Operator space fragmentation in perturbed Floquet-Clifford circuits

arXiv:2408.01545

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in collaboration with C. J. Turner, L. Masanes & A. Pal

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Why study random circuits?

1. **Analytically tractable limit of many-body dynamics**
 [Fisher et al. (2023) Ann. Rev. Of Cond. Matter Phys.]

2. Tools borrowed from quantum pseudo-randomness, k-designs, frame potentials…etc.
 [Roberts & Yochida (2017) JHEP]

3. Classical mappings for entropy growth and operator spreading
 [Nahum, Vijay & Haah (2017) PRX]

4. The role of symmetries in limiting ergodicity
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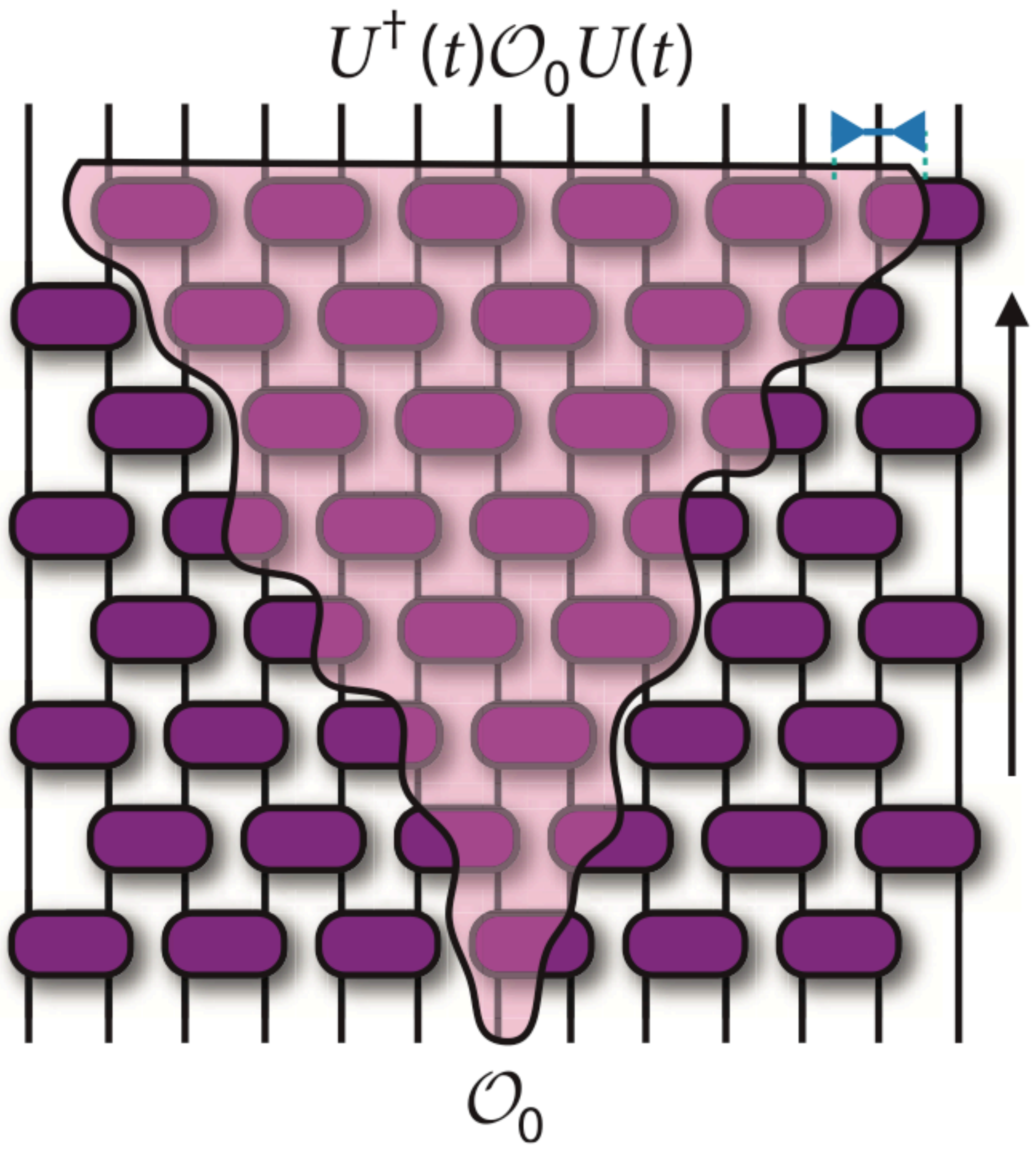

Figure adapted from M. P. Fisher et al. Ann. Rev. of Cond. Matter Phys. (2023). https://doi.org/10.1146/annurev-conmatphys-031720-030658

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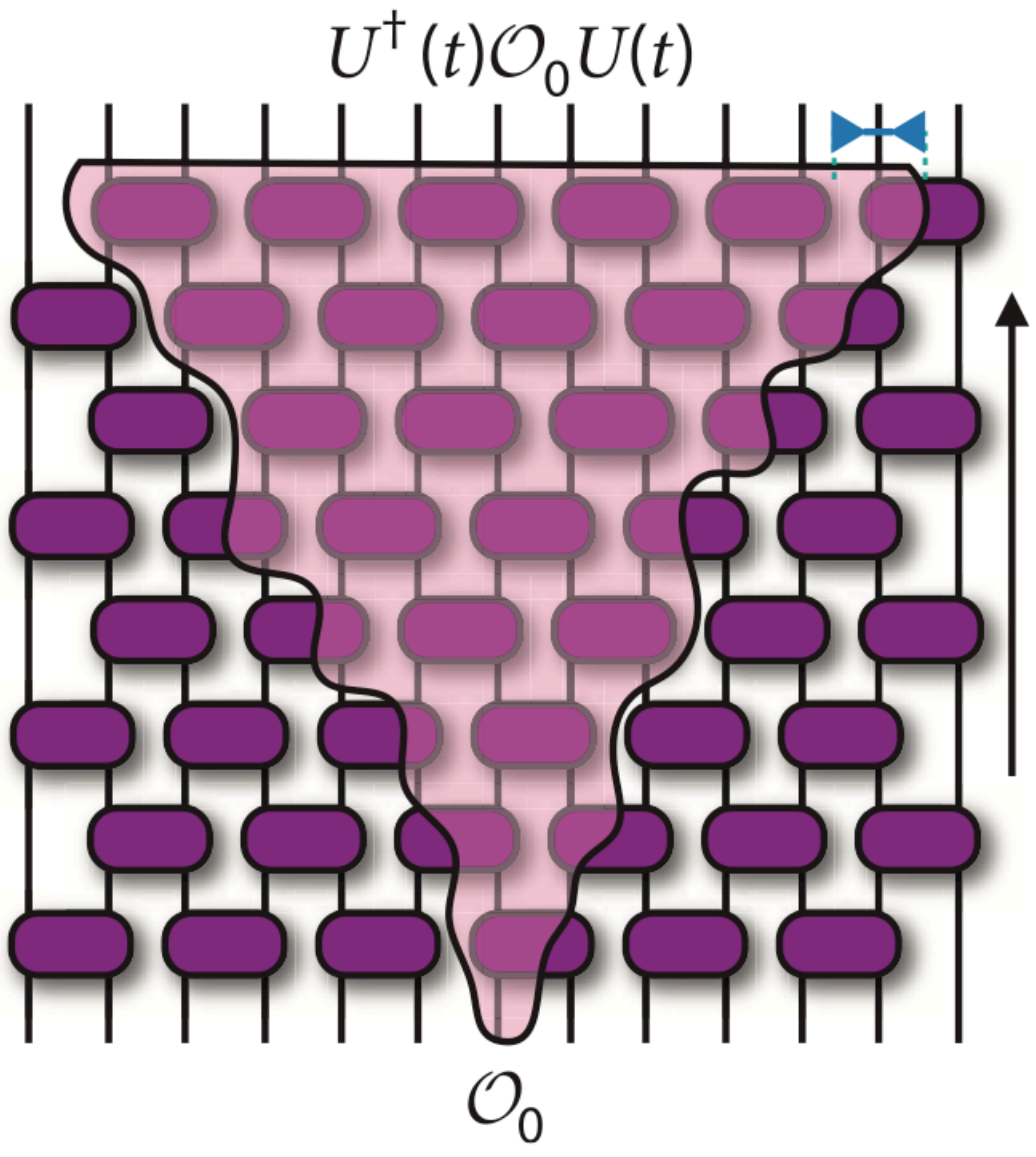

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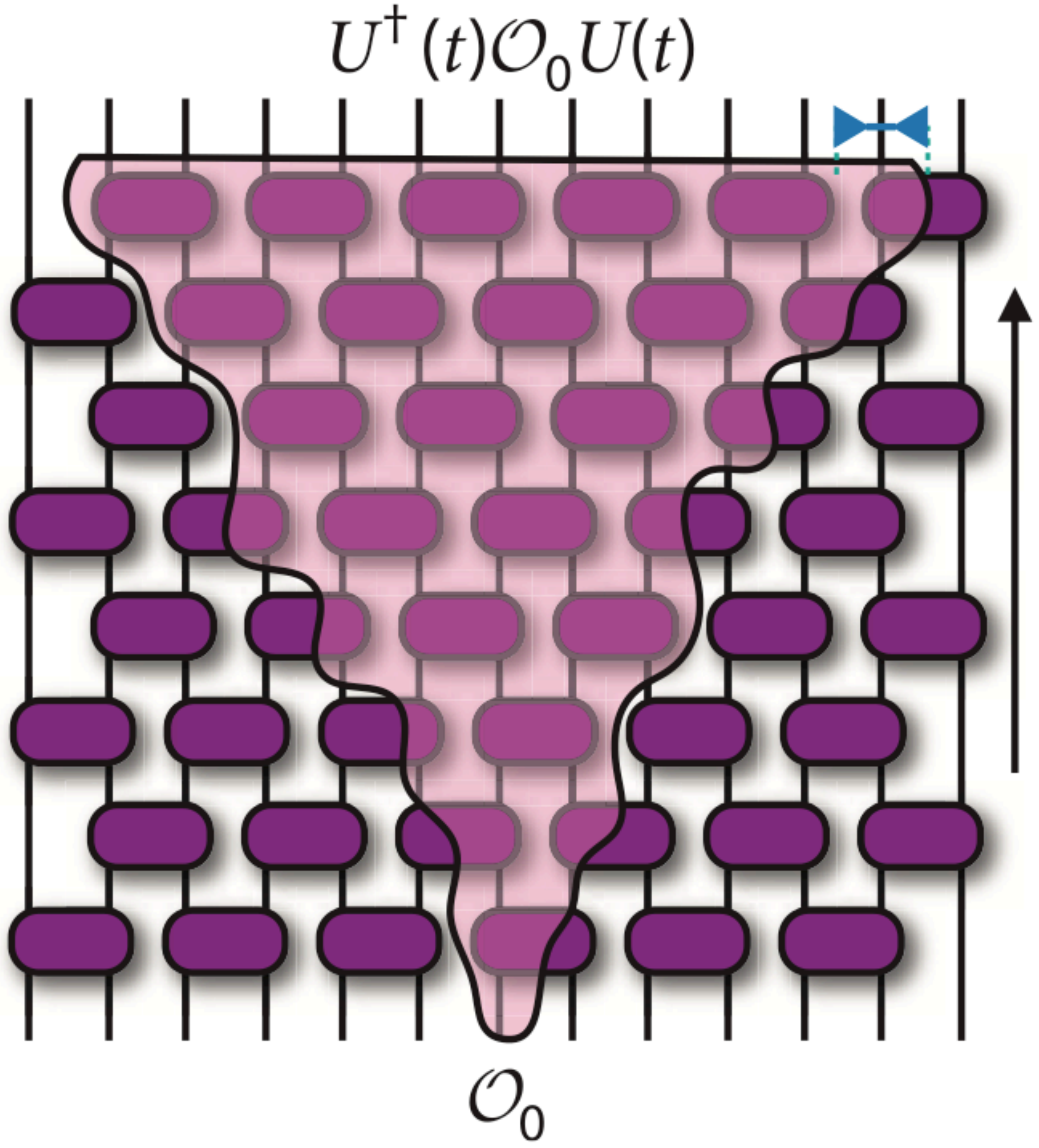

