How much secure randomness is in a quantum state?

Young Quantum Information Scientists 2024

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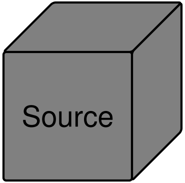
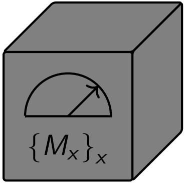
Problem setup





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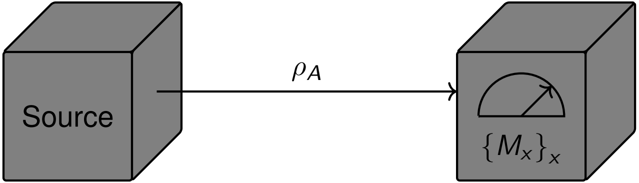

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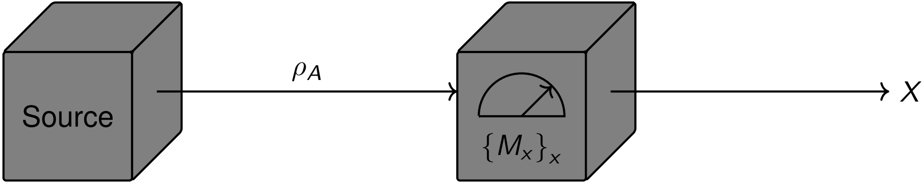

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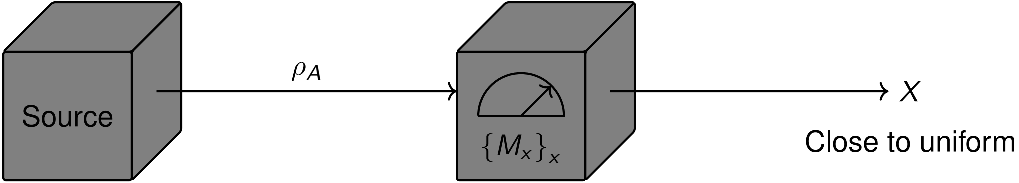

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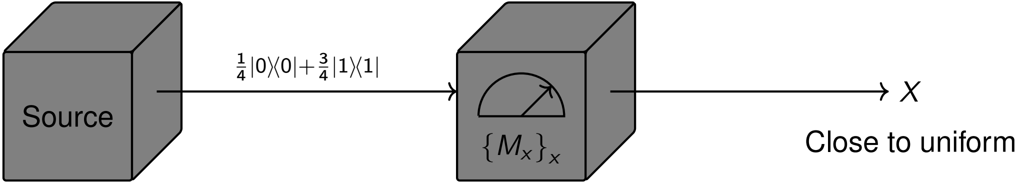

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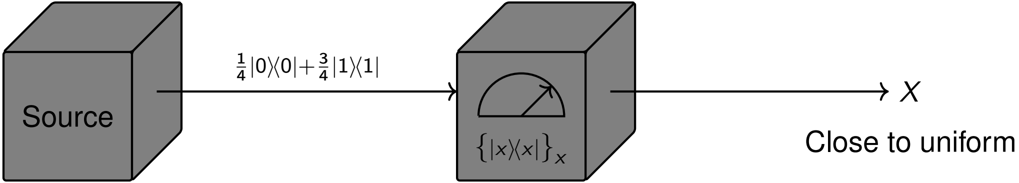

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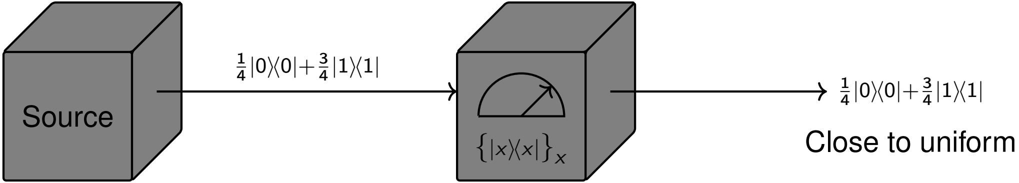

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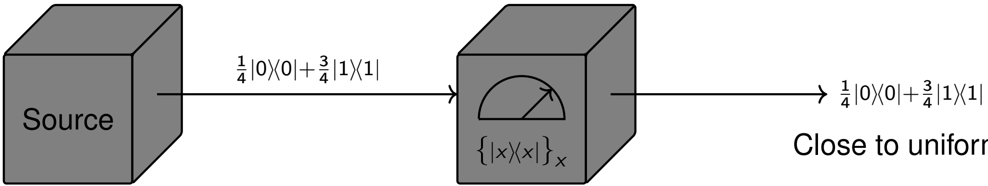

Close to uniform ✗

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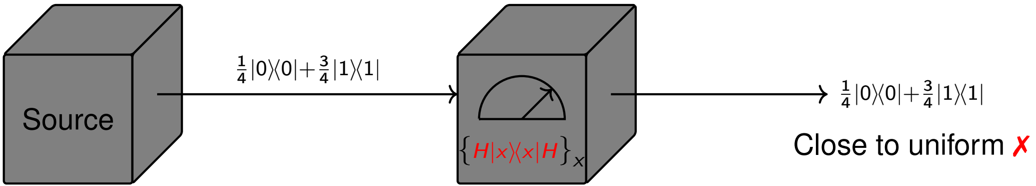

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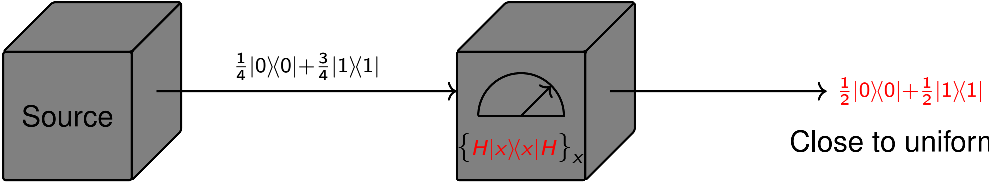

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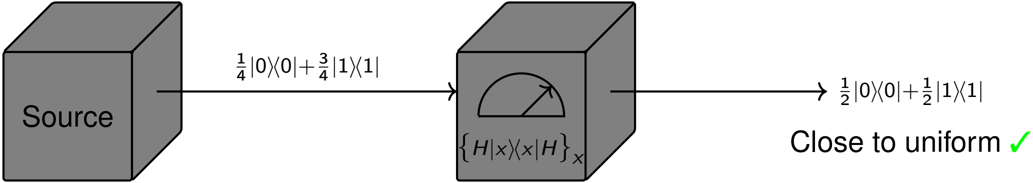
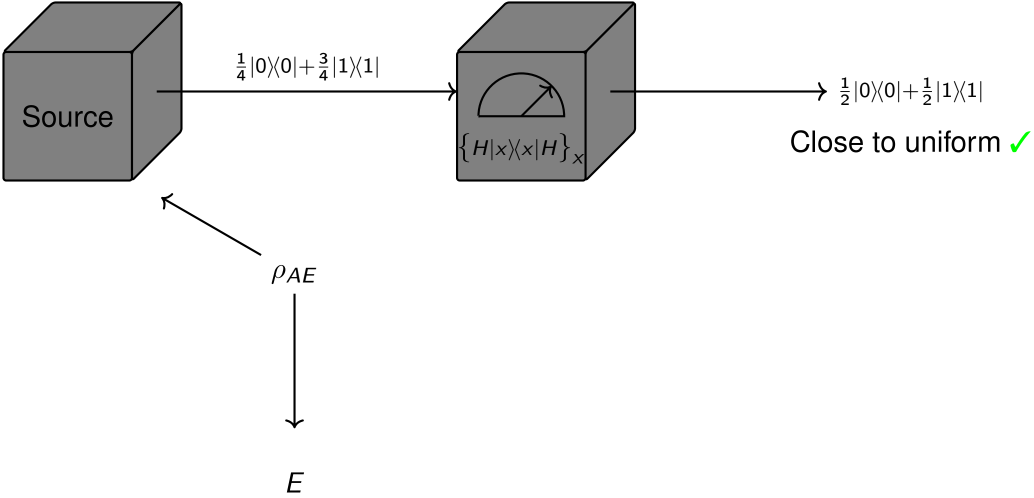


But what if an adversary had information about the source?

