// YQIS – Paris INRIA

Early Fault-Tolerant Quantum Algorithms in Practice

Oriel **Kiss**,

November 6th, 2024

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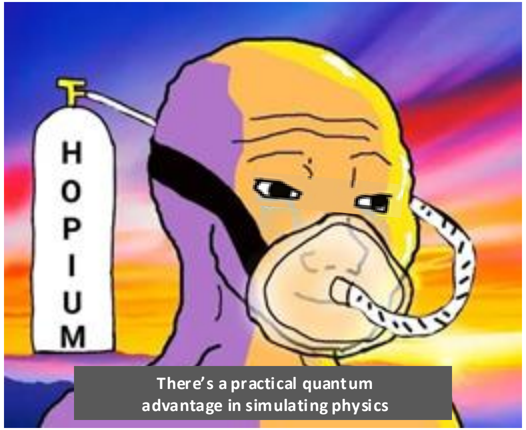

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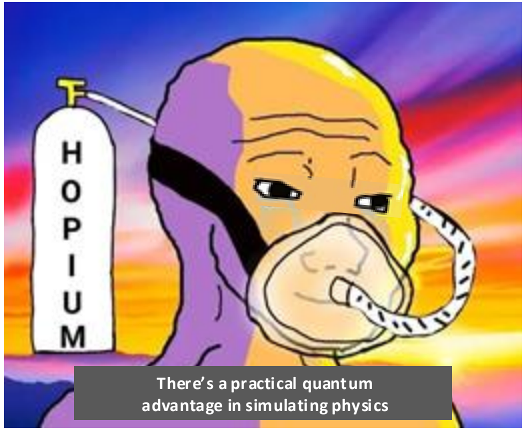

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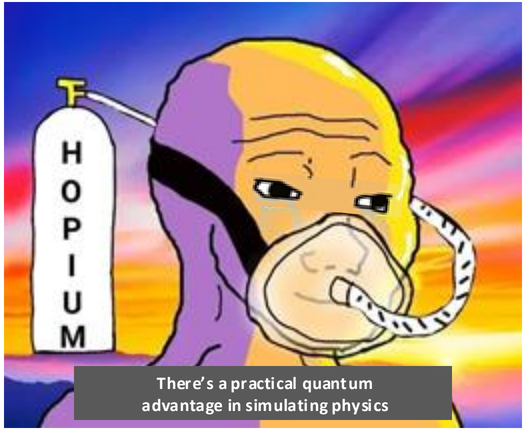

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Variational Algorithms (VQE, QAOA)

No theoretical arguments for quantum advantage

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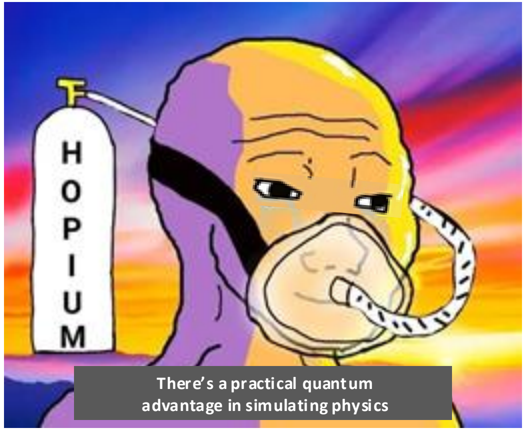

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Fully-FTQC

Phase Estimation
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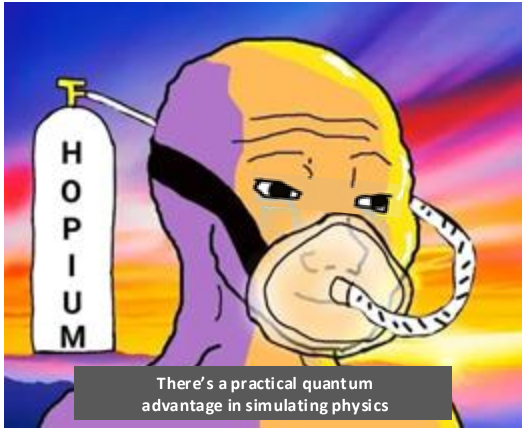

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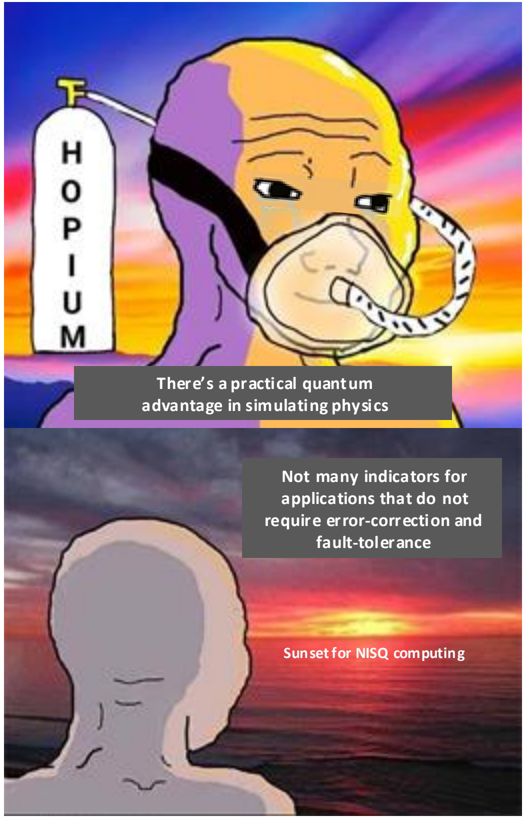