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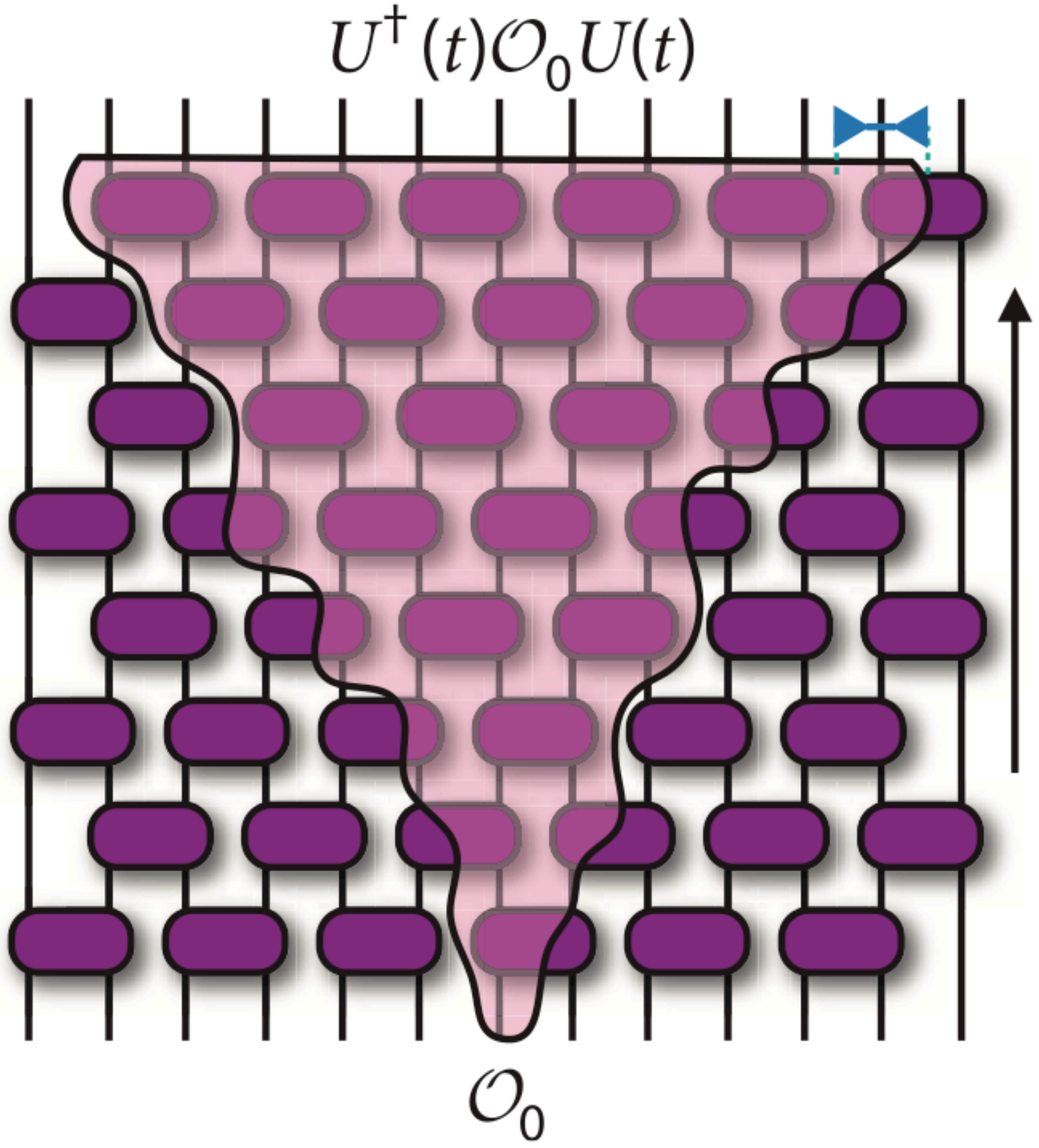

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Ergodicity-breaking in literature

1. **Time-periodicity permits ergodicity breaking via biased sampling of the unitary 'bricks'**
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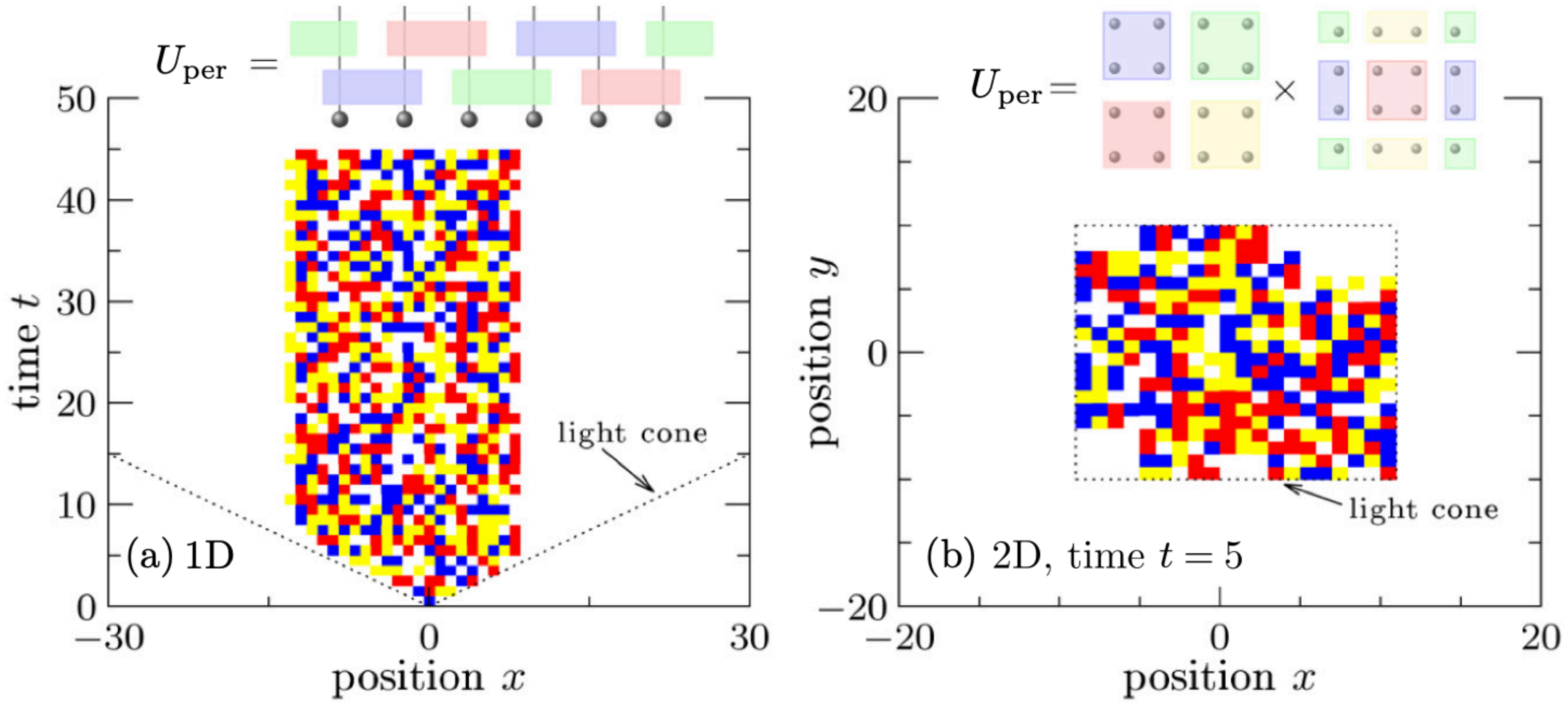

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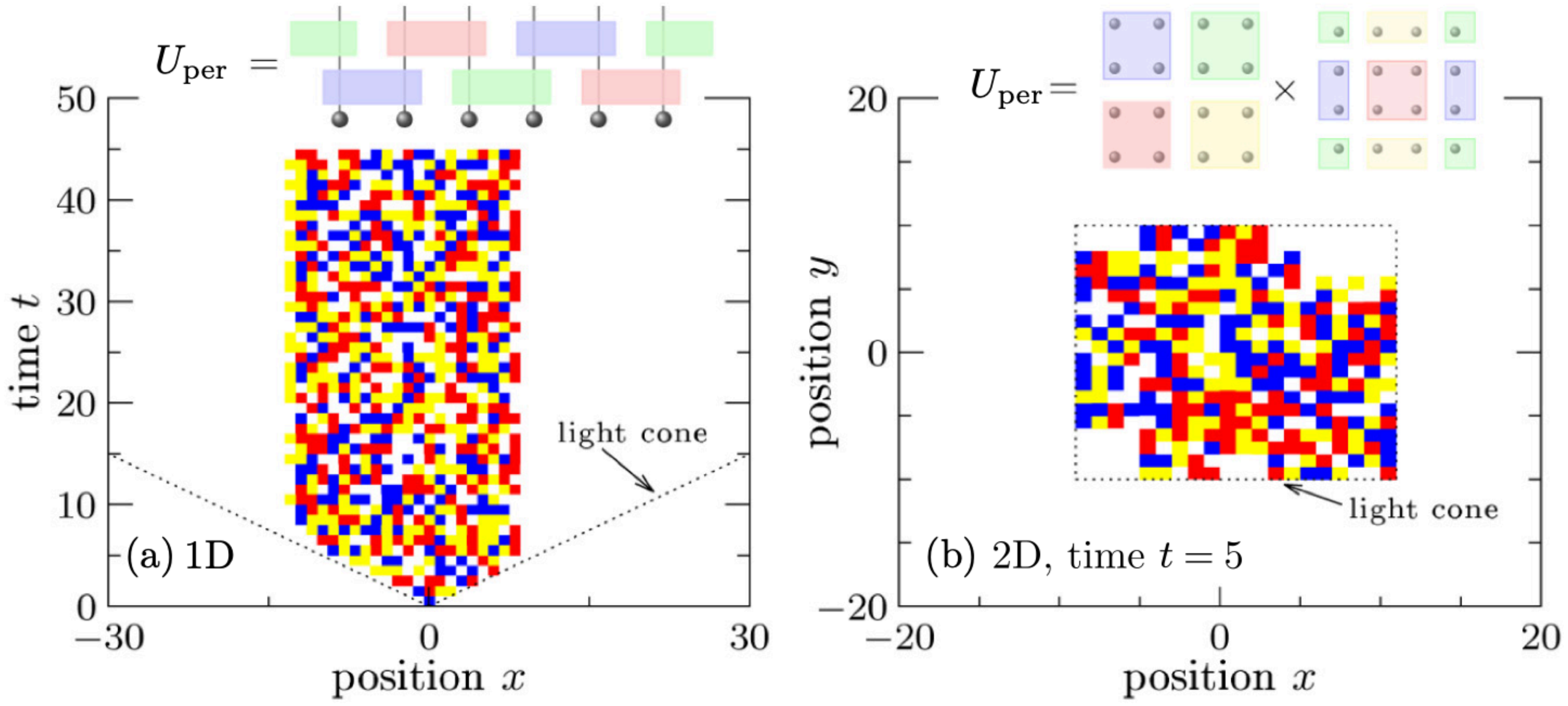

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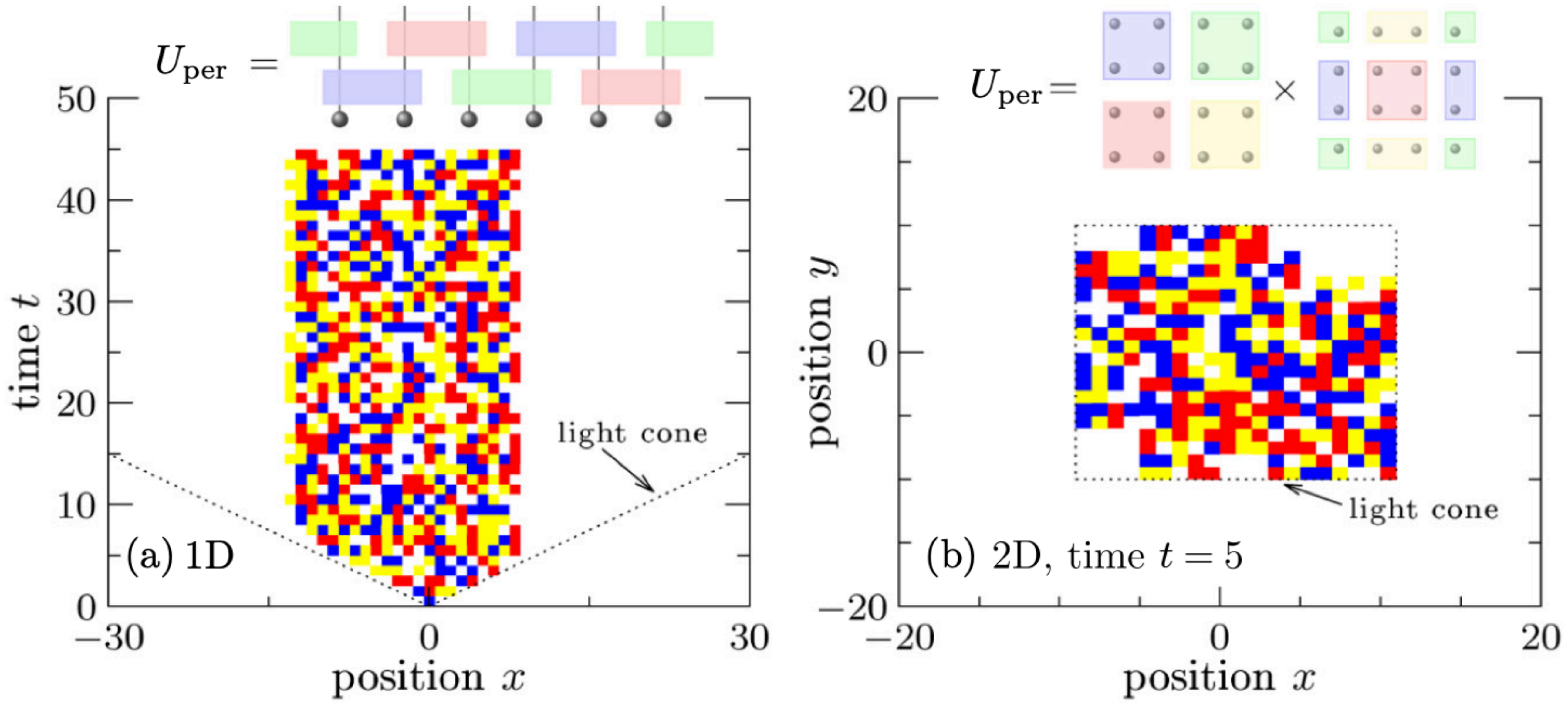