Problem setup



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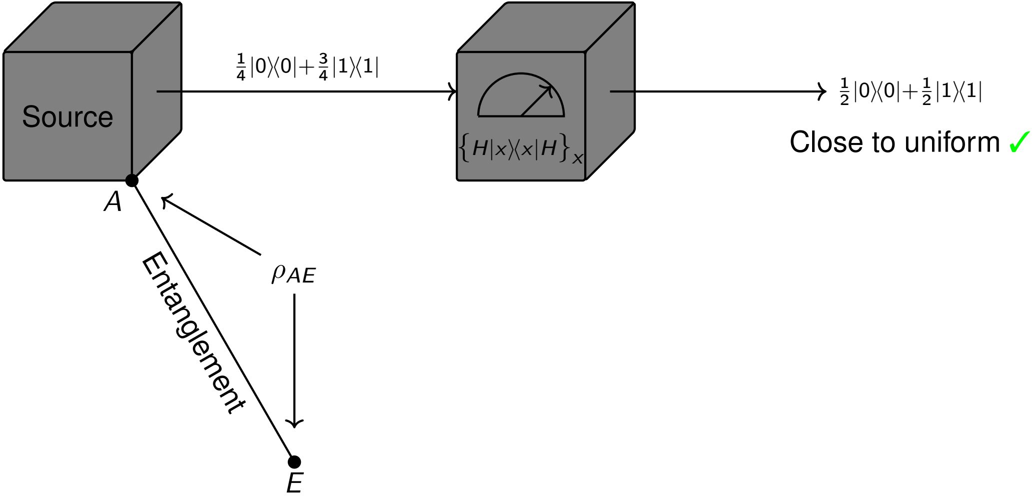

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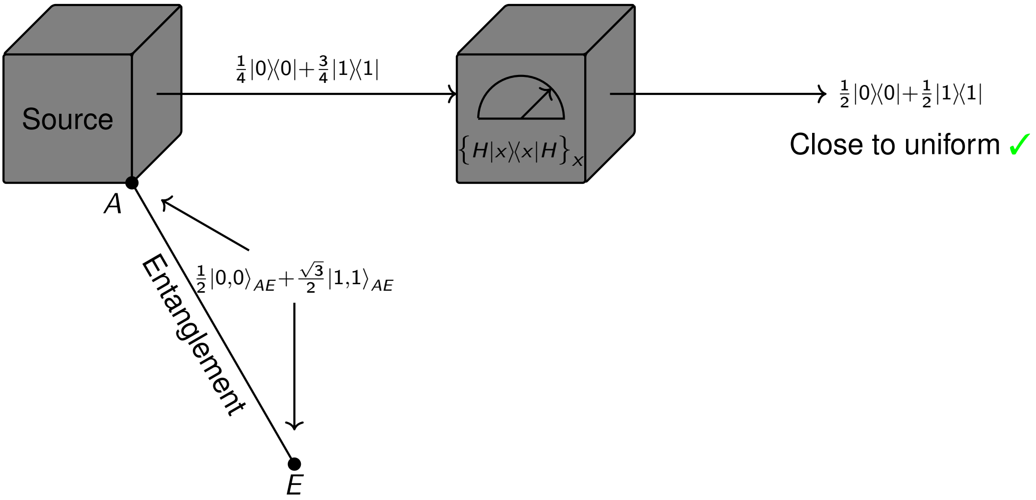

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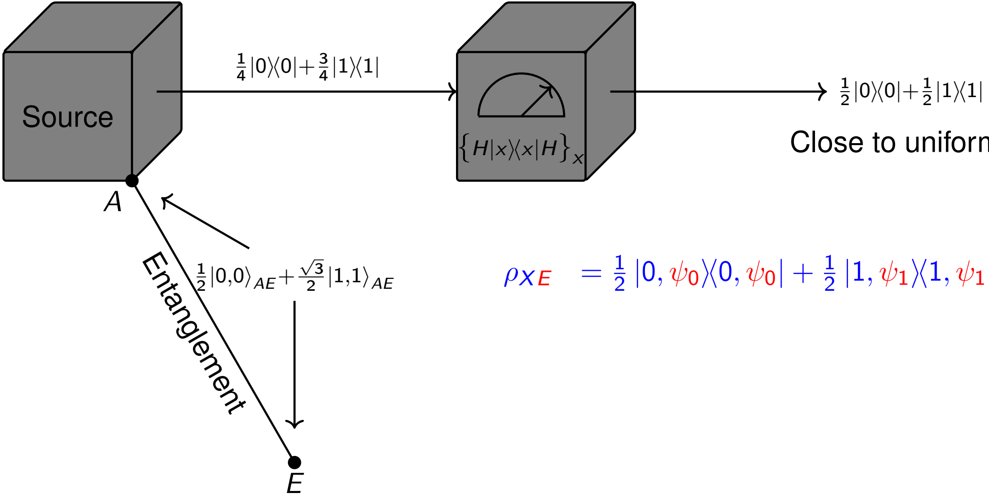

Close to uniform ✓

$$\rho_{XE} = \frac{1}{2}|0,\psi_0\rangle\langle 0,\psi_0| + \frac{1}{2}|1,\psi_1\rangle\langle 1,\psi_1|$$

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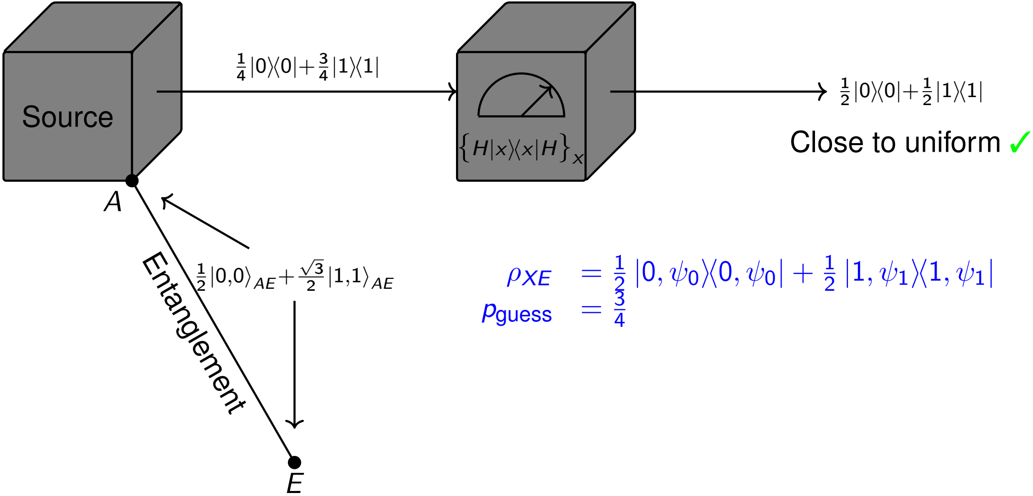

$$
\begin{aligned}
\rho_{XE} &= \frac{1}{2}\,|0,\psi_0\rangle\langle 0,\psi_0| + \frac{1}{2}\,|1,\psi_1\rangle\langle 1,\psi_1| \\
p_{\text{guess}} &= \frac{3}{4}
\end{aligned}
$$

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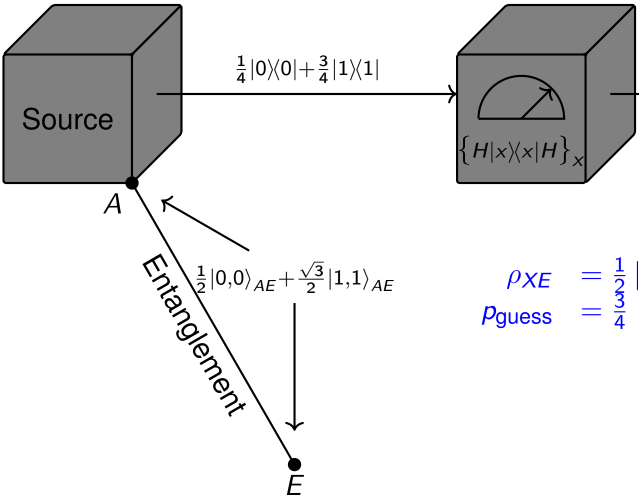

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Secret ✗

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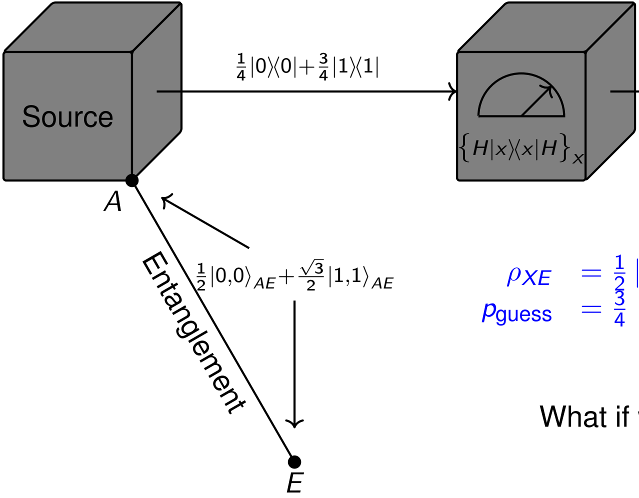

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What if we try to be clever?

