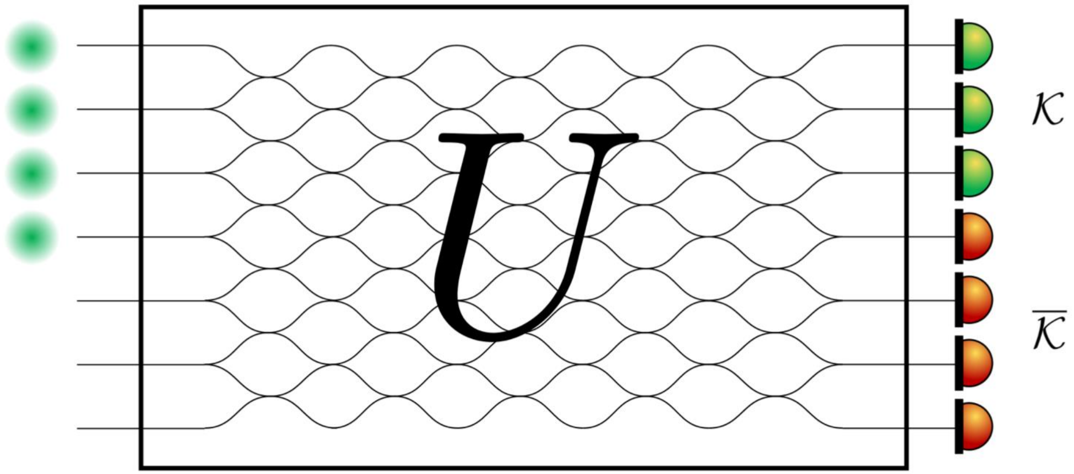
Anomalous bunching of nearly indistinguishable bosons

Léo Pioge, Benoit Seron, Leonardo Novo, Nicolas J. Cerf



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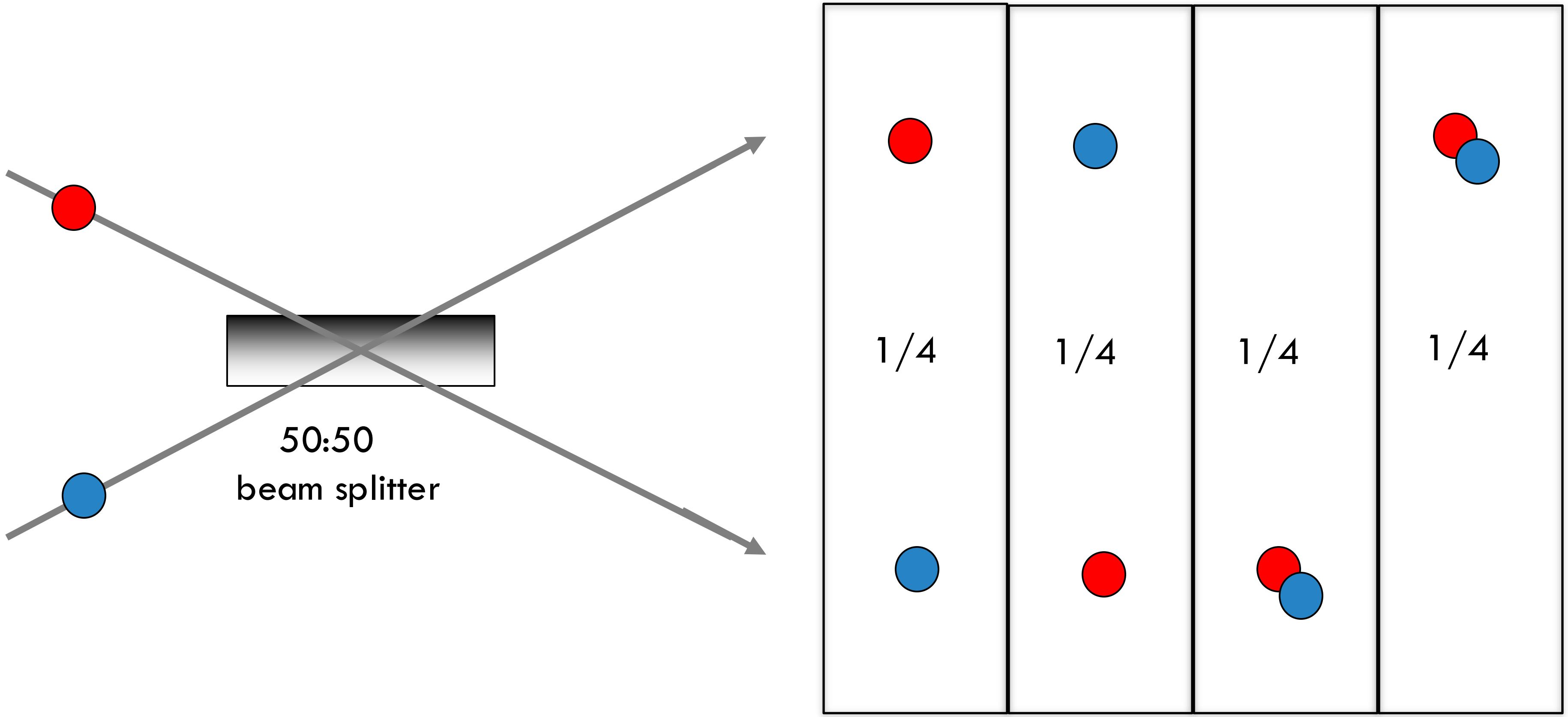
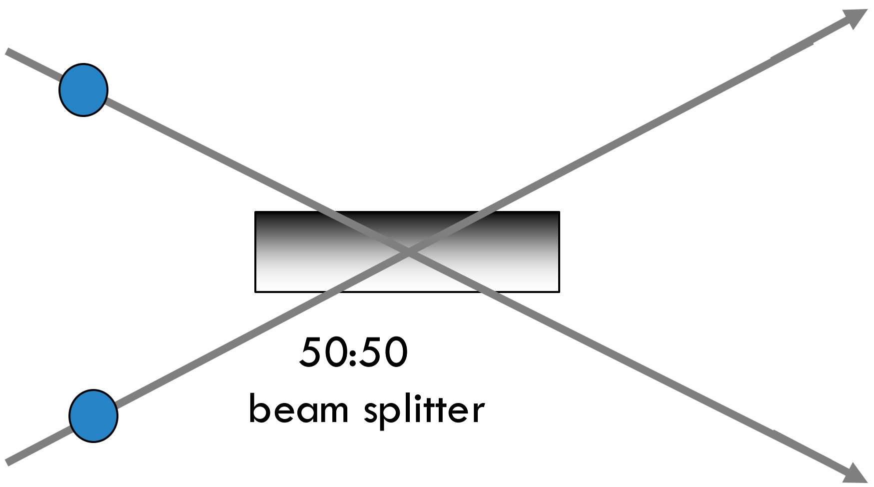
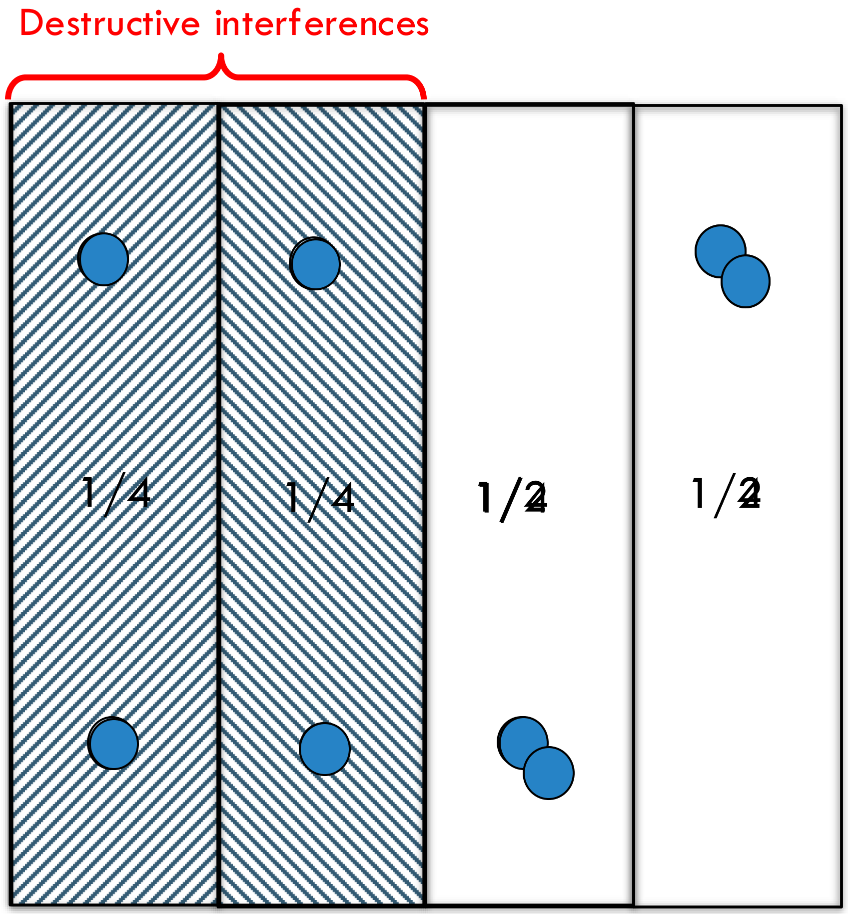
Hong-Ou-Mandel effect: distinguishable photons