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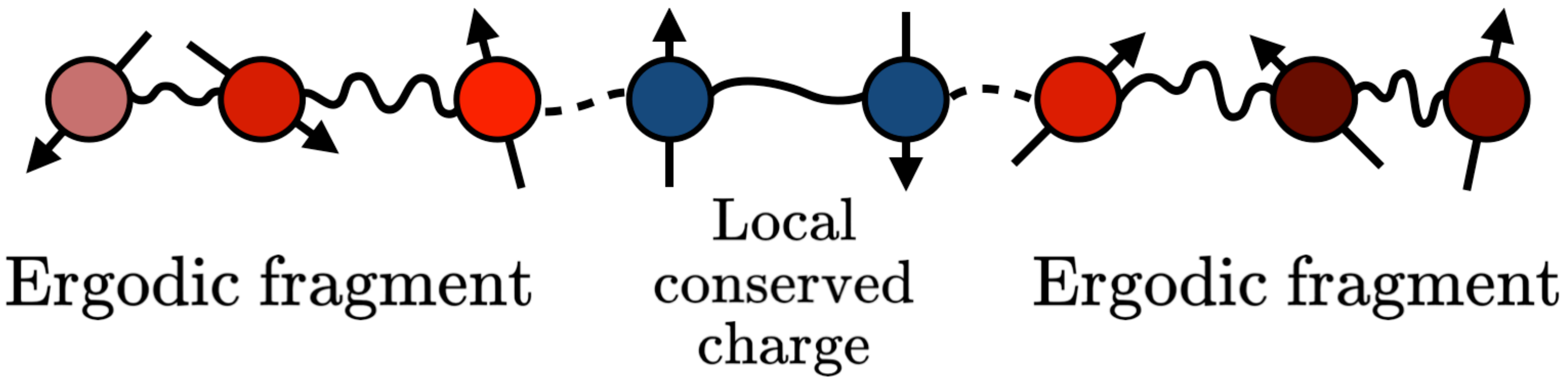
Some open questions

1. Analytical models for random circuit localisation (…in Clifford circuits)?

2. What’s the stability of localisation against perturbations?

3. Can ergodicity and localisation coexist in many-body dynamics?

4.* Is this the same phenomena as many-body localisation?



Today's talk

1. Floquet model of Cliffords with perturbations

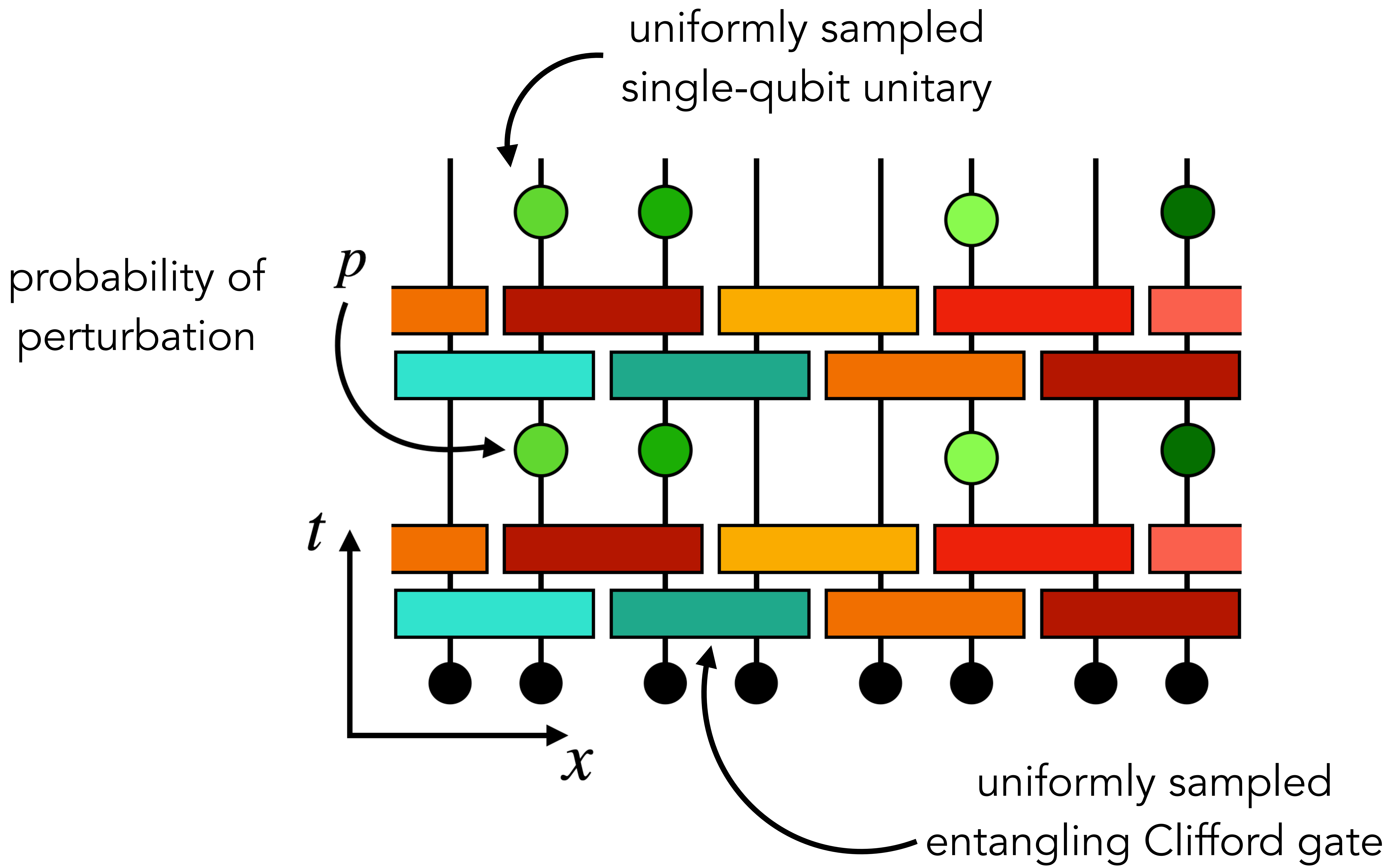
2. Wall configurations

3. Stability of fragmentation

4. Entanglement signatures of non-ergodicity

5. Further work & conclusions

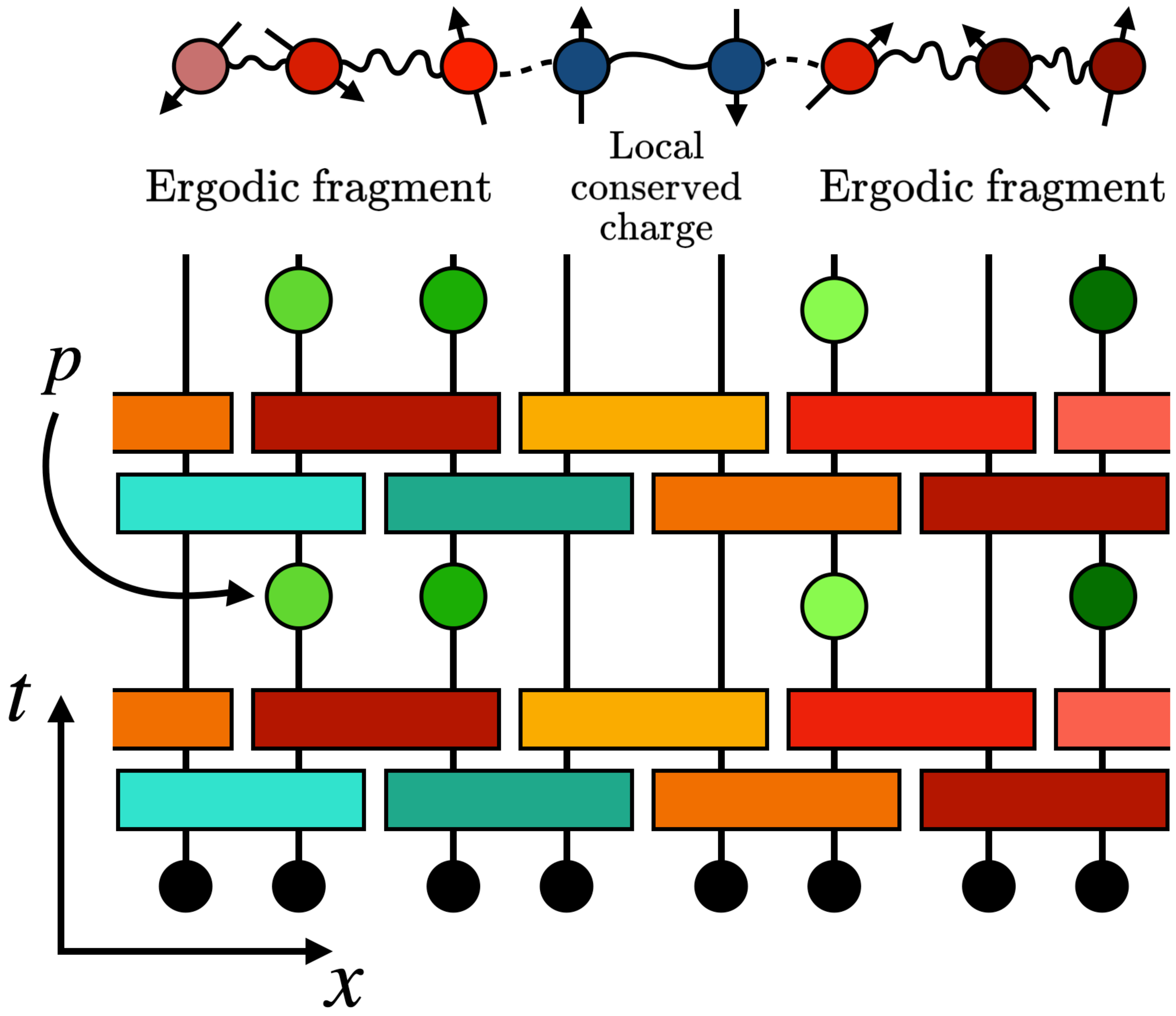
Disordered Floquet model



Floquet symmetry = periodicity in discrete time

Spatial disorder by i.i.d sampling gates

Summary of results



1. The infinite chain fragments in operator space for $p < 1$
2. Fragments are locally ergodic
3. Atypical localising regions harbour local conserved quantities
4. Entanglement is limited across fragments boundaries
5. Percolation transition at $p \to 1$