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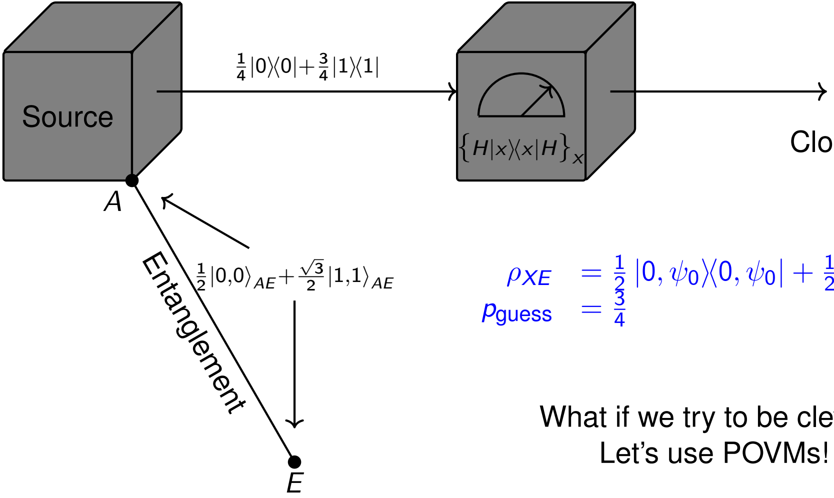

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What if we try to be clever?
Let's use POVMs!

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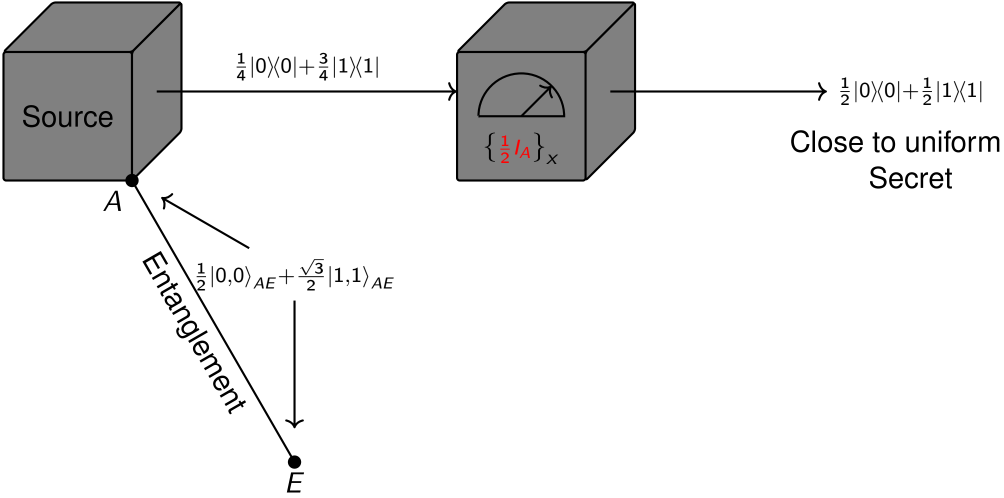

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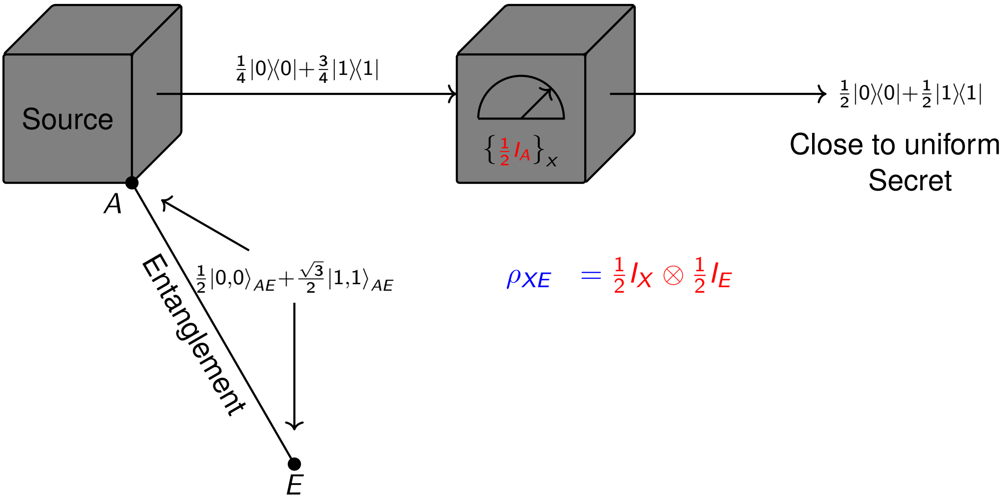

$$\rho_{XE} = \frac{1}{2}I_X \otimes \frac{1}{2}I_E$$

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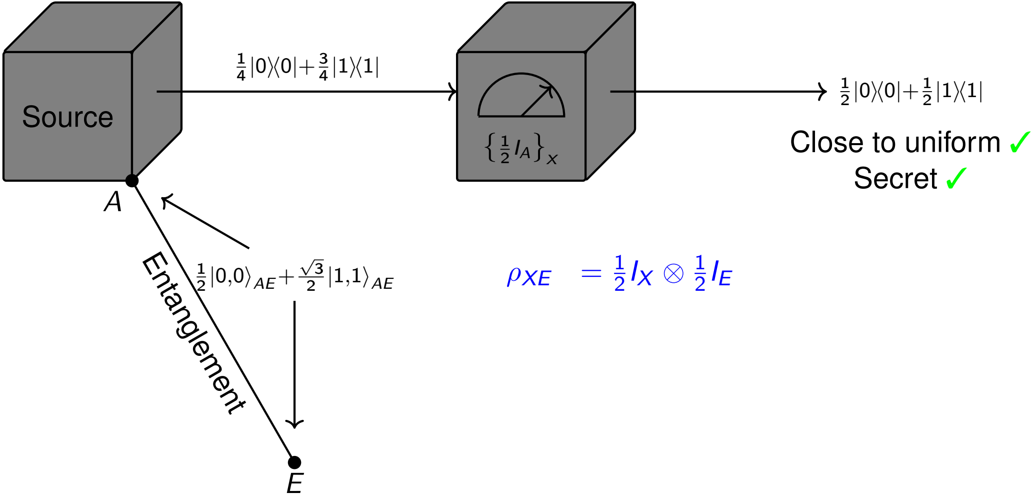

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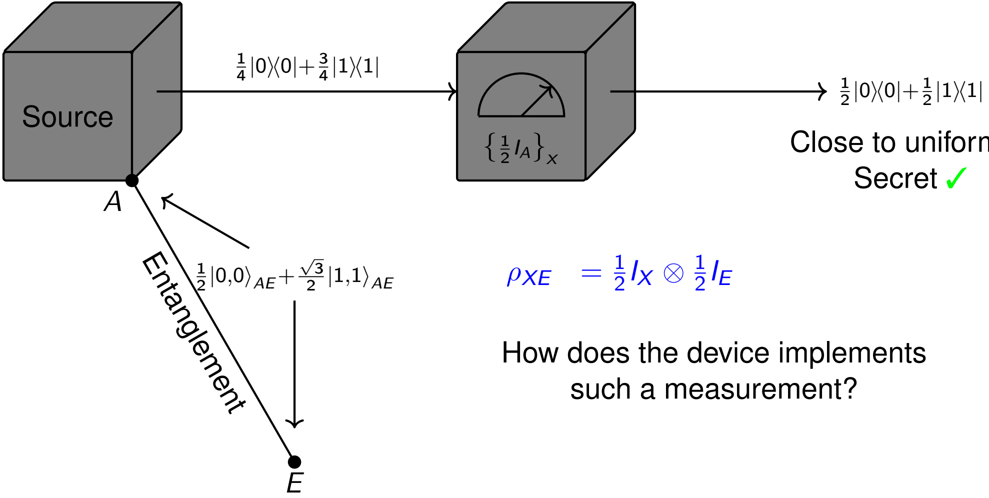