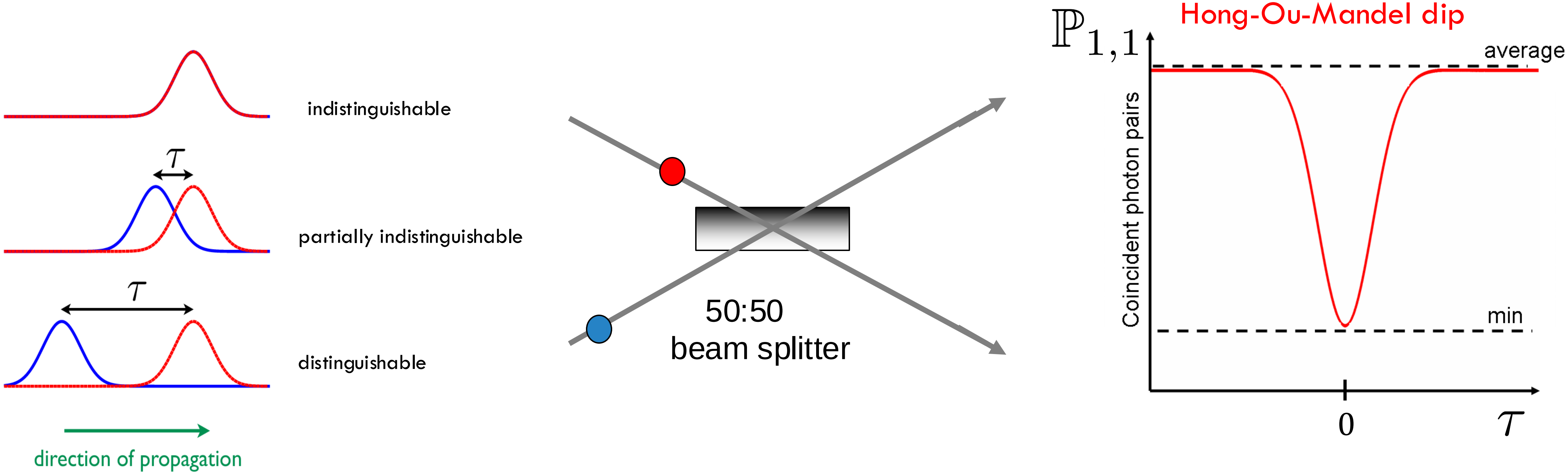
Hong-Ou-Mandel effect: indistinguishable photons