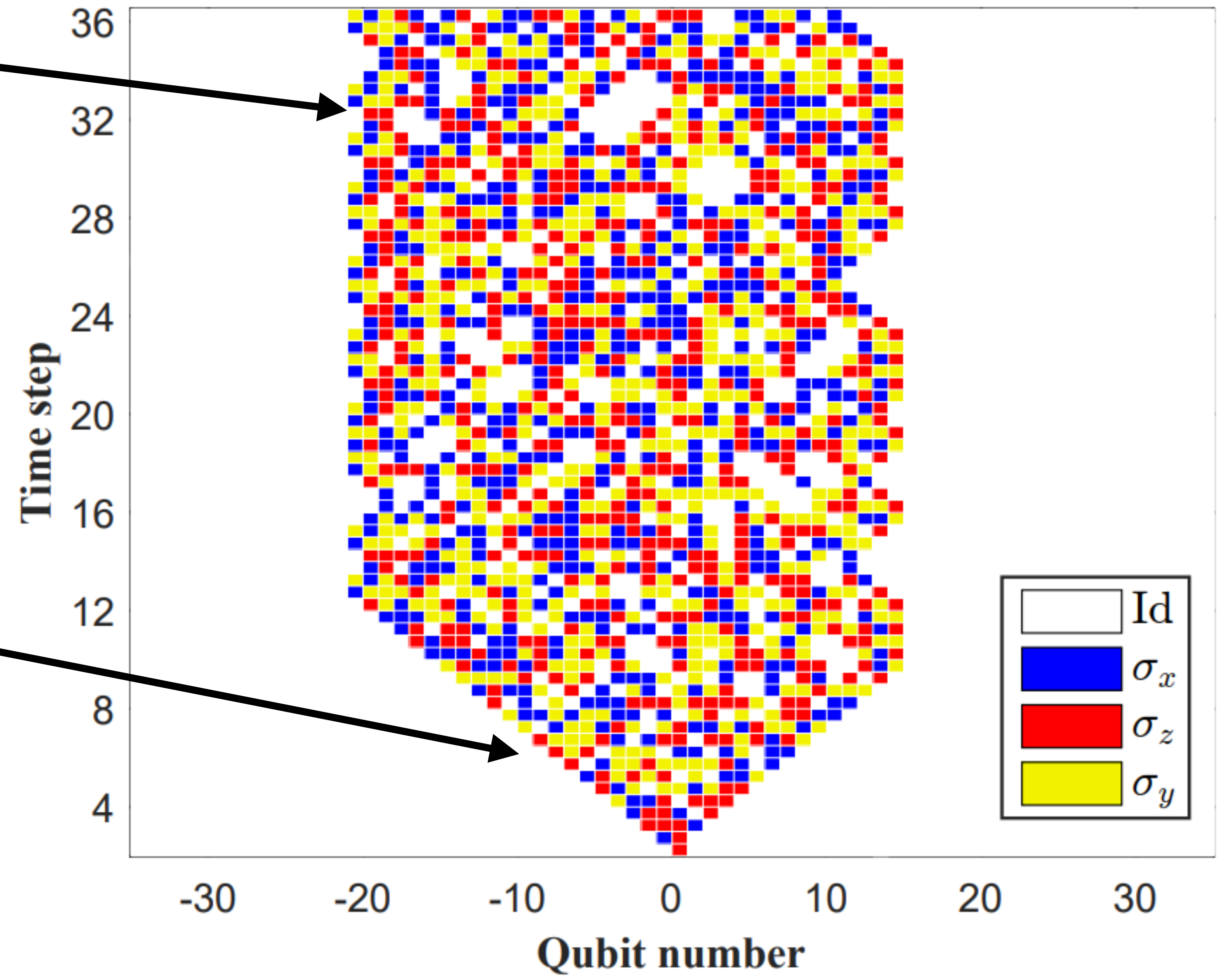
The Clifford-limit

$$P \to U^t P U^{-t}$$

'Walls' arresting operator spreading

Pauli mixing within lightcone

Ballistic light cone



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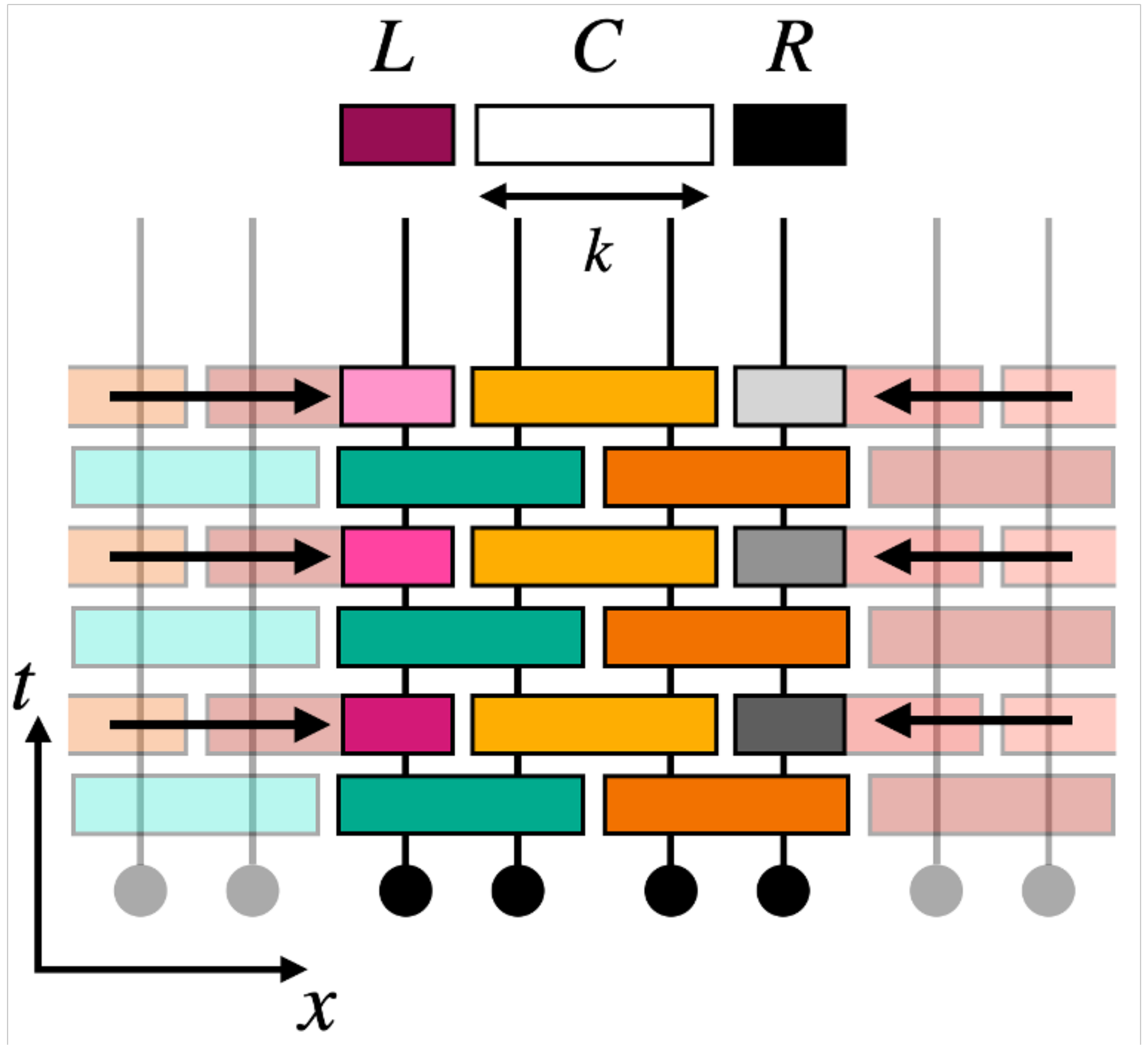
https://doi.org/10.1063/5.0054863

1. **Structure of localising regions in Clifford?**
2. **Is this behaviour Clifford specific?**

Operator localisation

k-walls: Clifford gate configurations that arrest the spreading of arbitrary operators in k steps

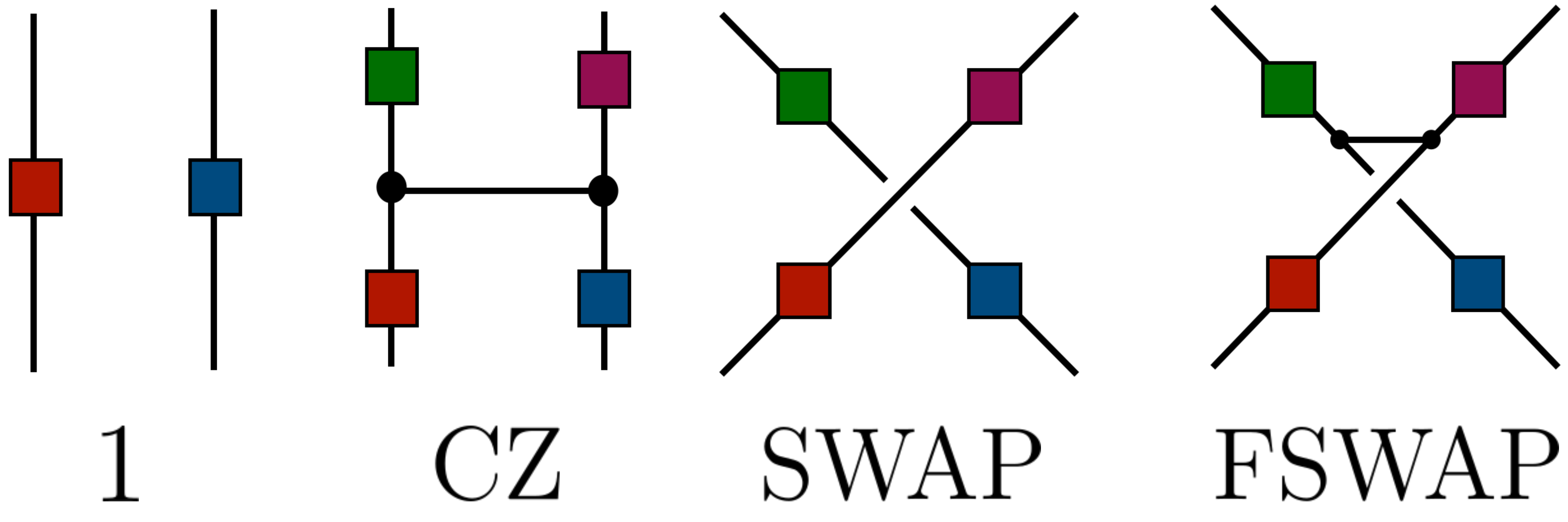
$$U^t(A \otimes \mathbf{1}_C \otimes \mathbf{1}_R)U^{-t} = A'(t) \otimes \mathbf{1}_R$$