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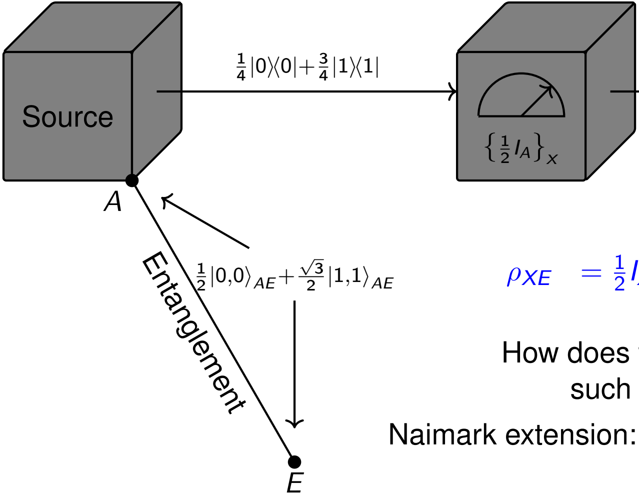
How does the device implements such a measurement?

Problem setup



$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$

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Naimark extension: measures first qubit of $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$

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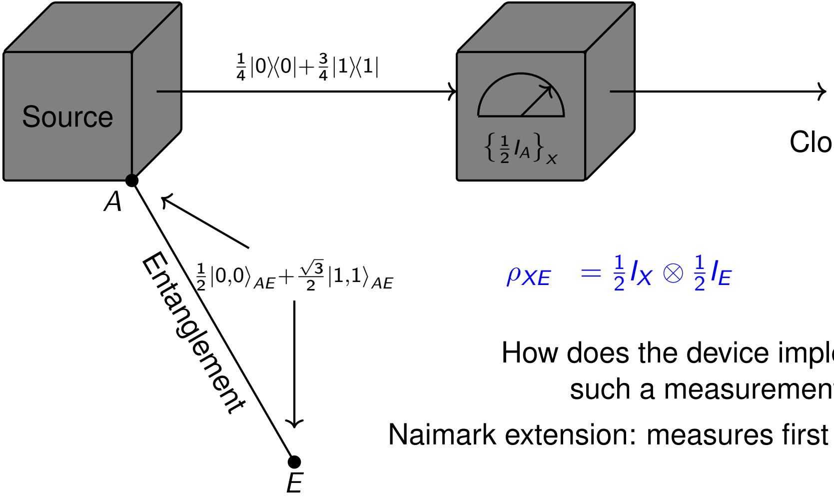

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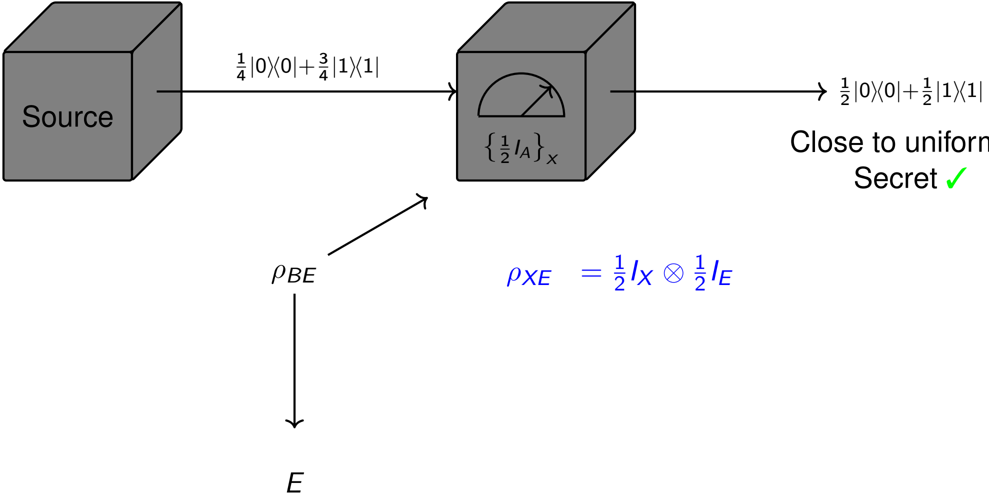

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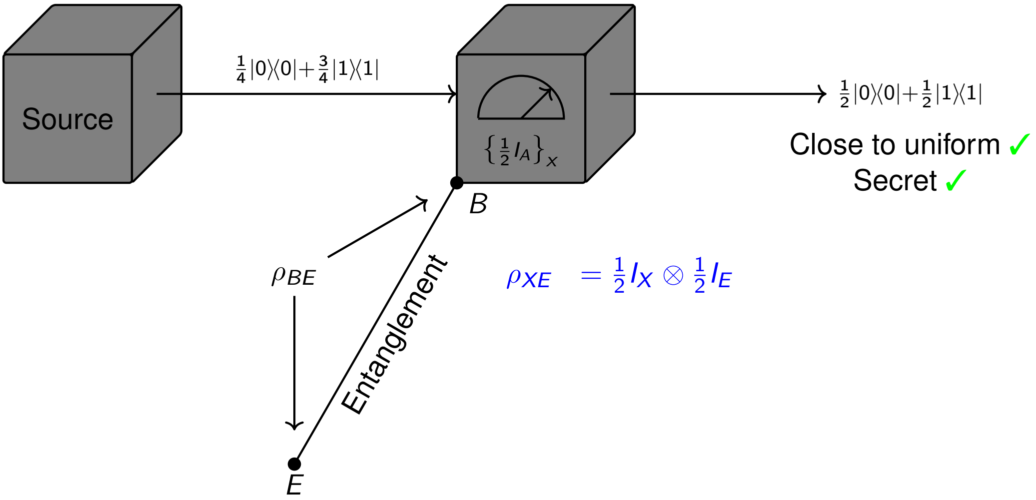

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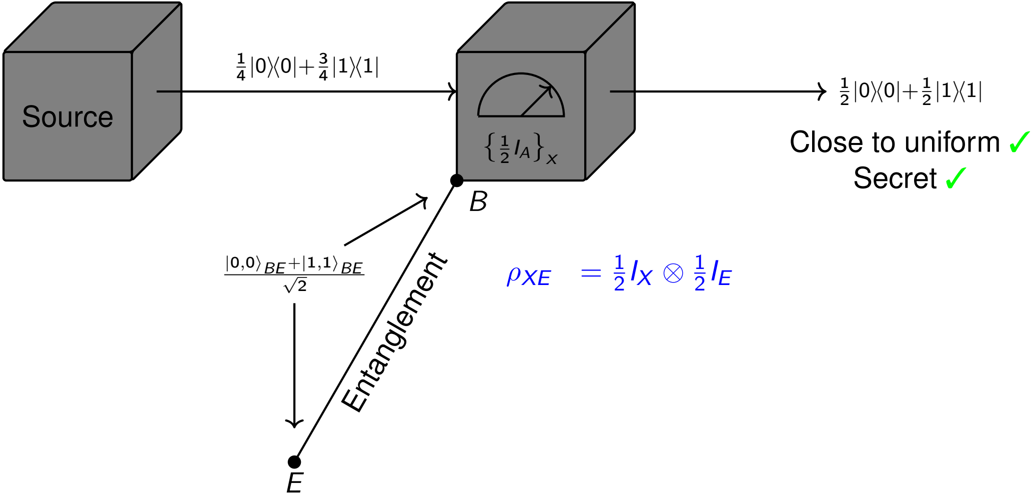