- Destructive interferences lead to **Boson Bunching**

Hong-Ou-Mandel effect: partially distinguishable photons

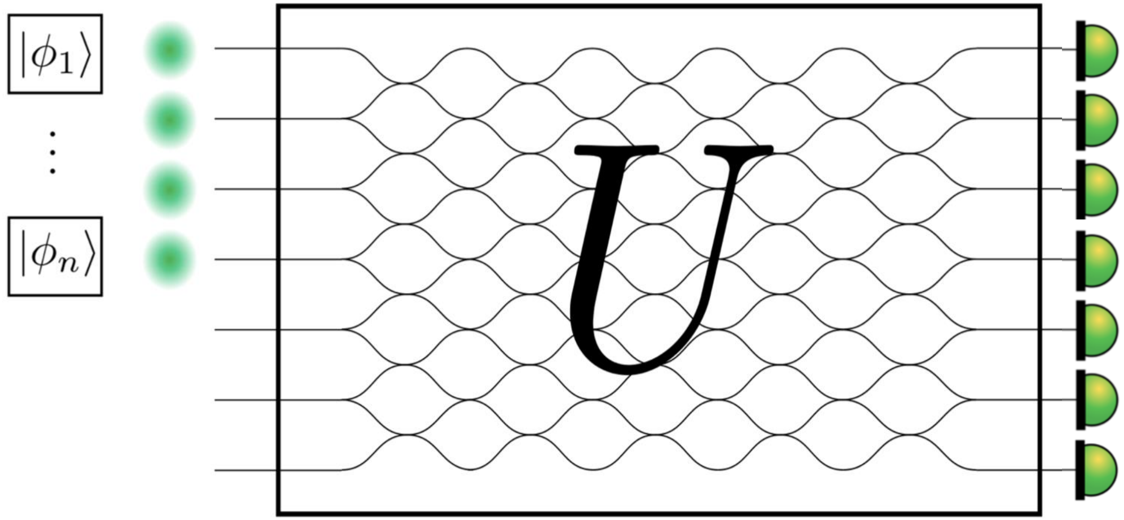


- The more photons are indistinguishable, the more they **bunch!**

Many-particle interferences

1. Input state: single photons
   - Pure, non-entangled

$$|\Psi\rangle_{in} = \prod_{j=1}^{n} \hat{a}^{\dagger}_{j,\phi_j} |0\rangle$$



2. Evolution of the quantum state

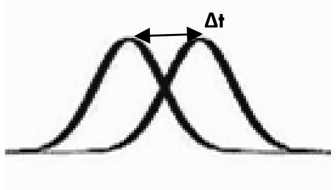
$$\hat{a}^{\dagger}_{j,\phi} \quad \rightarrow \quad \hat{U}\, \hat{a}^{\dagger}_{j,\phi}\, \hat{U}^{\dagger} = \sum_{k=1}^{m} U_{k,j}\, \hat{a}^{\dagger}_{k,\phi}, \quad \forall j, \forall \phi$$

3. Photon-number-resolving detectors

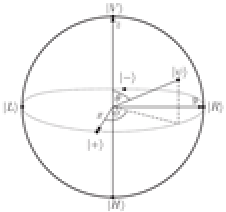
Distinguishability with many-particle

- Sources of the distinguishability:

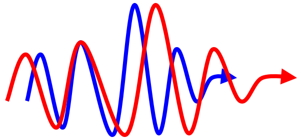
Time delay:



Polarization:



Frequency:



- Distinguishability matrix: $S_{i,j} = \langle \phi_i | \phi_j \rangle$
  - Three <u>in</u>distinguishable photons

$$S = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

  - Three distinguishable photons

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

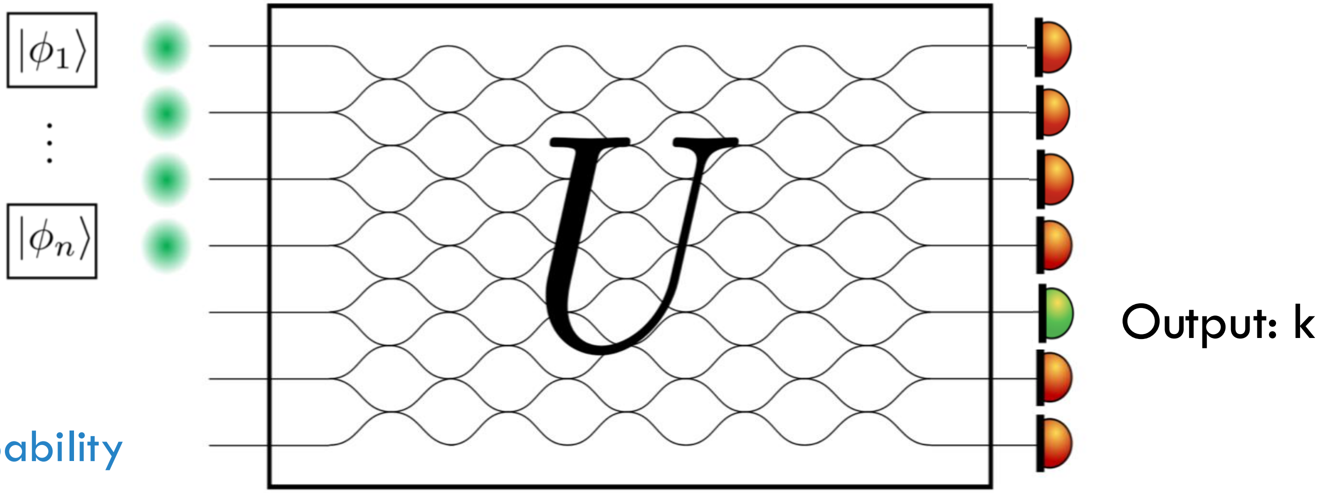
Single-mode bunching

- All photons are measured in a single output mode

- Single-mode bunching probability:
  - $$P_b = \text{perm}\left(S\right) \prod_{i=1}^{n} \left|U_{i,k}\right|^2$$