0-walls are product unitaries that prevent the spreading of any operator

$$U = u_1 \otimes u_2$$

Clifford group equivalence classes w.r.t. product unitaries

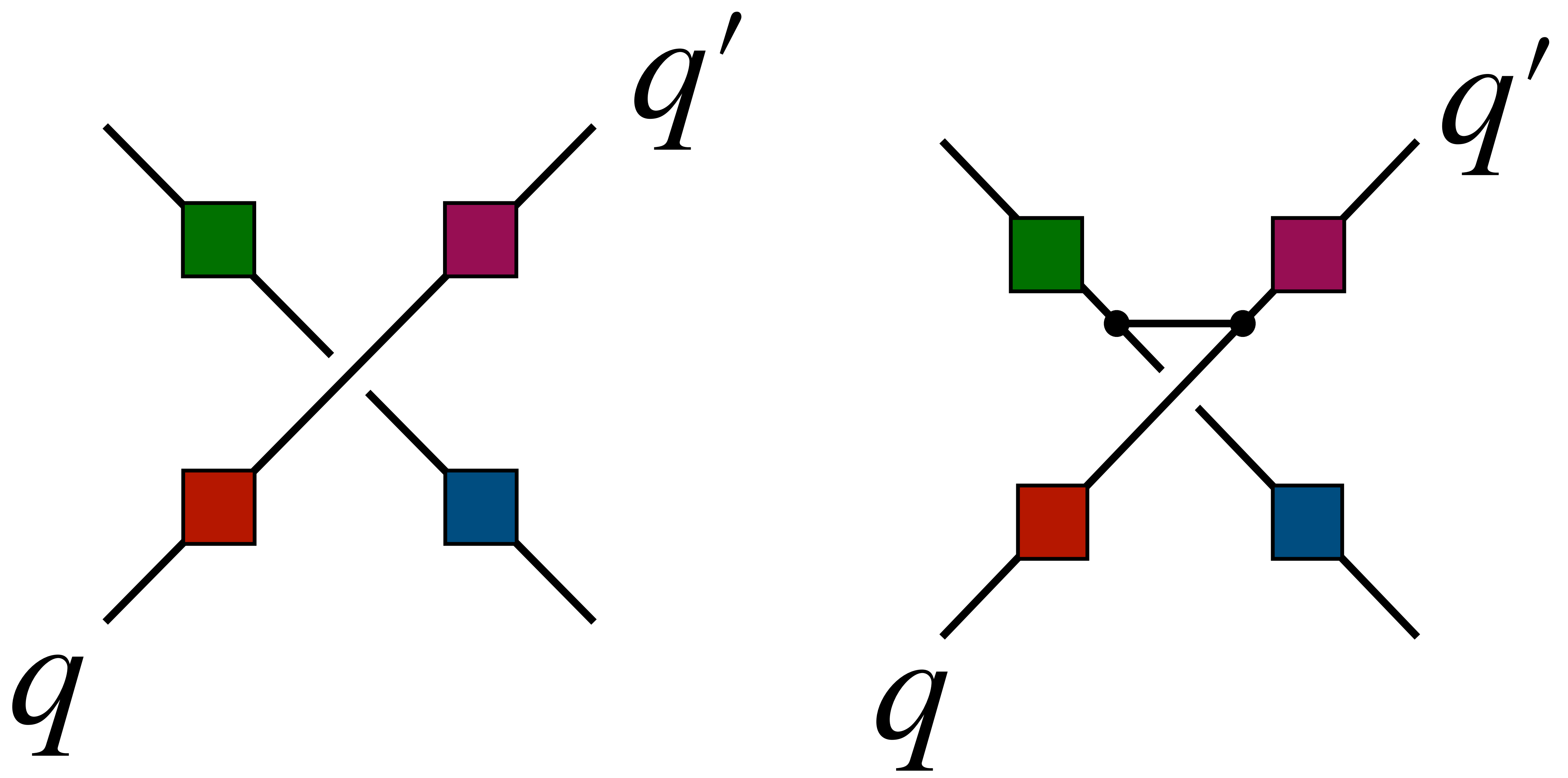


Controlled-class

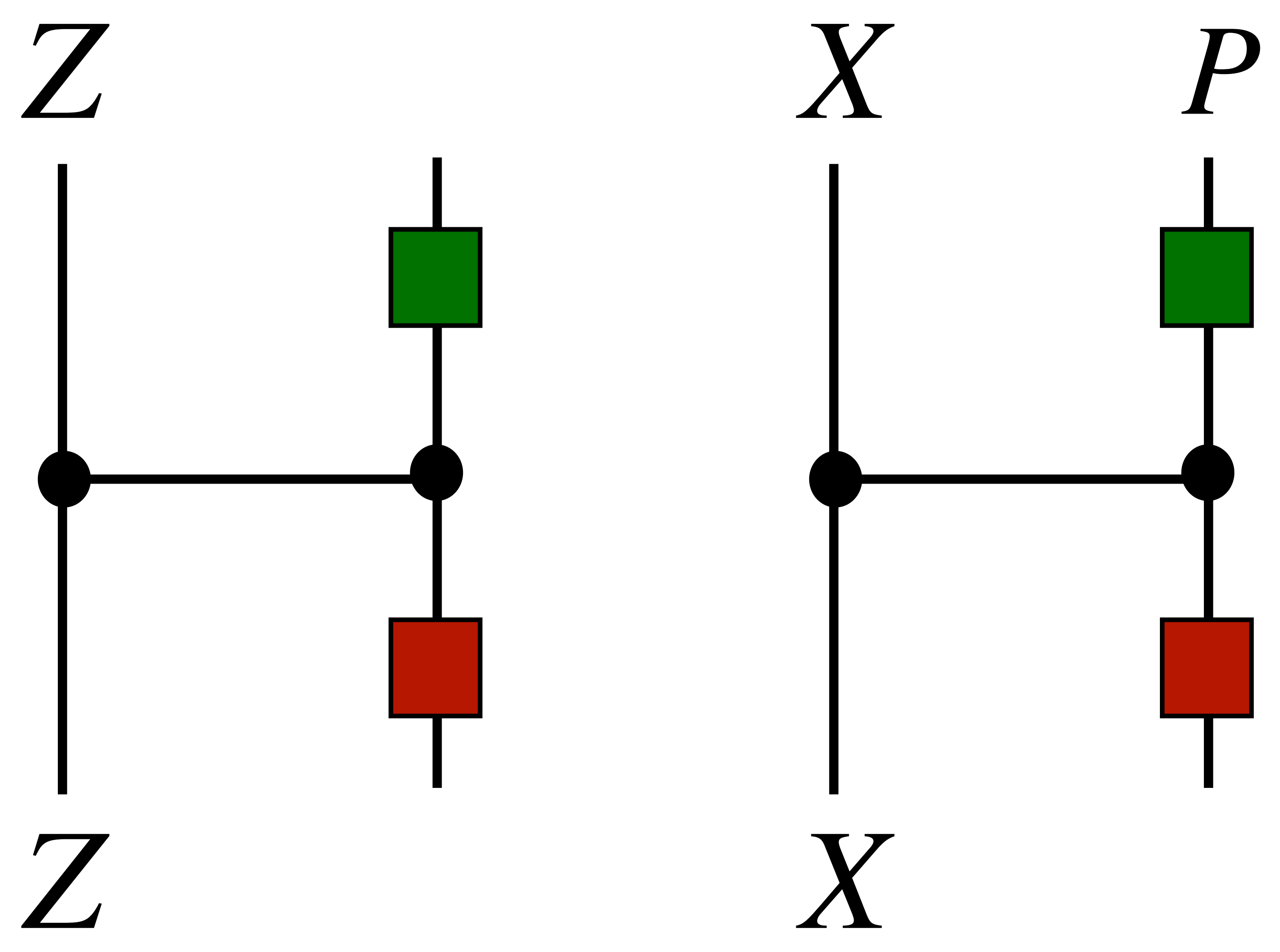
Dual unitary classes

1-walls in the Clifford group

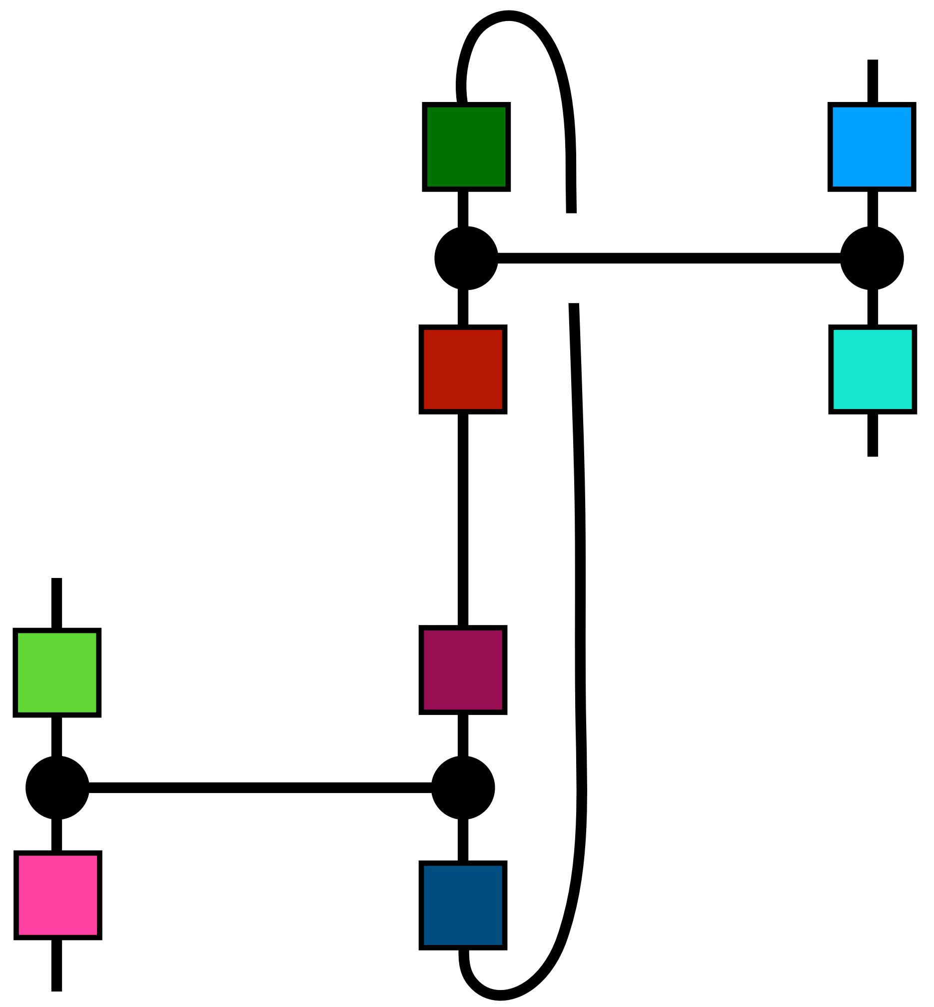
1-walls cannot contain dual-unitary classes due to ballistic spreading of arbitrary operators



Controlled-class can have local conservation laws, with restricted spreading



1-walls in the Clifford group



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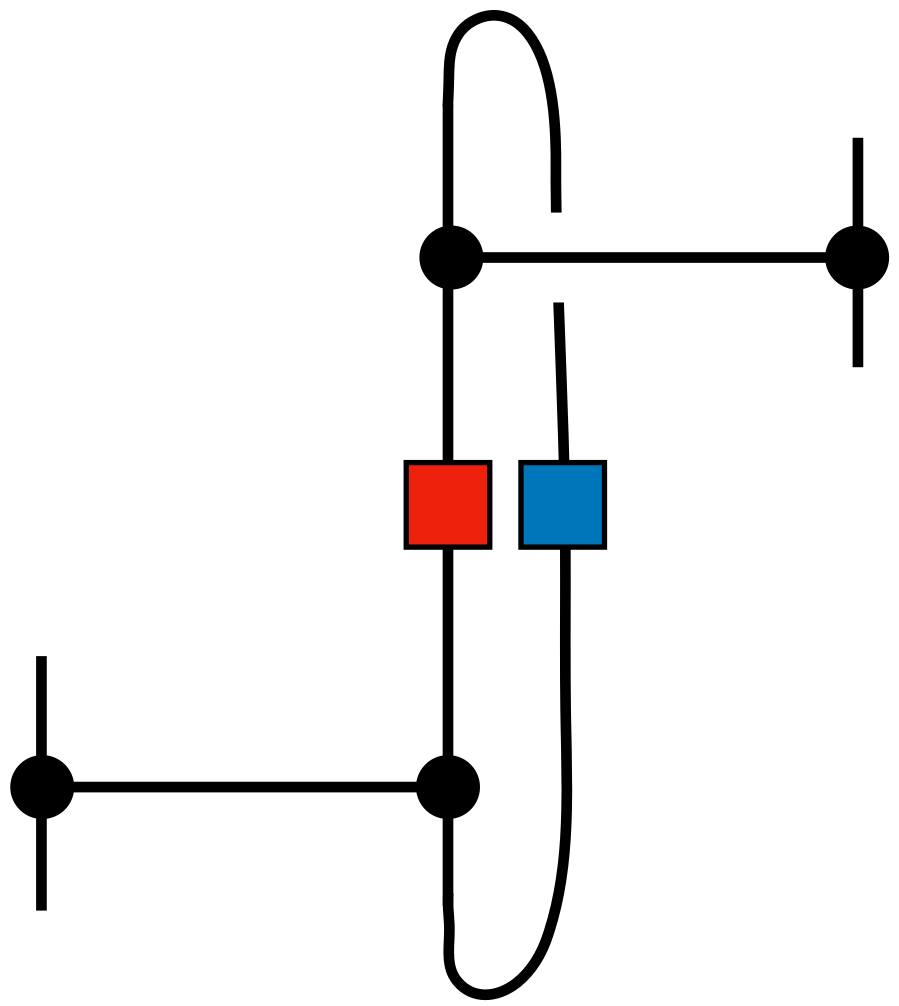


1. Left-right equivalence
2. Local conserved charge hosted in the centre
3. Near wall-boundaries one must have CZ-like gates

Multiplicity of walls in Haar sampling via loop diagrams

$$\mathbf{P}(1\text{-wall}) \approx 2\%$$

$\Rightarrow$

Infinite chain hosts many fragments with high probability

Fragmentation

Left/right invariant subspaces:

$$\mathcal{L} = A \otimes \{\mathbf{1}, \sigma_c\} \otimes \mathbf{1}_R$$

$$\mathcal{R} = \mathbf{1}_L \otimes \{\mathbf{1}, \sigma_c\} \otimes A$$

Fragments commute and are closed under multiplication

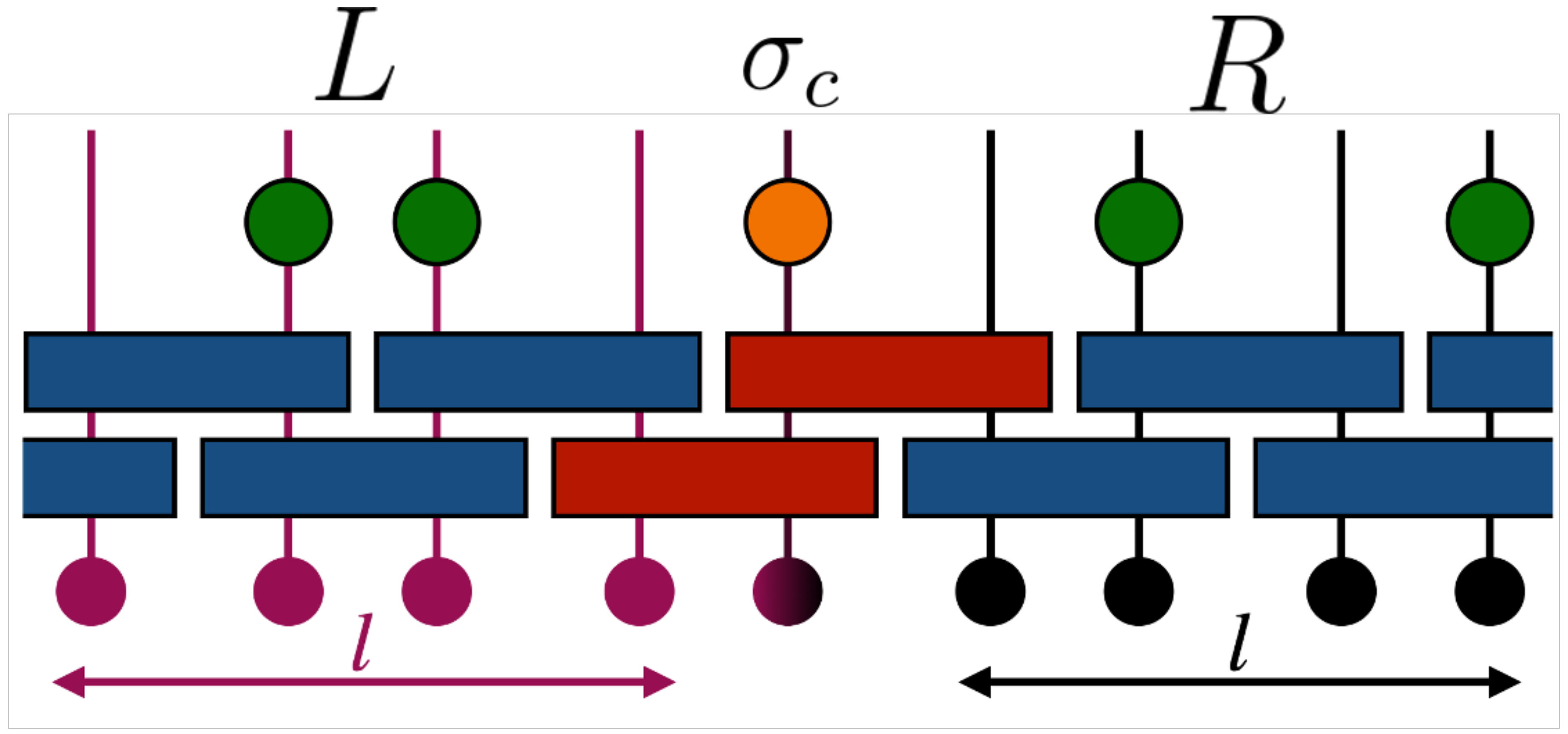
$$\Downarrow$$

No. fragments $N_f \sim n/\mu$ as $n \to \infty$

$$\Downarrow$$

Fragment space is exponential:

$$\dim \mathcal{F} \sim \exp\left(n/\mu\right)$$



Bulk perturbations preserve localisation

Edge perturbations destabilise walls

Exponential wall distribution in space:

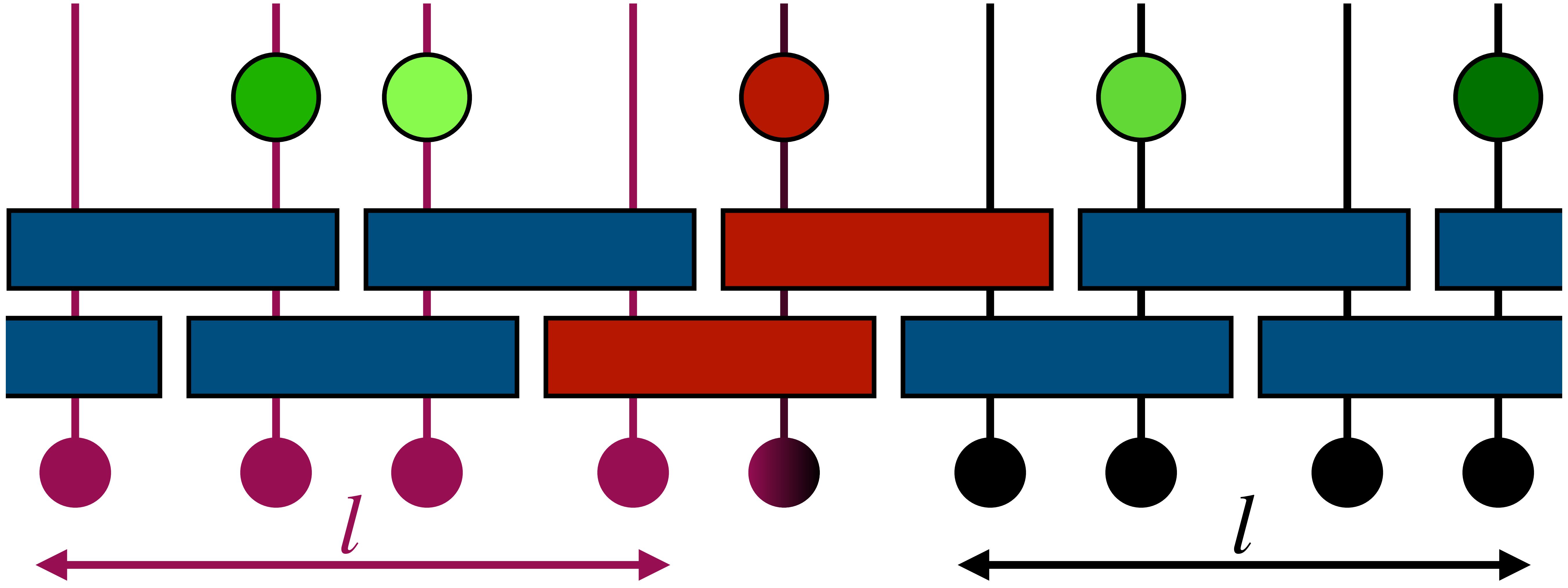
$$\mathbf{P}(x) \sim \exp\left(-x/\mu\right)$$

Tunable localisation length:

$$\mu = 1/|\log(1 - \mathbf{P}(\text{walls}))| \sim 44/(1-p)$$

Operator percolation transition as $p \to 1^-$

Numerical setup



1. **Transport:** no 1-walls in the circuit, perturbations everywhere with prob. p
2. **Perturbed wall:** random 1-wall perturbed, fragments perturbed with prob. p
3. **Localisation:** random 1-wall without perturbations, fragments perturbed with prob. p

Are fragments chaotic?

Are fragments stable?

Signatures of localisation?

Entanglement signature

Entropy for randomly sampled 1-wall

$$\rho(t) = \mathbf{Tr}_L[U^t(|0\rangle\langle 0|)^{\otimes n}U^{-t}]$$

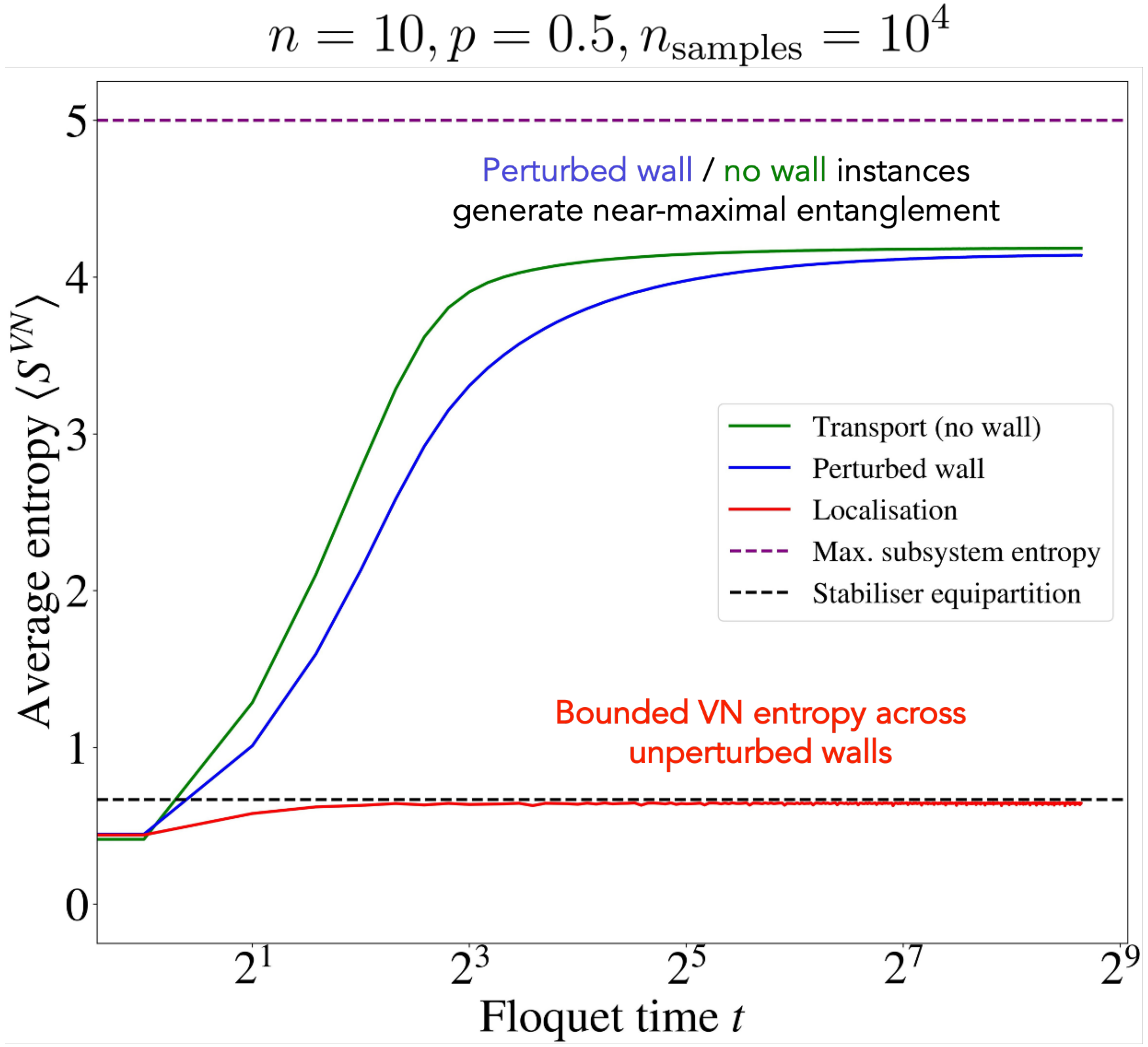
$$S^{\mathrm{VN}} = -\mathbf{Tr}[\rho \log_2 \rho]$$

Stabiliser rank decreases by at most unity across wall due to conserved central charge:

$$0 \leq S^{\mathrm{VN}} \leq 1$$

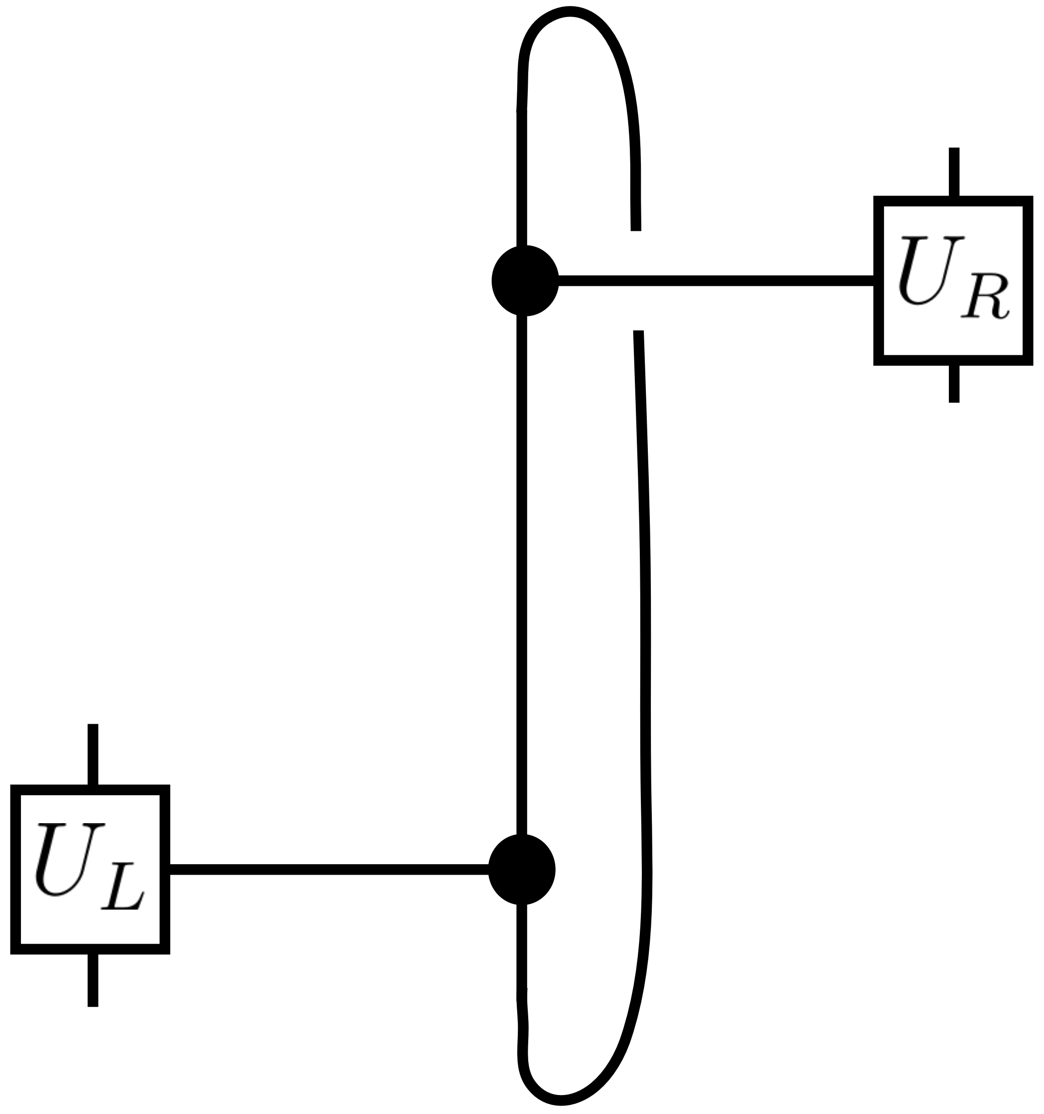
Perturbations generate stabilizer equipartition across disorder instances:

$$\langle S^{\mathrm{VN}} \rangle \leq 2/3$$

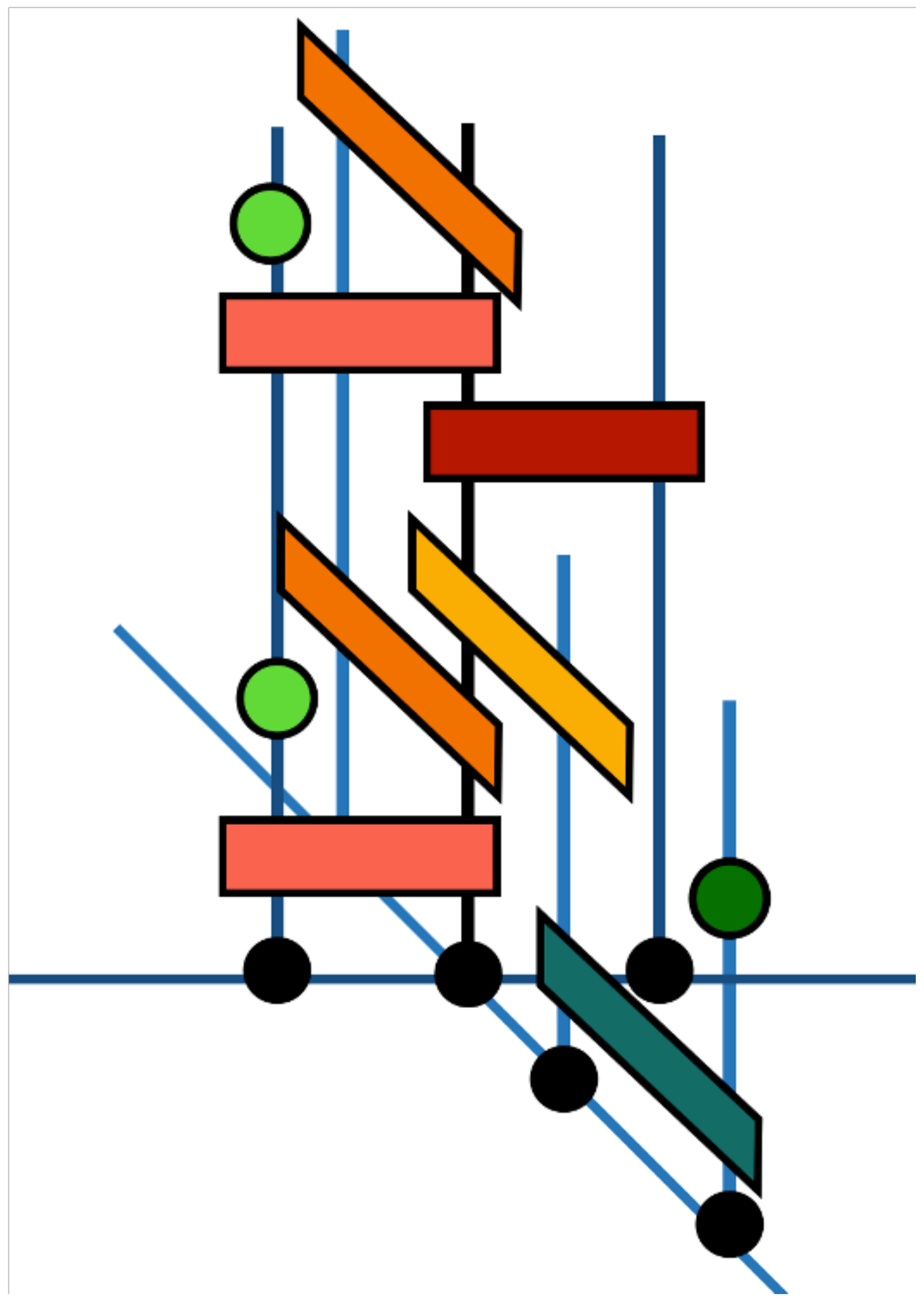


Further work

What is the form of general form of k-walls?



Qudits & higher dimensions?



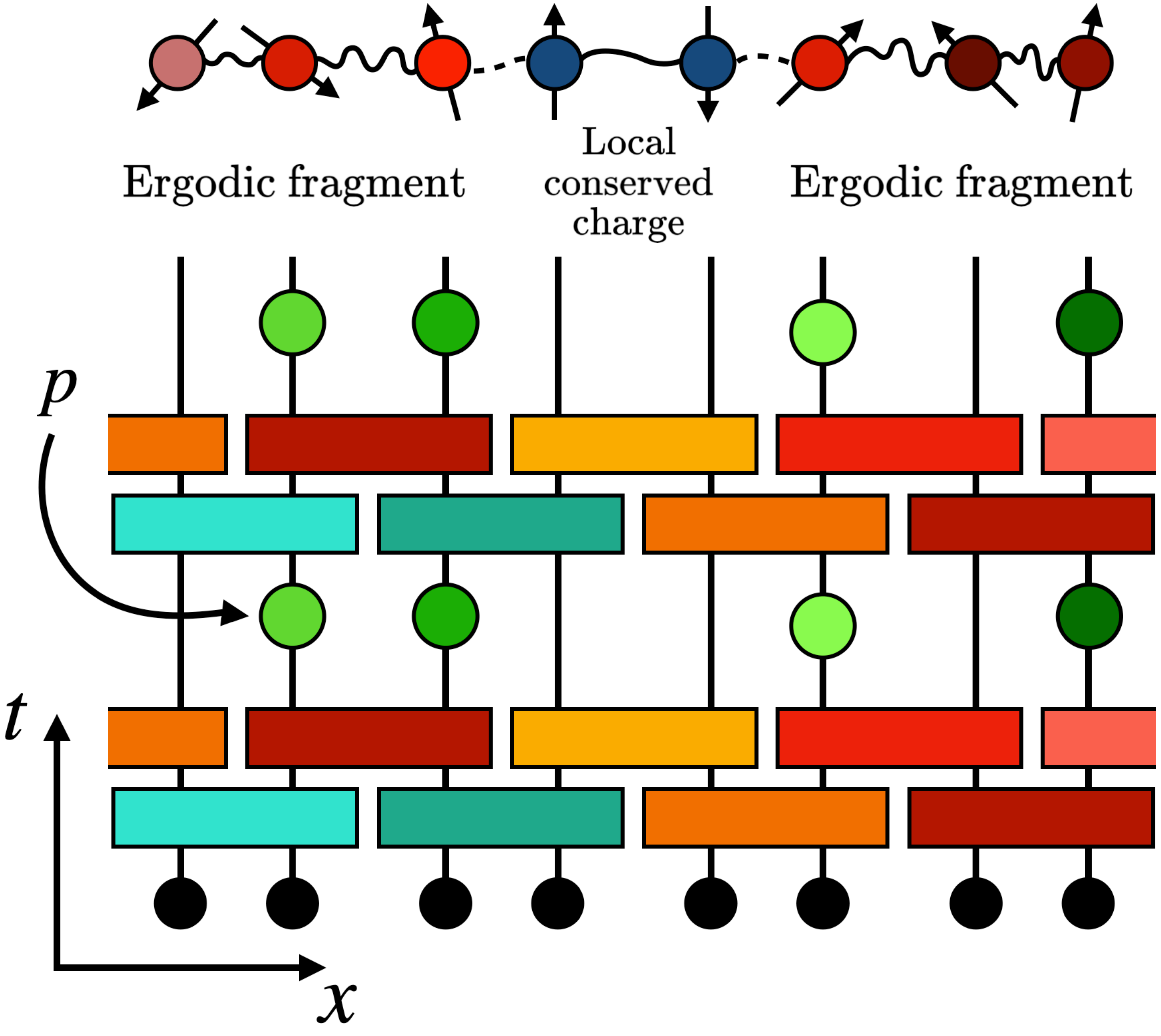
Conclusion

1. Robust non-ergodicity in the thermodynamic limit

2. Emergent symmetries fragment an interacting system

3. Localised regions are weakly entangled

4. Spectral signatures of chaos…

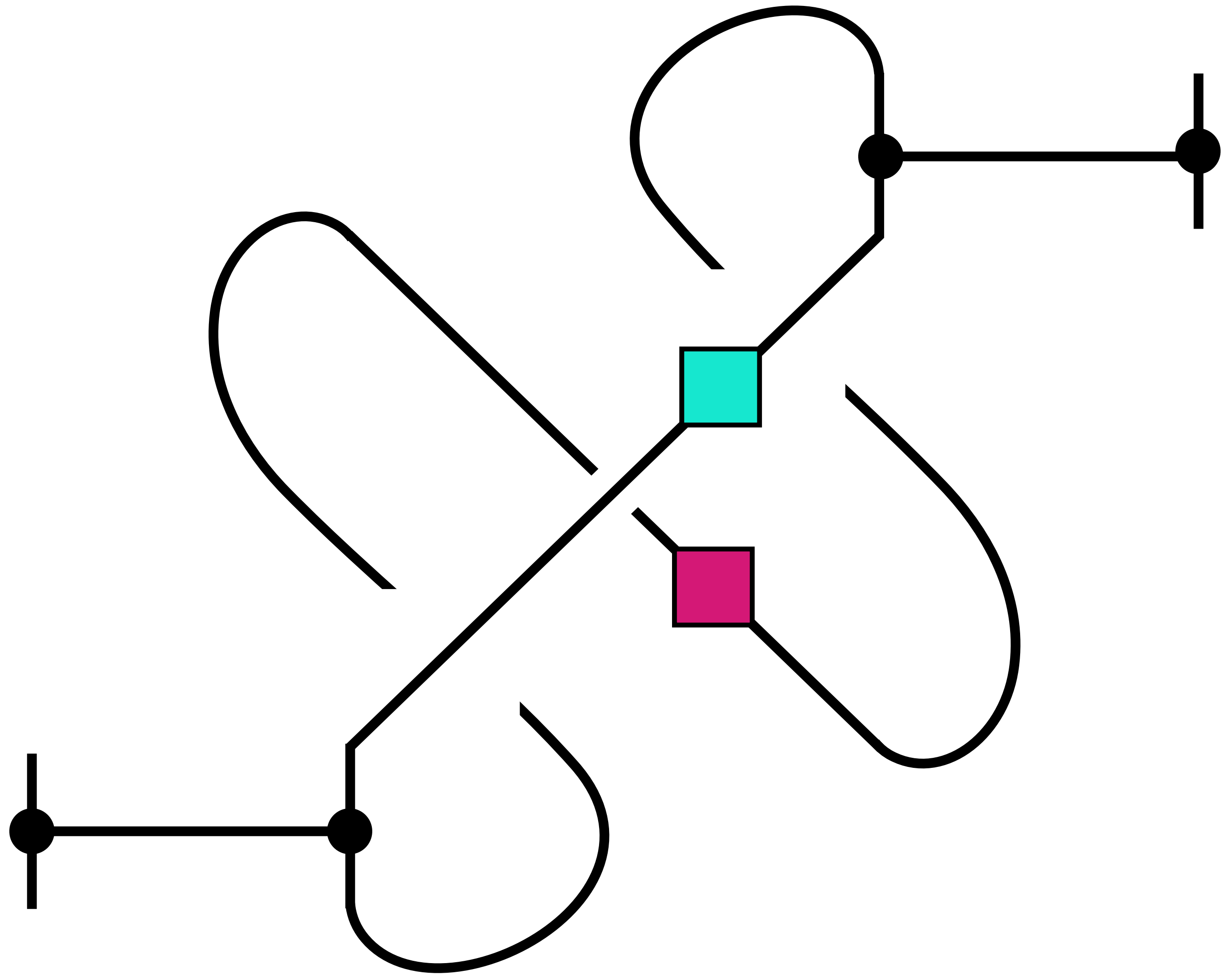


Thank you for your attention!