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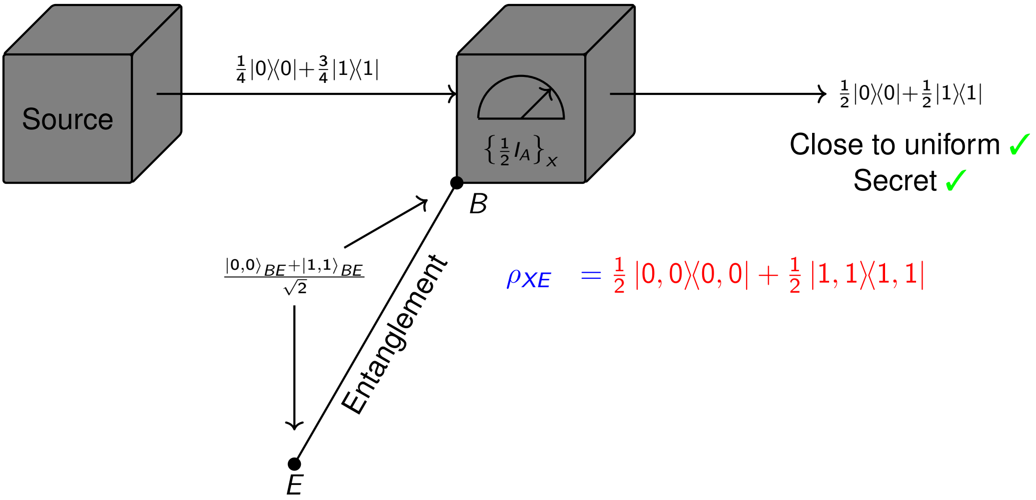

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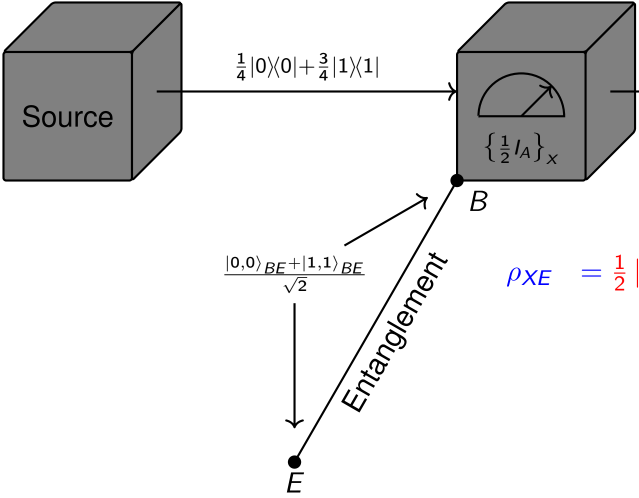

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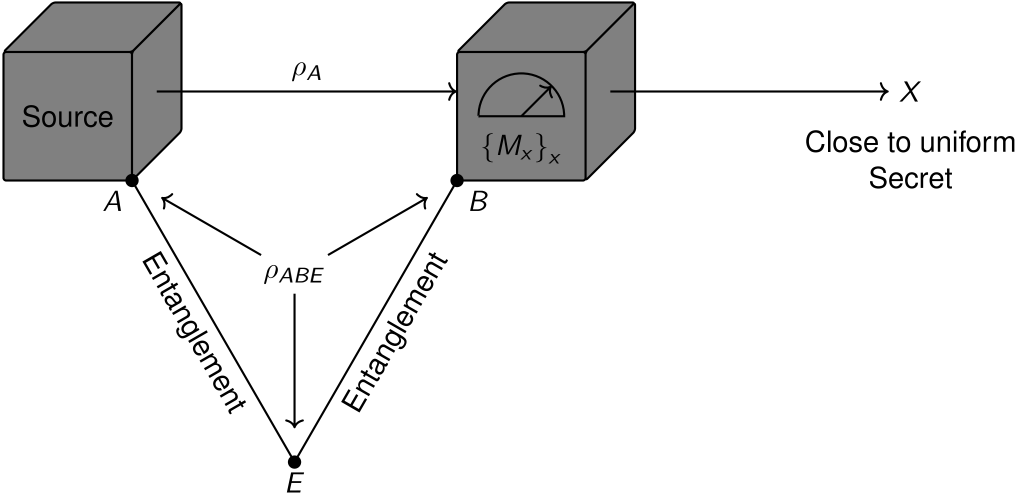

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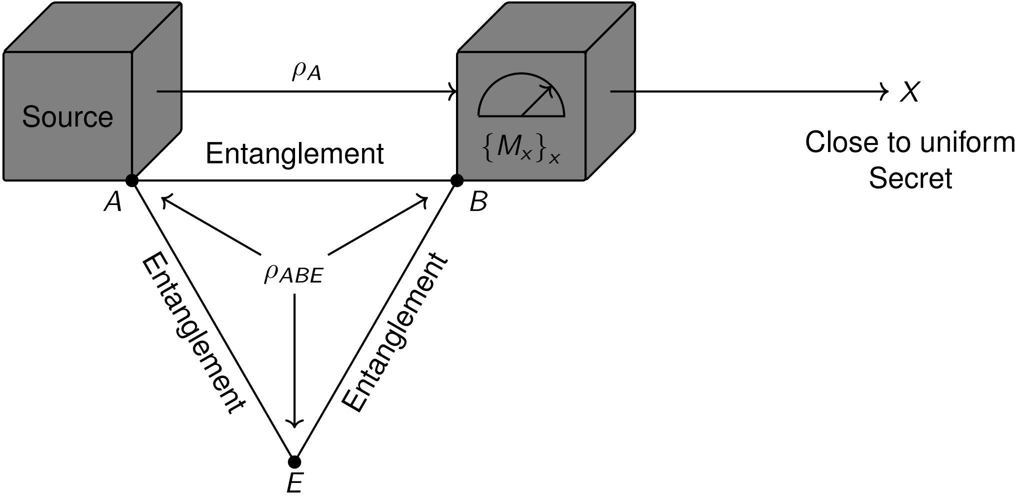

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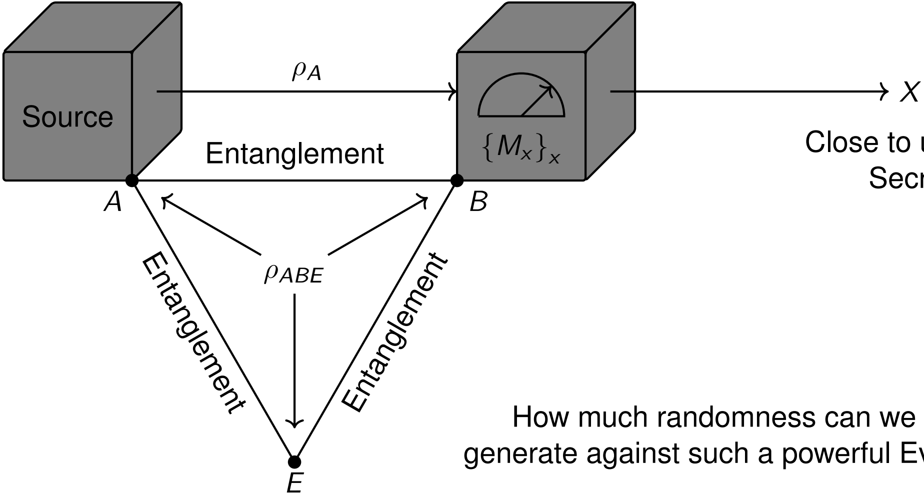
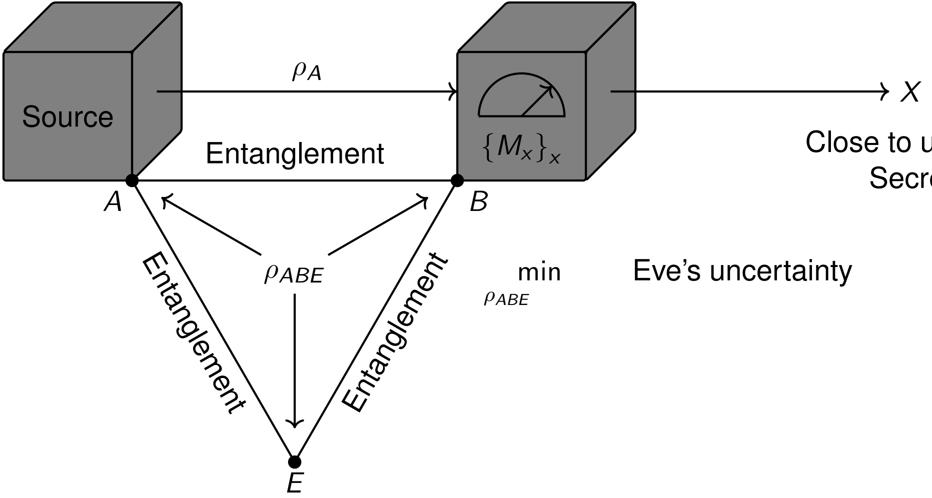


Close to uniform
Secret

How much randomness can we generate against such a powerful Eve?

