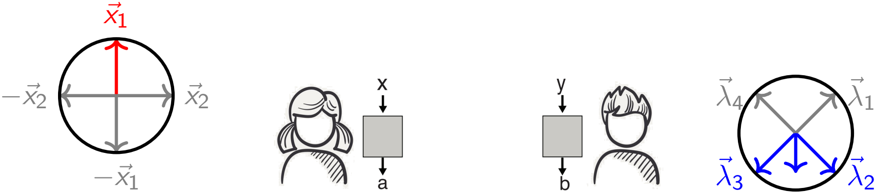

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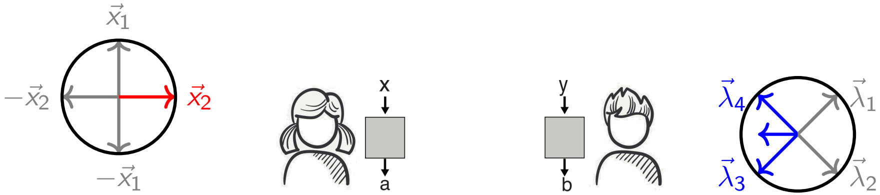

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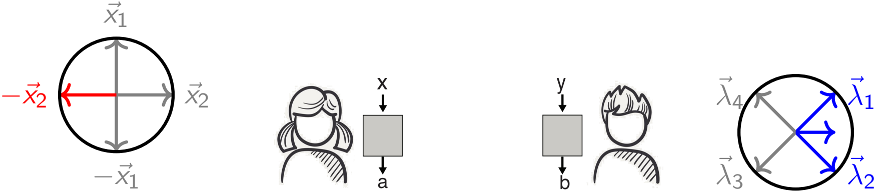

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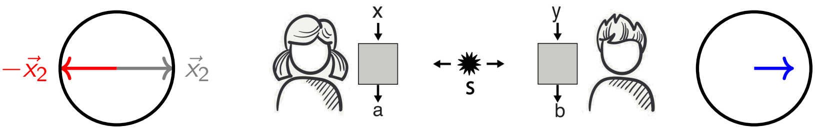
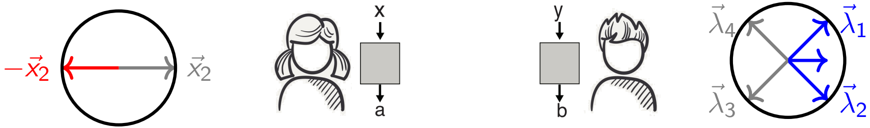
Measurements on real Werner state:



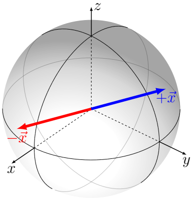
Local Hidden State model:

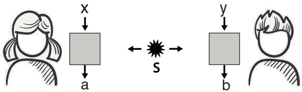


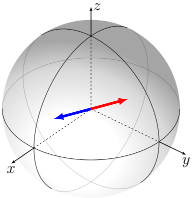
$$\Longrightarrow \; tr[(A_{a|x} \otimes B_{b|y}) \; \rho_W^{\eta}] = \sum_{i=1}^{4} \; \frac{1}{4} p_A(a|x, \vec{\lambda}_i) \; tr[B_{b|y} \; \rho_{\vec{\lambda}_i}]$$

Werner's model

$$\rho_W^{1/2} = \frac{1}{2}\left|\Psi^-\right\rangle\left\langle\Psi^-\right| + \frac{1}{2}\ \frac{\mathbb{1}}{4}$$





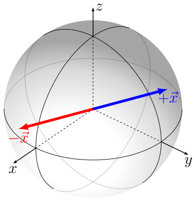


$$A_{\pm|x} = \frac{1}{2}(\mathbb{1} \pm \vec{x}\cdot\vec{\sigma})$$

$$\rho_{\pm|x} = \frac{1}{2}(\mathbb{1} \mp \frac{1}{2}\vec{x}\cdot\vec{\sigma})$$

$$p(a,b|x,y) = tr[(A_{a|x}\otimes B_{b|y})\rho_W^{1/2}] = \int_{S_2} \mathrm{d}\vec{\lambda}\ p_A(a|x,\vec{\lambda})\ tr[B_{b|y}\ \rho_{\vec{\lambda}}]$$