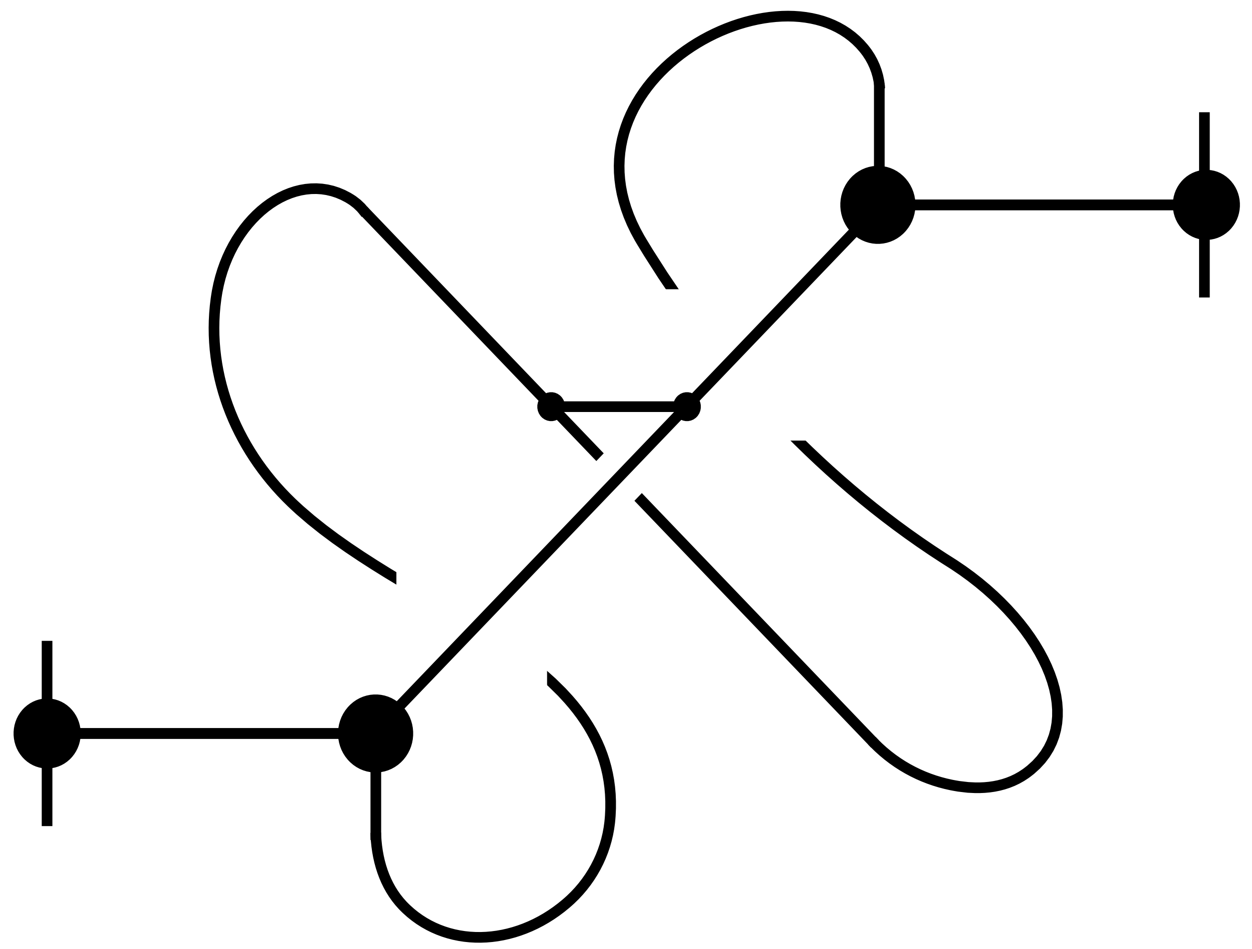
Additional results

1. Higher-order walls
2. Spectral probes of chaos
3. Restoring ergodicity and approximate Haar randomness

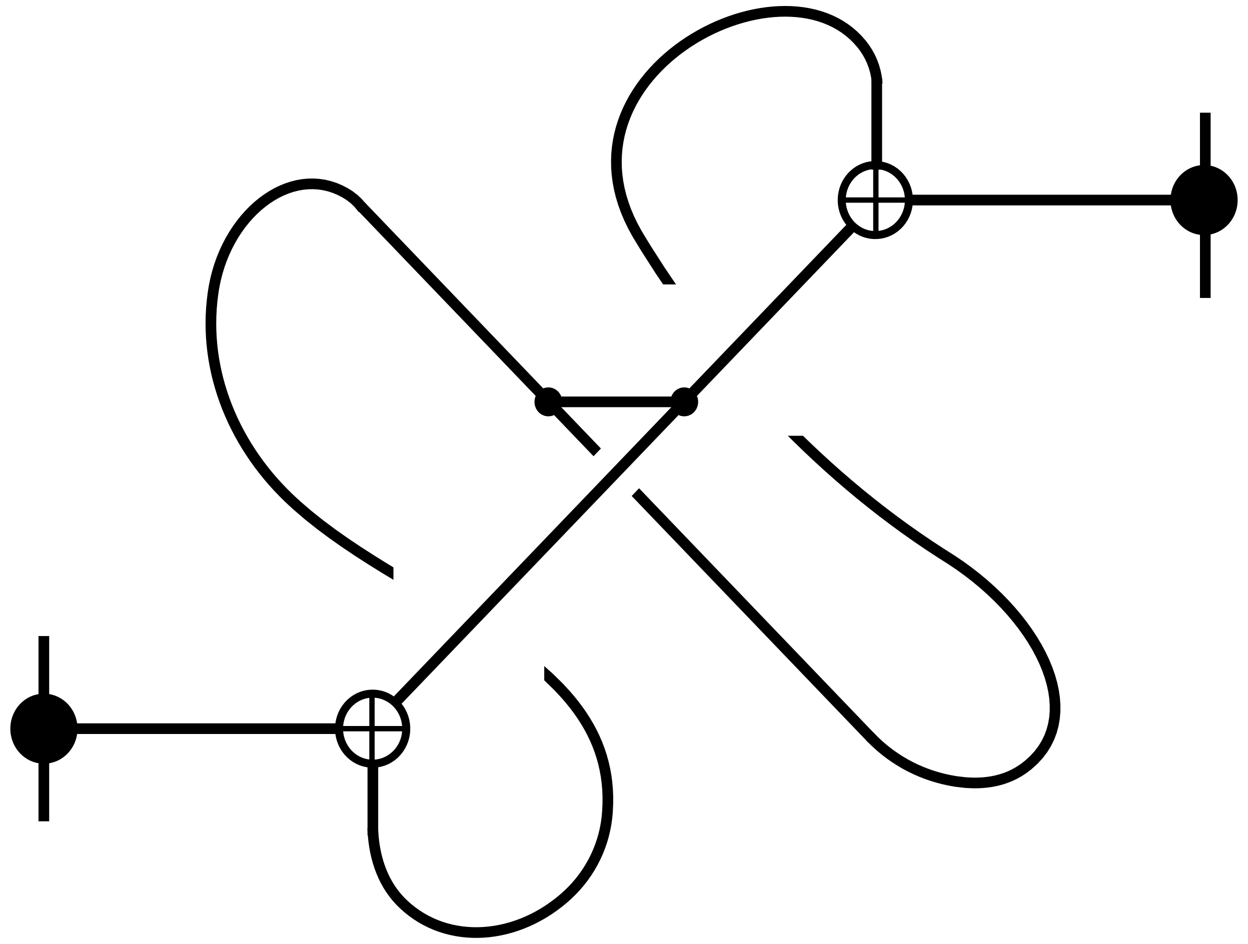
Higher-order walls



SWAP-like 2-walls have local charge oscillating in central space



Non-interfering FSWAP-like 2-walls host local charges



Interference permits localisation without conserved charges

Sampling long walls are exponentially suppressed:

$$\left(\frac{9}{19}\right)^2 \frac{1}{9}\left(\frac{6}{19}\right)^{k-1} \le \mathbf{P}(k\text{-wall}) \le \left(\frac{9}{19}\right)^2\left(\frac{10}{19}\right)^{k-1}$$

Spectral probes of chaos

Spectral form factor

$$K(t) = \langle |\mathbf{Tr} U^t|^2 \rangle = \left\langle \sum_{a,b=1}^{\dim \mathcal{H}} e^{i(\theta_a - \theta_b)t} \right\rangle_\theta = \left\langle \sum_P \mathbf{Tr}[U^t P U^{-t} P] \right\rangle_U$$

Probes level repulsion in quasi-energy window t

Ensemble average of Pauli auto-correlators

For Haar-ensemble, form factor is exactly:

$$K_{\text{Haar}}(t) = \begin{cases} D^2 & \text{if } t = 0 \\ t & \text{if } t \leq D \\ D & \text{if } t > D \end{cases}$$

'Dip'

'Ramp'

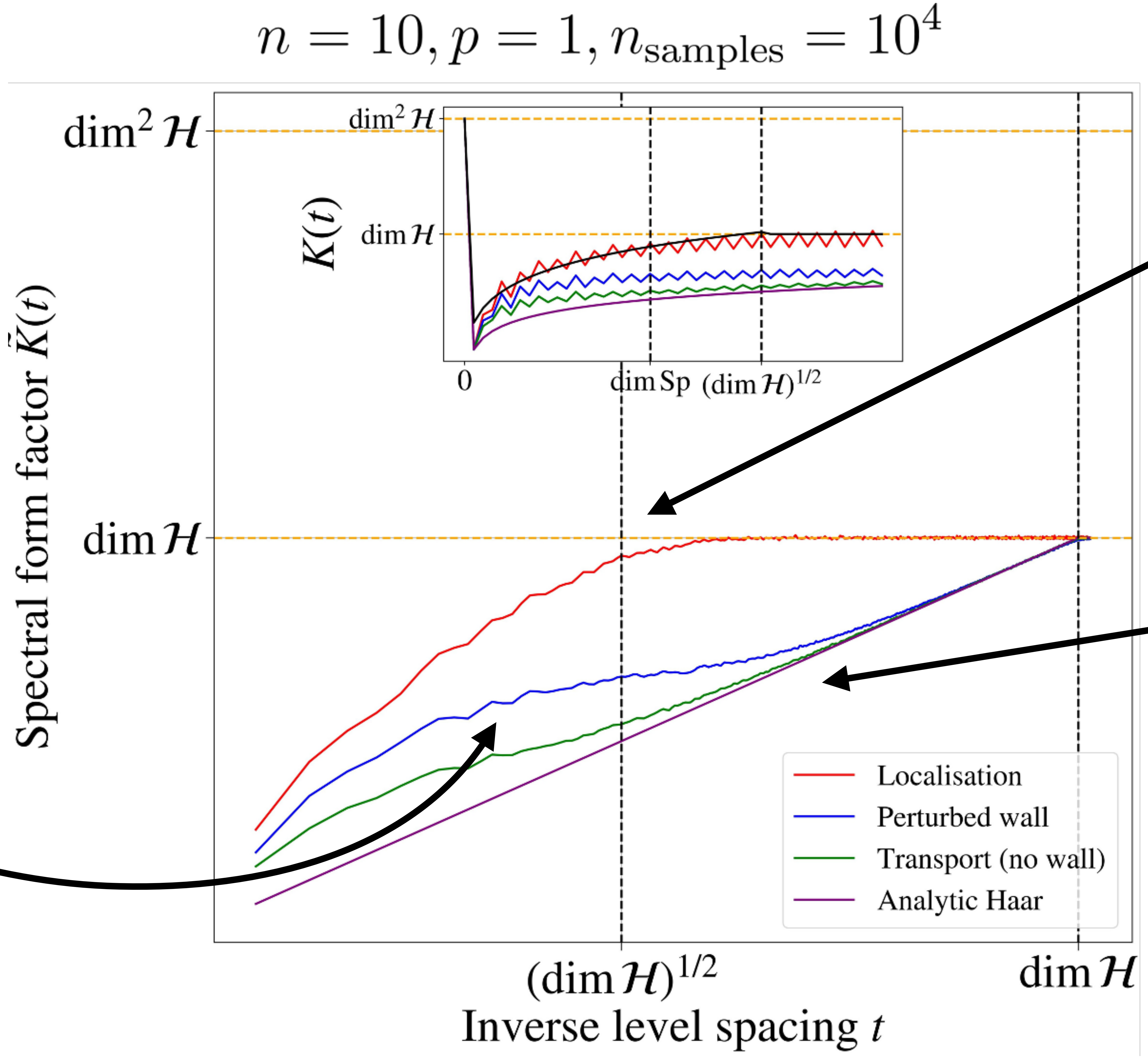
'Plateau'

$$D = \dim \mathcal{H}$$

Form factor respects Pauli fragmentation, hence for two Haar-fragments we expect:

$$K(t) \approx t^2 + \mathcal{O}(t)$$

Fragmentation and restoring ergodicity



Localised system has approximately quadratic ramp

Emergent (Thouless)-timescale beyond which level correlations are Haar-like

Perturbed walls lead to transient localisation