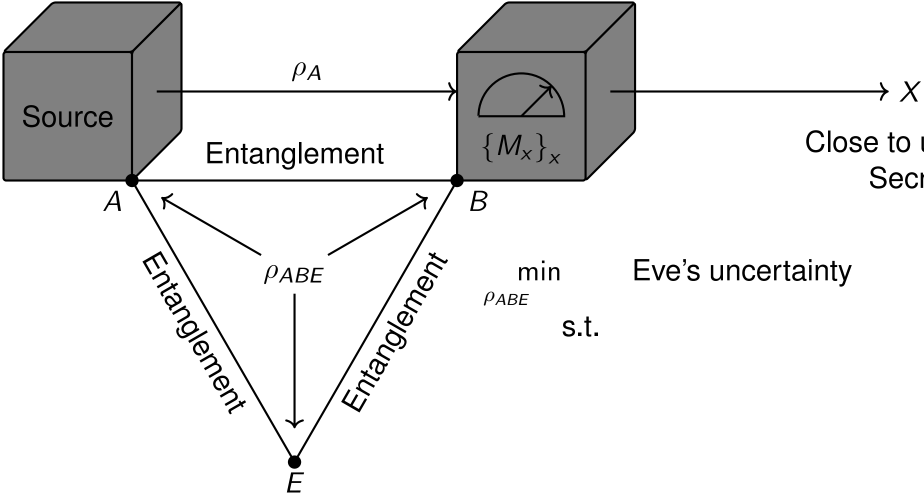
Problem setup



Close to uniform
Secret

$\min_{\rho_{ABE}}$ Eve's uncertainty

Problem setup

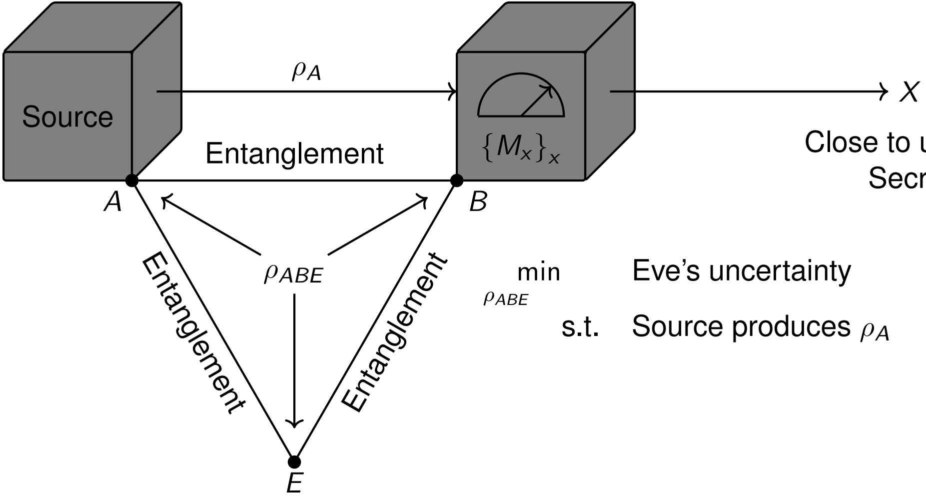


Close to uniform
Secret

$\min_{\rho_{ABE}}$ Eve's uncertainty

s.t.

Problem setup



Close to uniform
Secret

$$\min_{\rho_{ABE}} \quad \text{Eve's uncertainty}$$
$$\text{s.t.} \quad \text{Source produces } \rho_A$$

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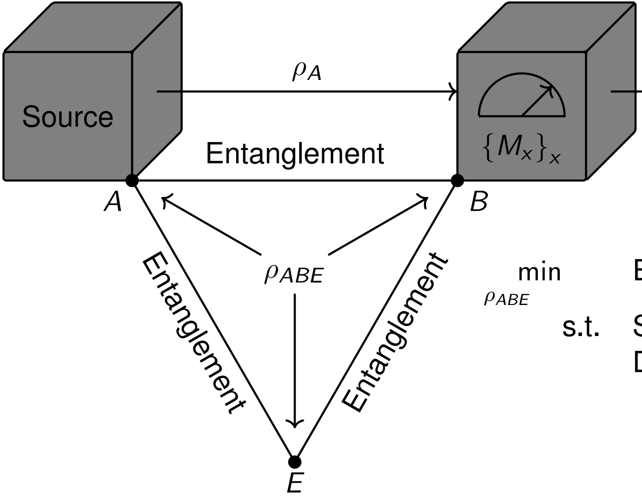
Problem setup



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$$\min_{\rho_{ABE}} \quad \text{Eve's uncertainty}$$

s.t. Source produces ρ_A
Device implements $\{M_x\}_x$

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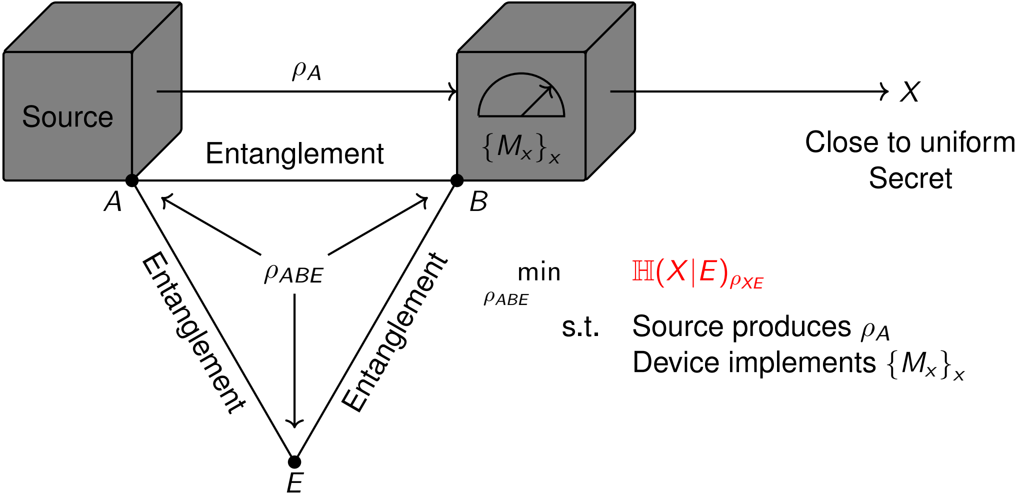

$$\min_{\rho_{ABE}} \quad \mathbb{H}(X|E)_{\rho_{XE}}$$

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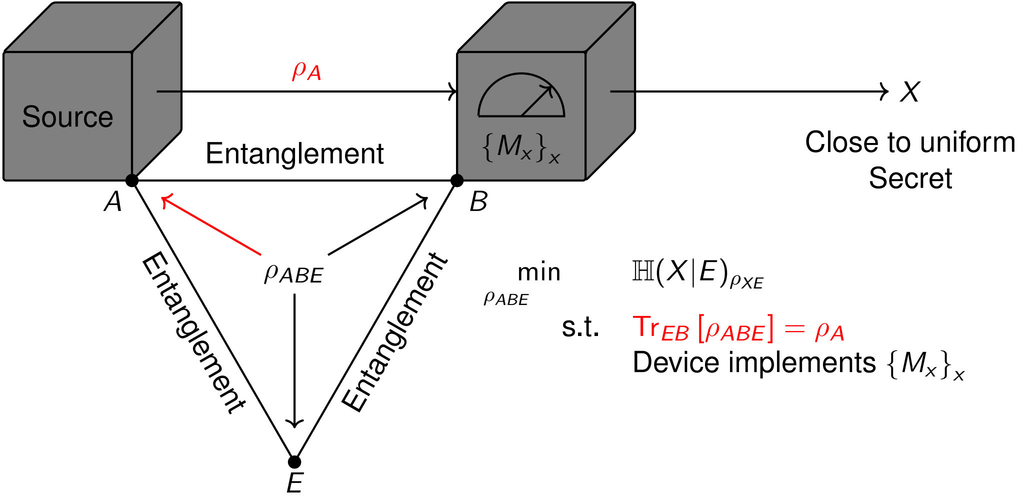