Bosonic enhancement

Classical bunching probability



  - $1 \leq \text{perm}\left(S\right) \leq n!$

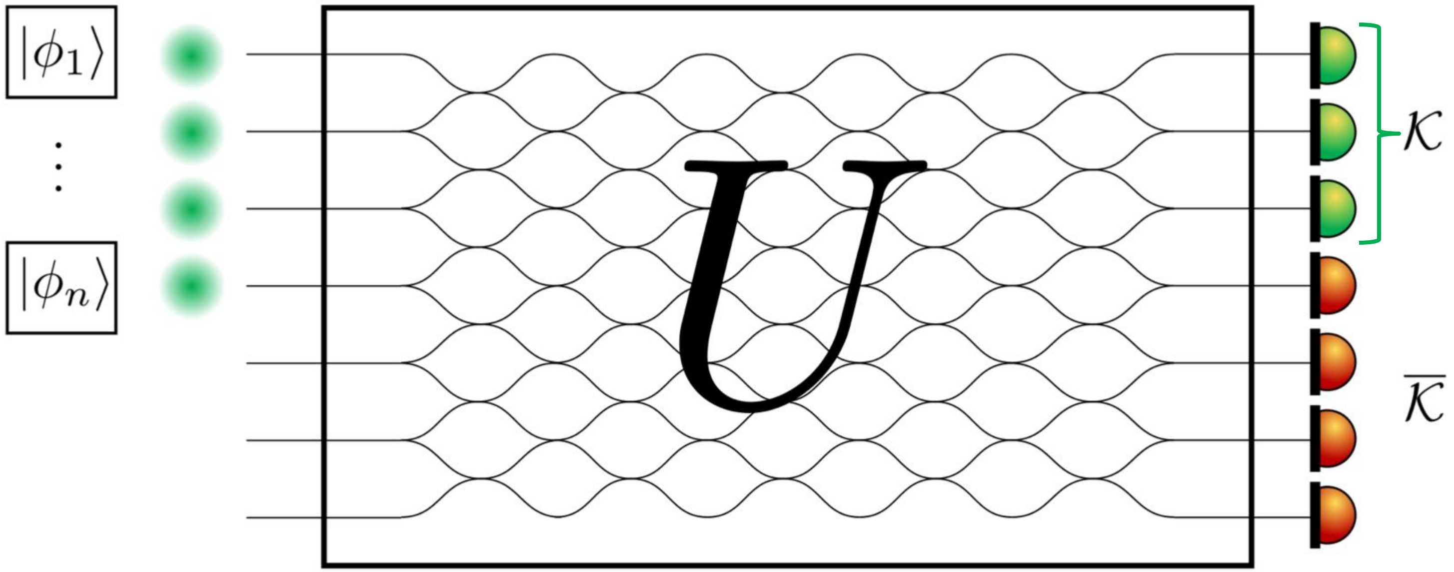
Distinguishable case

Indistinguishable case

- The more photons are indistinguishable, the more they **bunch!**

Multimode boson bunching

- Multimode bunching probability:
  - Probability that all photons are measured in a subset of the output spatial mode



- Which equation governs the multimode bunching probability?

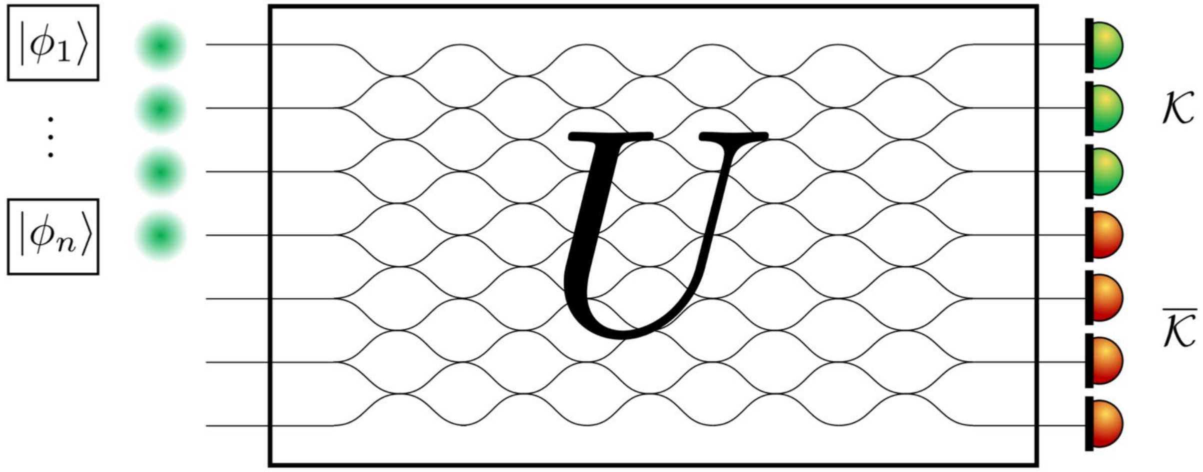
Multimode boson bunching

- Multimode bunching probability: $P_b = \mathrm{perm}\left(H \odot S\right)$

  - $H_{i,j} = \sum_{k \in \mathcal{K}} U^*_{k,i} U_{k,j}$

  - $S_{ij} = \langle \phi_i | \phi_j \rangle$

  - $(H \odot S)_{i,j} = H_{i,j}\, S_{i,j}$



- What is the impact of the distinguishability on the <u>multimode</u> bunching probability?

Do indistinguishable photons bunch more than partially distinguishable ones?

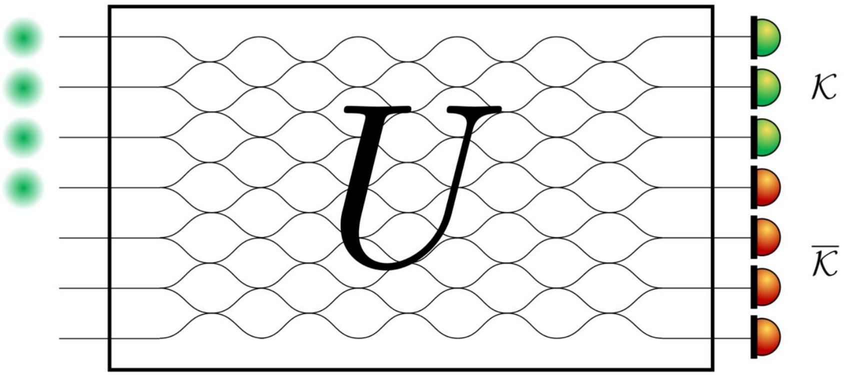
Physical conjecture P1: multimode bunching conjecture (Shchesnovich 2016)

For any interferometer and any subset of output mode,

the multimode bunching probability is maximized when the photons are indistinguishable.

$$P_b = \mathrm{perm}\,(H \odot S) \overset{?}{\leq} \mathrm{perm}\,(H)$$

- Evidences:
  - *Numerical evidence*
  - *Physical intuition*
  - *Valid for single-mode boson bunching*
  - *Valid for the fermionic equivalent of the boson bunching*
  - *Valid for 3 photons or less*
  - *Bapat-Sunder conjecture (1985)*



Mathematical conjecture M1

Mathematical conjecture M1: (Bapat-Sunder 1985)

Let A and B be $n \times n$ positive semi-definite Hermitian matrices, then

$$\mathrm{perm}\,(A \odot B) \leq \mathrm{perm}\,(A) \prod_{i=1}^{n} B_{ii}$$

$$\mathrm{perm}\,(A \odot B) \leq \mathrm{perm}\,(A) \prod_{i=1}^{n} B_{ii} \xrightarrow[]{\substack{A = H \\ B = S}} \mathrm{perm}\,(H \odot S) \leq \mathrm{perm}\,(H)$$

- Bapat-Sunder conjecture (1985) $\Longrightarrow$ Multimode bunching conjecture (2016)
- This physical conjecture is likely true!