Werner's model



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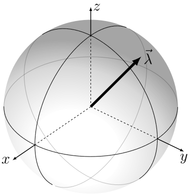

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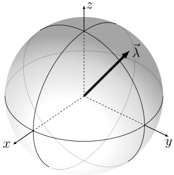
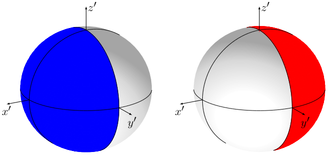
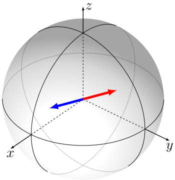
1. Hidden state $\vec{\lambda}$



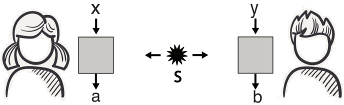
2. $p_A(\pm|\vec{x},\vec{\lambda}) = -sign(\vec{x}\cdot\vec{\lambda})$



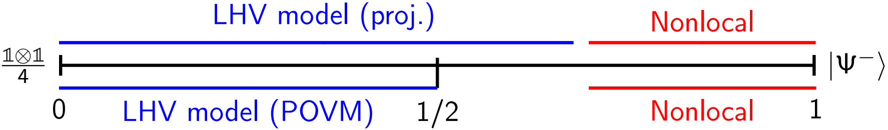
3. Average



Non-locality of two-qubit Werner states



$$\rho_W^\eta = \eta \left|\Psi^-\right\rangle \left\langle\Psi^-\right| + (1-\eta)\, \mathbb{1}/4$$



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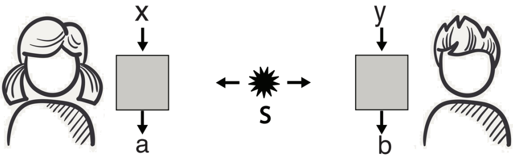
Open question: Role of POVMs in nonlocality



Does there exists a state ρ_{AB} such that:

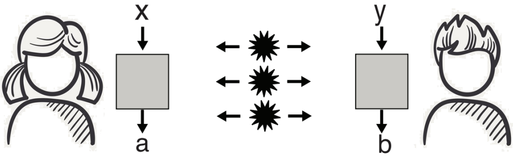
Local for all projective measurements ($A_{a|x} = A^2_{a|x}$, $B_{b|y} = B^2_{b|y}$):

$$p(a, b|x, y) = tr[(A_{a|x} \otimes B_{b|y})\rho_{AB}] = \int \mathrm{d}\lambda \; p_A(a|x, \lambda) \; p_B(b|y, \lambda)$$

Non-local for general POVMs:

$$p(a, b|x, y) = tr[(A_{a|x} \otimes B_{b|y})\rho_{AB}] \neq \int \mathrm{d}\lambda \; p_A(a|x, \lambda) \; p_B(b|y, \lambda)$$

Super-activation of quantum nonlocality

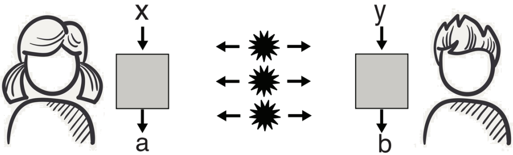


- Single copy: Local model for all measurements
- Joint measurements on several qubits $\Longrightarrow$ Bell violation

Can all entangled states demonstrate Bell nonlocality?

C. Palazuelos, Phys. Rev. Lett. 109, 190401 (2012)
D. Cavalcanti, A. Acin, N. Brunner, T. Vertesi, Phys. Rev. A 87, 042104 (2013)

Super-activation of quantum nonlocality



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