$$\min_{\rho_{ABE}} \quad \mathbb{H}(X|E)_{\rho_{XE}}$$

$$\text{s.t.} \quad \mathrm{Tr}_{EB}\left[\rho_{ABE}\right] = \rho_A$$

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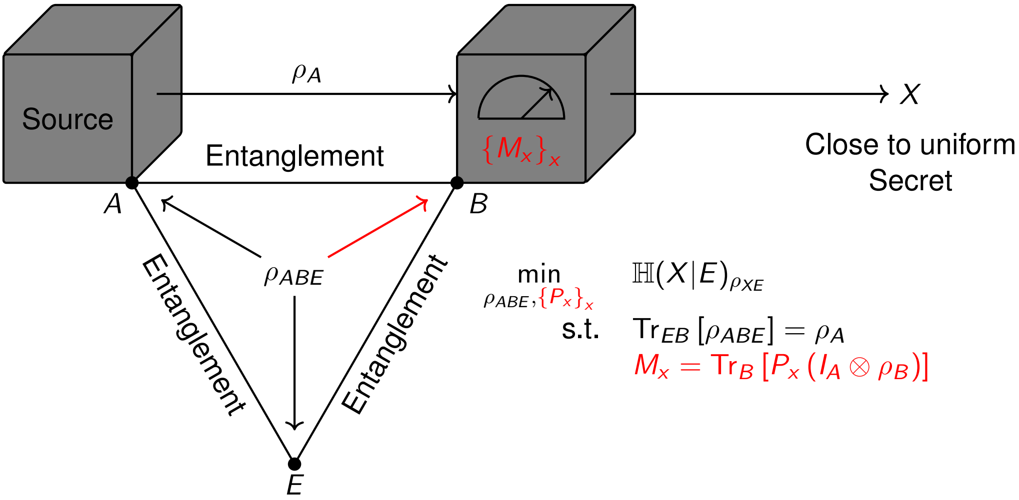

$$\min_{\rho_{ABE},\{P_x\}_x} \quad \mathbb{H}(X|E)_{\rho_{XE}}$$
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$$M_x = \mathrm{Tr}_B[P_x(I_A \otimes \rho_B)]$$

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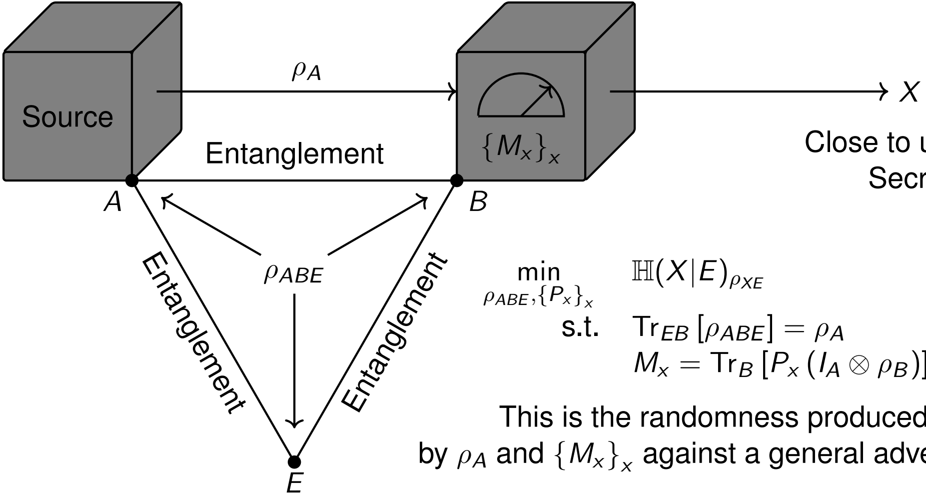

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$$M_x = \mathrm{Tr}_B\left[P_x\left(I_A \otimes \rho_B\right)\right]$$

This is the randomness produced by ρ_A and $\{M_x\}_x$ against a general adversary!

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- Randomness hand-waved using entropies

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$$\rho_{XE} \approx_\varepsilon \frac{1}{d_X} I_X \otimes \rho_E$$

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$$\min_{\rho_{AEB}, \{P_x\}_x} \mathbb{H}(X|E)_{\rho_{XE}} = \qquad \text{randomness produced by } \rho_A \text{ and } \{M_x\}_x$$

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- $\dim(E)$ arbitrary

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- Optimize over all Naimark extensions

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- "$\max\min$" problem
- $\dim(E)$ arbitrary

- Optimize over all Naimark extensions
  - $\dim(B)$ arbitrary
- $\implies$ Not trivial!

Reductions

$$
\begin{aligned}
\sup_{\text{POVM } \{M_x\}_x} \min_{\substack{\rho_{ABE} \\ \text{PVM } \{P_x\}_x}} \quad & \mathbb{H}(X|E)_{\rho_{XE}} \\
\text{s.t.} \quad & \mathrm{Tr}_{BE}\left[\rho_{ABE}\right] = \rho_A \\
& M_x = \mathrm{Tr}_B\left[P_x\left(I_A \otimes \mathrm{Tr}_{AE}\left[\rho_{ABE}\right]\right)\right]
\end{aligned}
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Reductions

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$$M_x = \mathrm{Tr}_B\left[P_x\left(I_A \otimes \mathrm{Tr}_{AE}\left[\rho_{ABE}\right]\right)\right]$$

- Eve: purifies ρ_{AB}

Reductions

$$\sup_{\text{POVM } \{M_x\}_x} \min_{\substack{|\psi_{ABE}\rangle \\ \text{PVM } \{P_x\}_x}} \mathbb{H}(X|E)_{\rho_{XE}}$$

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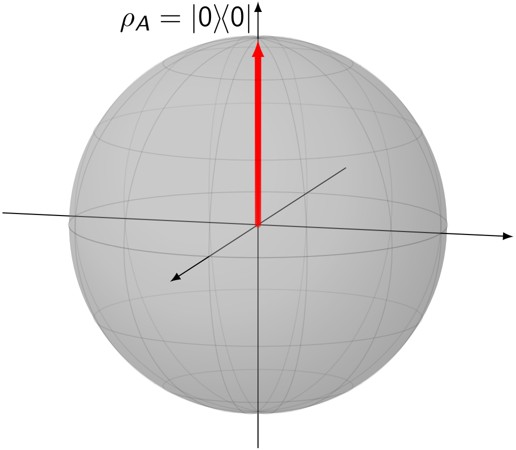
$\implies$ We only need to find the best rank-1 extremal POVM!

A simple case



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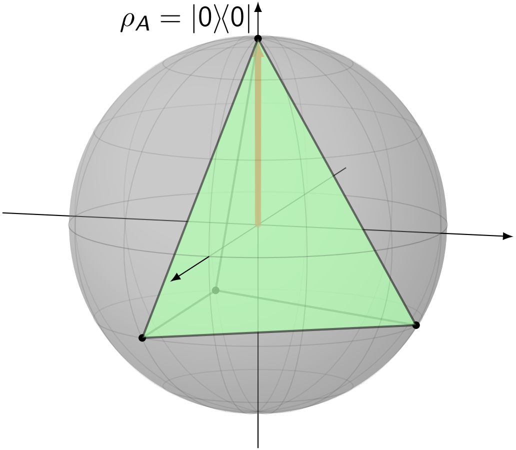
Extremal



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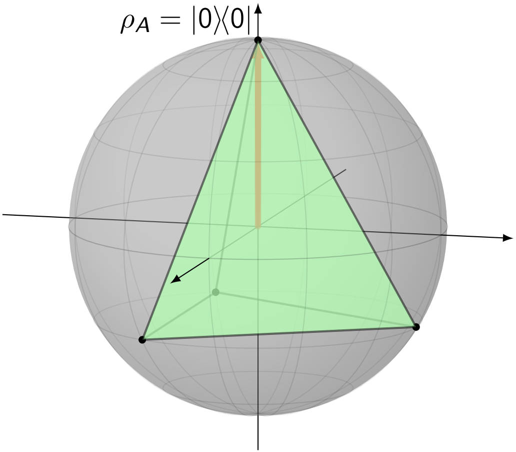