Bapat-Sunder (1985) is false!

- In 2016, Stephen Drury found a 7 x 7 counterexample to Bapat-Sunder (1985)

$$A = \begin{pmatrix} 1 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 1 & \frac{1}{\sqrt{2}}e^{\frac{4}{5}i\pi} & \frac{1}{\sqrt{2}}e^{\frac{2}{5}i\pi} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{-\frac{2}{5}i\pi} & \frac{1}{\sqrt{2}}e^{-\frac{4}{5}i\pi} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{-\frac{4}{5}i\pi} & 1 & \cos\left(\frac{1}{5}\pi\right)e^{-\frac{1}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{-\frac{2}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{\frac{2}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{\frac{1}{5}i\pi} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{-\frac{2}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{\frac{1}{5}i\pi} & 1 & \cos\left(\frac{1}{5}\pi\right)e^{-\frac{1}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{-\frac{2}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{\frac{2}{5}i\pi} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \cos\left(\frac{2}{5}\pi\right)e^{\frac{2}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{\frac{1}{5}i\pi} & 1 & \cos\left(\frac{1}{5}\pi\right)e^{-\frac{1}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{-\frac{2}{5}i\pi} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{\frac{2}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{-\frac{2}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{\frac{2}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{\frac{1}{5}i\pi} & 1 & \cos\left(\frac{1}{5}\pi\right)e^{-\frac{1}{5}i\pi} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{\frac{4}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{-\frac{1}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{-\frac{2}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{\frac{2}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{\frac{1}{5}i\pi} & 1 \end{pmatrix}$$

- $$\frac{\operatorname{perm}\left(A \odot A^T\right)}{\operatorname{perm}\left(A\right)} = \frac{1237}{1152} > 1$$

- Can the counterexample to M1 be implemented in a physical setup?

Multimode bunching conjecture is false!

- **Distinguishability can enhance boson bunching! (anomalous bunching)**
- Seven photons counterexample to the multimode bunching conjecture

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Boson bunching is not maximized by indistinguishable particles

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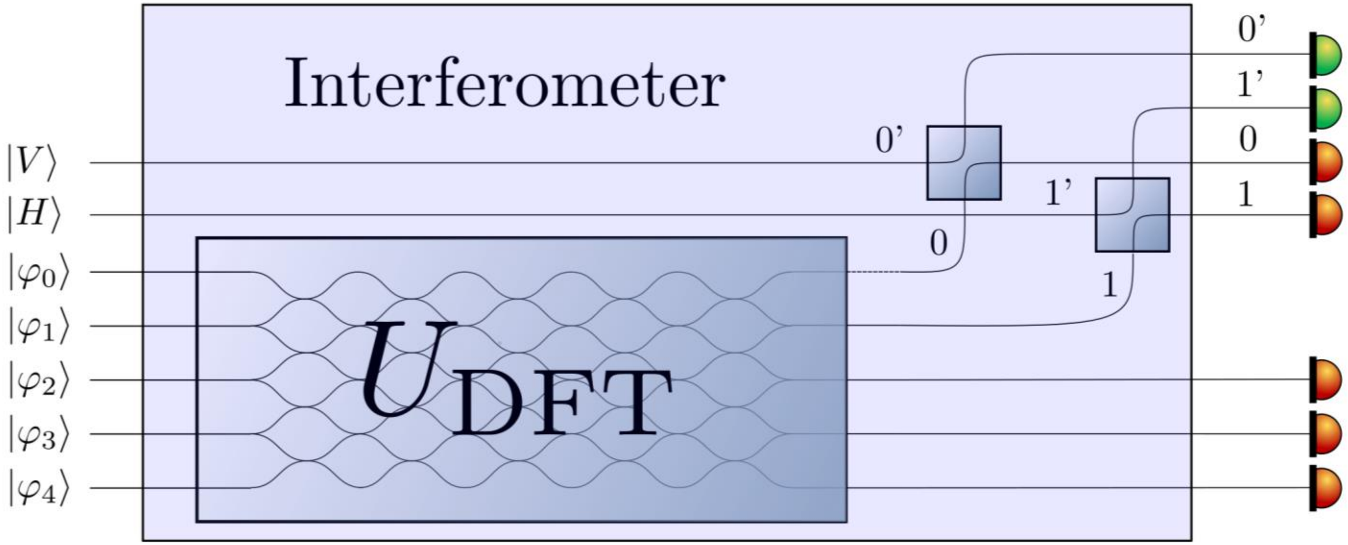
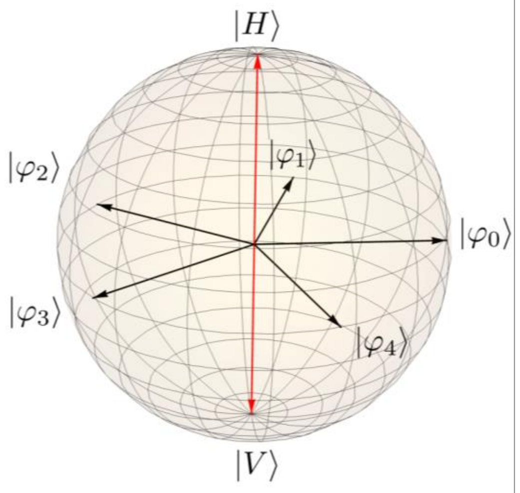