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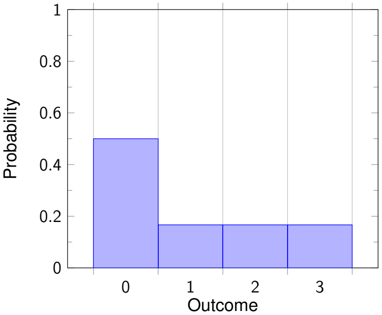
$\min \mathbb{H}(X|E) \approx 1.792$



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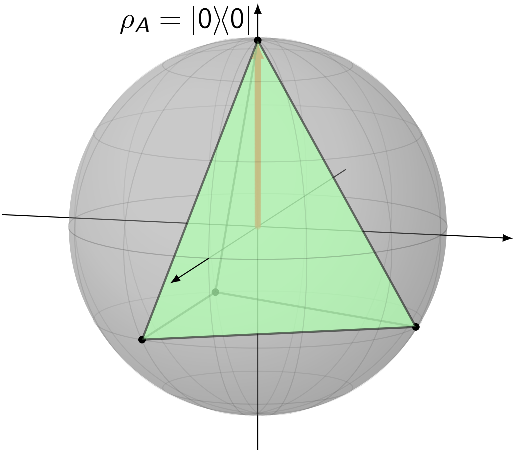

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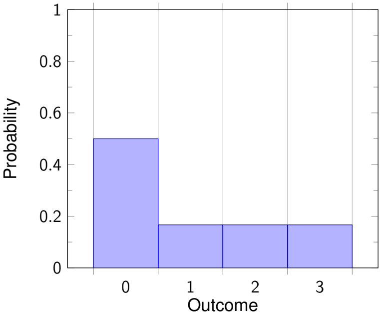

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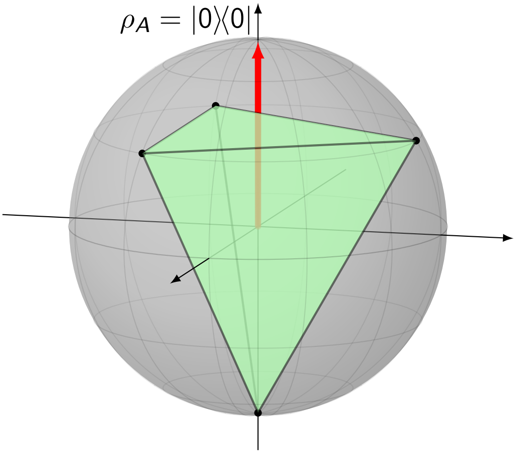

Extremal

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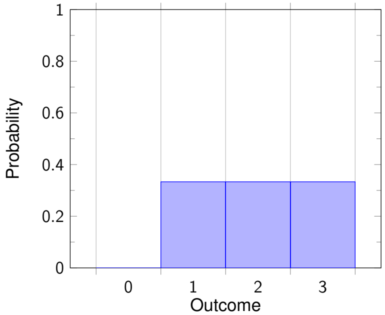
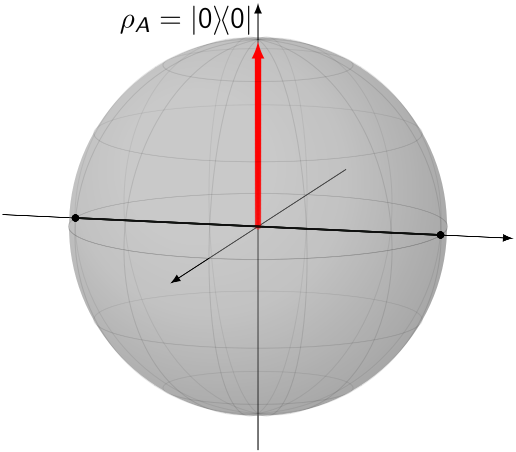
SIC-POVM

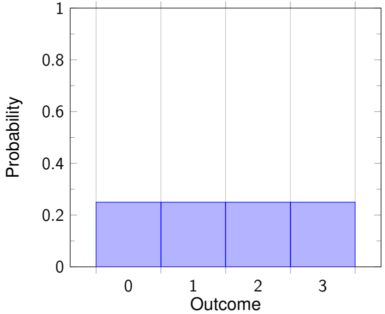


A simple case

Not extremal

$\min \mathbb{H}(X|E) = 1$





$$M = \left\{ \tfrac{1}{2} \left|+\right\rangle\!\left\langle+\right| , \tfrac{1}{2} \left|+\right\rangle\!\left\langle+\right| , \tfrac{1}{2} \left|-\right\rangle\!\left\langle-\right| , \tfrac{1}{2} \left|-\right\rangle\!\left\langle-\right| \right\}$$

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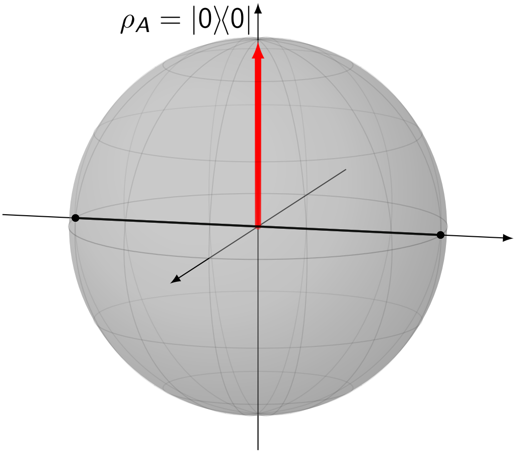

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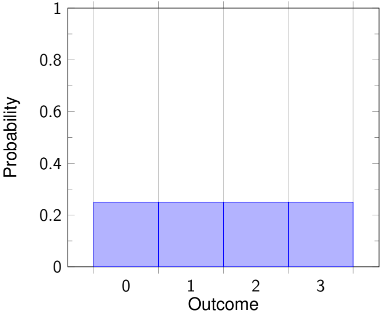

Key result: extremal rank-1 POVMs are dense within rank-1 POVMs

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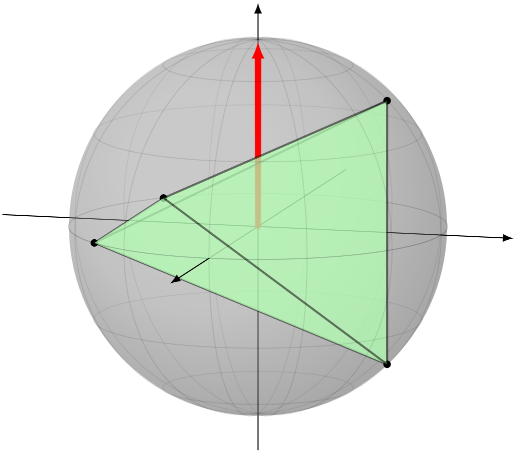
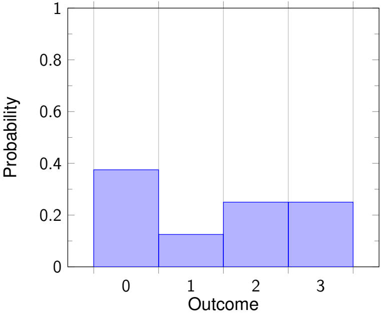


Extremal

$\min \mathbb{H}(X|E) \approx 2$



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Solve exactly for (operationally-relevant) sandwiched Rényi entropies H_α

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Recovers the results of (Meng et al., 2024) if:

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Thanks!

Bibliography I

Dupuis, Frédéric (2023). “Privacy Amplification and Decoupling Without Smoothing”. In: *IEEE Transactions on Information Theory* 69.12, pp. 7784–7792. DOI: 10.1109/TIT.2023.3301812. arXiv: 2105.05342.

Meng, Shuyang et al. (July 2024). “Maximal intrinsic randomness of a quantum state”. In: *Phys. Rev. A* 110 (1), p. L010403. DOI: 10.1103/PhysRevA.110.L010403. arXiv: 2307.15708. URL: https://link.aps.org/doi/10.1103/PhysRevA.110.L010403.