Photons' polarization states

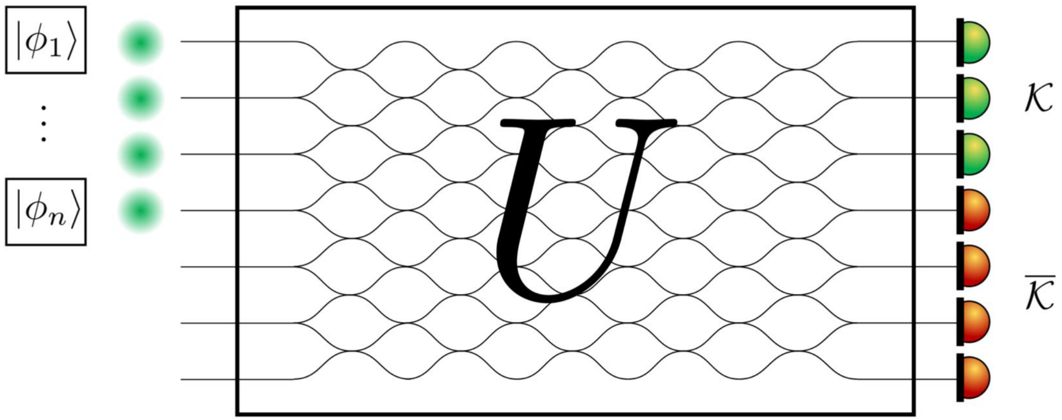


The arrows point in different directions

The special status of indistinguishable bosons

- Question: does the state of indistinguishable bosons still have a special status?
- Answer: **yes!** The bunching probability of indistinguishable bosons is stationary!

$$|\phi_i\rangle = \frac{1}{\alpha_i}\left(|\phi_0\rangle + \epsilon v_i\,|\eta_i\rangle\right)$$



- Bunching probability:

$$P_b(\epsilon) = \mathrm{perm}\left(H \odot S(\epsilon)\right)$$

- Stationarity, for every H:

$$\left.\frac{\partial P_b}{\partial \epsilon}\right|_0 = 0$$

Local version of the multimode bunching conjecture

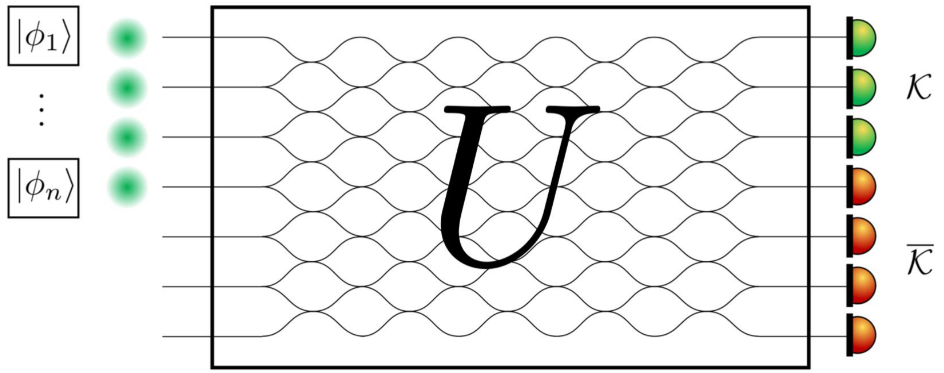
- Knowing the bunching probability is stationary,

Physical conjecture P2: a local version of the multimode bunching conjecture.

For any interferometer and any subset of output mode,
the state of indist. boson is a local maximum of the multimode bunching probability.

$$P_b(0) \geq P_b(\epsilon), \quad \epsilon \ll 1$$

- Evidences:
  - *Physical intuition*
  - *Bunching probability is stationary (indist. bosons)*
  - *The counterexample of Drury is "far" from indist. bosons*
  - *Bapat-Sunder conjecture (1986)*



Local version of the multimode bunching conjecture

- Let's consider the case of the polarization

$$|\phi_i\rangle = \frac{1}{\alpha_i}(|H\rangle + \epsilon\, v_i\,|V\rangle) \qquad S_{i,j} = \langle\phi_i|\phi_j\rangle = \frac{1+\epsilon^2 v_i^* v_j}{\sqrt{(1+\epsilon^2|v_i|^2)(1+\epsilon^2|v_j|^2)}}$$

- Perturbative calculation of the multimode bunching probability

$$P_b(\epsilon) = \underbrace{P_b(0)}_{\text{Indist. bosons}} + \epsilon^2 \underbrace{\left(\lambda_{max}(F) - \text{perm}\,(H)\right)}_{\overset{?}{\leq} 0} + O(\epsilon^4)\text{, with } F_{i,j} = H_{i,j}\text{perm}\,(H(i,j))$$

- The physical conjecture P2 is true if $\lambda_{max}(F) \leq \text{perm}\,(H)$ for every H

Mathematical conjecture M2:

Mathematical conjecture M2: (Bapat-Sunder 1986)

Let A be $n \times n$ positive semi-definite Hermitian matrix, then

$\lambda_{max}(F) = \mathrm{perm}\,(A)$, where $F_{i,j} = A_{i,j}\mathrm{perm}\,(A(i,j))$

$$P_b(\epsilon) = P_b(0) + \epsilon^2 \underbrace{\left(\boldsymbol{v}^\dagger F \boldsymbol{v} - \mathrm{perm}\,(H)\right)}_{\leq 0} + O(\epsilon^4)$$

- Local multimode bunching conjecture $\Longrightarrow$ Bapat-Sunder conjecture (1986)
- The physical conjecture P2 is likely true!

Bapat-Sunder (1986) is false!

- In 2018, Stephen Drury found a 8 x 8 counterexample to Bapat-Sunder (1986)

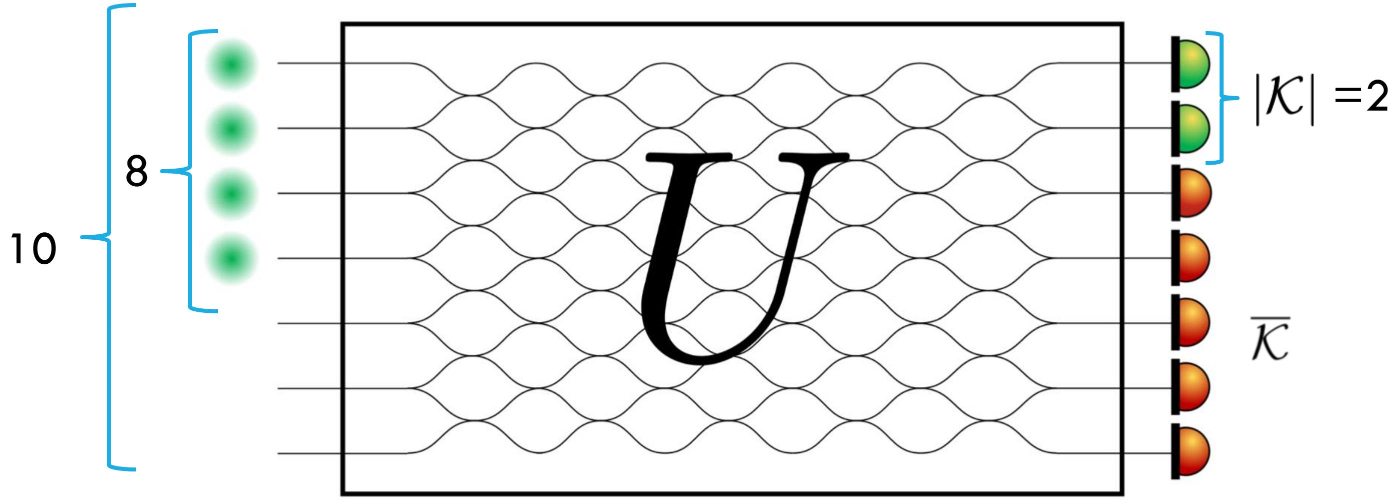
$$A = \begin{pmatrix} 106 & -91+6i & 28-38i & -53-59i & -1-81i & -15i & 66+9i & -48+29i \\ -91-6i & 107 & -76+30i & 24+77i & 30+67i & 17-36i & -19-29i & 58-64i \\ 28+38i & -76-30i & 108 & 38-76i & -30-16i & -24+82i & -50+58i & -48+64i \\ -53+59i & 24-77i & 38+76i & 90 & 22+14i & -43+24i & -76+13i & -36-27i \\ -1+81i & 30-67i & -30+16i & 22-14i & 102 & 37-56i & 38+33i & -85i \\ 15i & 17+36i & -24-82i & -43-24i & 37+56i & 97 & 52+62i & 77-7i \\ 66-9i & -19+29i & -50-58i & -76-13i & 38-33i & 52-62i & 101 & 18-23i \\ -48-29i & 58+64i & -48-64i & -36+27i & 85i & 77+7i & 18+23i & 99 \end{pmatrix}$$

$$\left.\begin{aligned} \lambda_{max}(F) &= 30280801509187248 11 \\ \text{perm}\,(A) &= 2977257622144118400 \end{aligned}\right\} \quad \frac{\lambda_{\max}(F)}{\text{perm}\,(A)} \approx 1.017$$

- Can the counterexample to M2 be implemented in a physical setup?

Physical conjecture P2 is false!

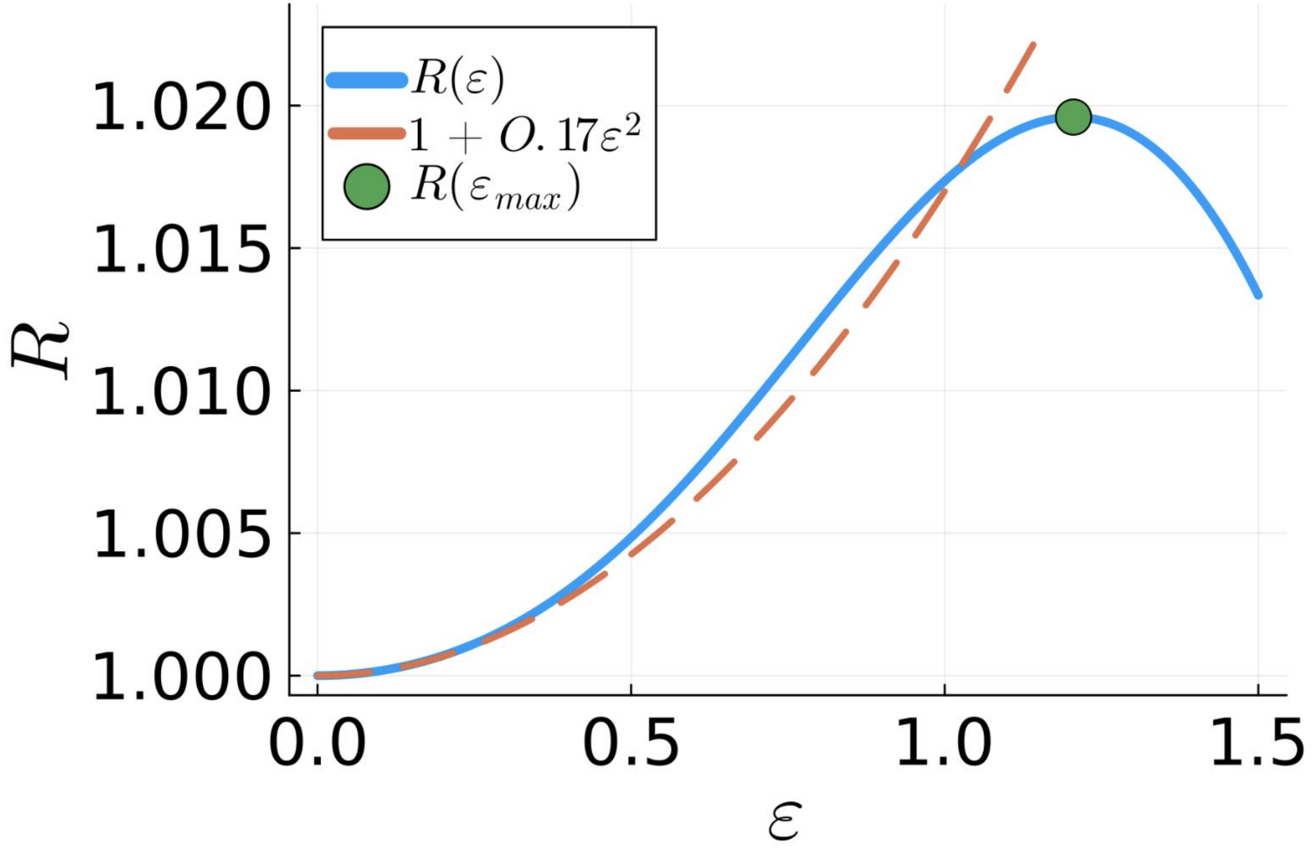
- Counterexample to P2:
  - 10 modes
  - 8 photons
  - Bunching in 2 output modes



- Internal state:
  - $|\phi_i\rangle = \frac{1}{\alpha_i}(|H\rangle + \epsilon\, (\boldsymbol{v}_{\max})_i\, |V\rangle)$
    - where $F\boldsymbol{v}_{max} = \lambda_{max}\boldsymbol{v}_{max}$, and $\lambda_{max} > \mathrm{perm}\,(H)$

- Interferometer:
  - We converted the Drury matrix into a H matrix and an interferometer

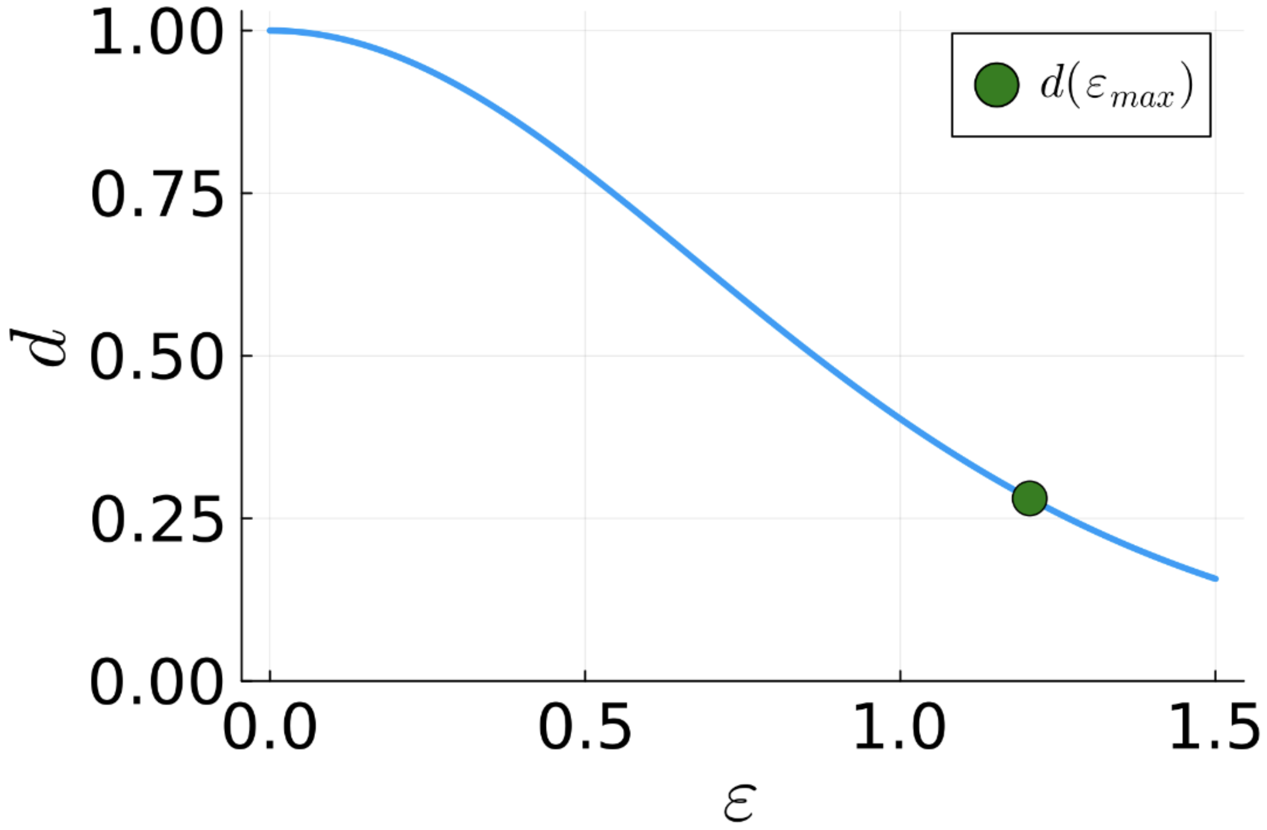
Violation ratio of the counterexample



$$R(\epsilon) = \frac{\mathrm{perm}\,(H \odot S(\epsilon))}{\mathrm{perm}\,(H)}$$
$$\approx 1 + \epsilon^2 \left(\frac{\lambda_{\max}(F)}{\mathrm{perm}\,(H)} - 1 \right)$$
$$\approx 1 + 0.017\,\epsilon^2$$

- Anomalous bunching with nearly indistinguishable photons!

Indistinguishability of the counterexample



- Measure of indistinguishability

$$d(\epsilon) = \frac{\mathrm{perm}\left(S(\epsilon)\right)}{n!}$$

- The less photons are indistinguishable, the more they **bunch!**

Summary of the conjectures

Conjecture P1

The bunching probability $P_b(S)$ has a <u>global maximum</u> for indistinguishable bosons (for all separable input states)

$\Longrightarrow$

Conjecture P2

The bunching probability $P_b(S)$ has a <u>local maximum</u> for indistinguishable bosons (for all separable input states)

$\Updownarrow$ (Conjecture P1 ⟺ Conjecture M1)

$\Downarrow$ (Conjecture P2 ⟹ Conjecture M2)

Conjecture M1

$$\mathrm{perm}(A \odot B) \leq \mathrm{perm}(A) \prod_{i=1}^{n} B_{ii}$$

$\Longrightarrow$

Conjecture M2

$$\lambda_{\max}(F) = \mathrm{perm}(A)$$

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arXiv > quant-ph > arXiv:2308.12226

Quantum Physics

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Anomalous bunching of nearly indistinguishable bosons

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How to increase/decrease the bunching?

- General perturbation:

$$|\phi_i\rangle = \frac{1}{\alpha_i}(|\phi_0\rangle + \epsilon v_i |\eta_i\rangle)$$

- Restriction:

$$\langle\phi_0|\eta_i\rangle = 0, \ \forall i$$

- Which $|\eta_i\rangle$ and $\boldsymbol{v}$ maximally increase the bunching?

$$|\phi_i\rangle = \frac{1}{\alpha_i}(|H\rangle + \epsilon\ (\boldsymbol{v}_{\max})_i\ |V\rangle)\ , \quad \text{with} \quad F\boldsymbol{v}_{max} = \lambda_{max}\boldsymbol{v}_{max}$$

- Which $|\eta_i\rangle$ and $\boldsymbol{v}$ maximally decrease the bunching?

$$|\phi_i\rangle = \frac{1}{\alpha_i}(|H\rangle + \epsilon\ (\boldsymbol{v}_{\min})_i\ |V\rangle), \quad \text{with} \quad F\boldsymbol{v}_{min} = \lambda_{min}\boldsymbol{v}_{min}$$