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1. Early fault-tolerant algorithms for GSEE: review of the Lin&Tong algorithm.
2. Applying the Lin&Tong algorithm in practice.
3. Numerical simulations.
4. Bonus: application on NISQ devices.

// Early-FTQC Algorithms for GSEE

Find the lowest eigenvalue of a **Hamiltonian H** describing a system with some error ϵ,

$$\tau H = \sum_k \tau_k |E_k\rangle\langle E_k| \text{ with } \|\tau H\| < \pi/2$$

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3. Some degree of robustness against algorithmic errors.

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Problem 1. *Given a precision $\delta > 0$ and lower bound on the overlap parameter $\eta > 0$, we seek to decide if*

$$Tr[\rho\Pi_{\leq x-\delta}] < \eta \quad or \quad Tr[\rho\Pi_{\leq x+\delta}] > 0. \tag{3}$$

Lin and Tong PRX Quantum 3, 010318 (2022) , Wan et al. Phys. Rev. Lett. 129, 030503 (2022)

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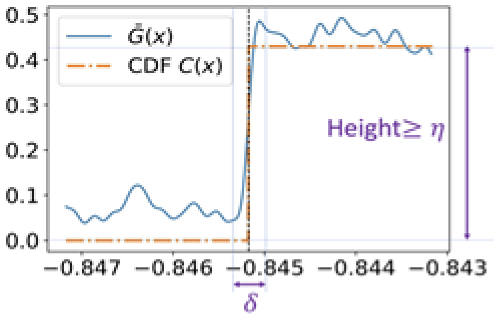

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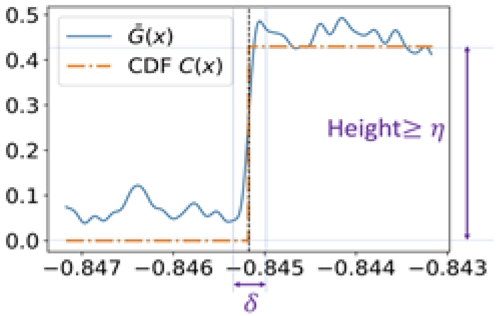

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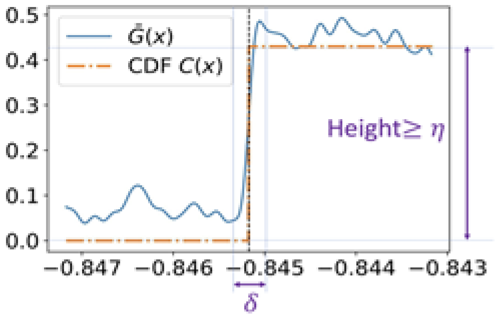

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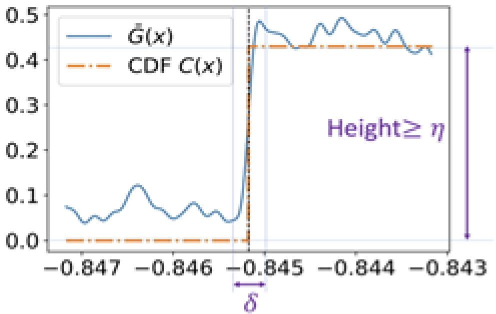

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$$\tilde{C}(x) = \int_{-\pi/2}^{\pi/2} p(y)F(x-y)dy$$
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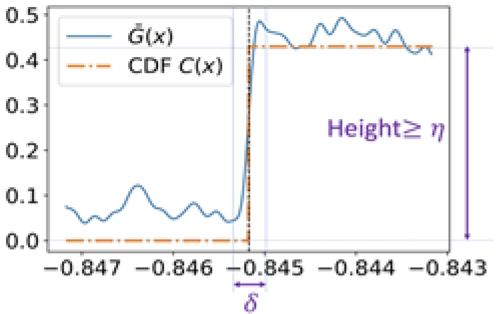

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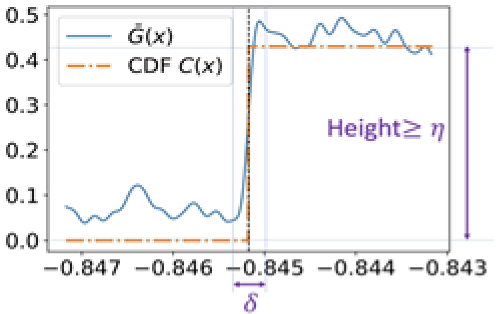

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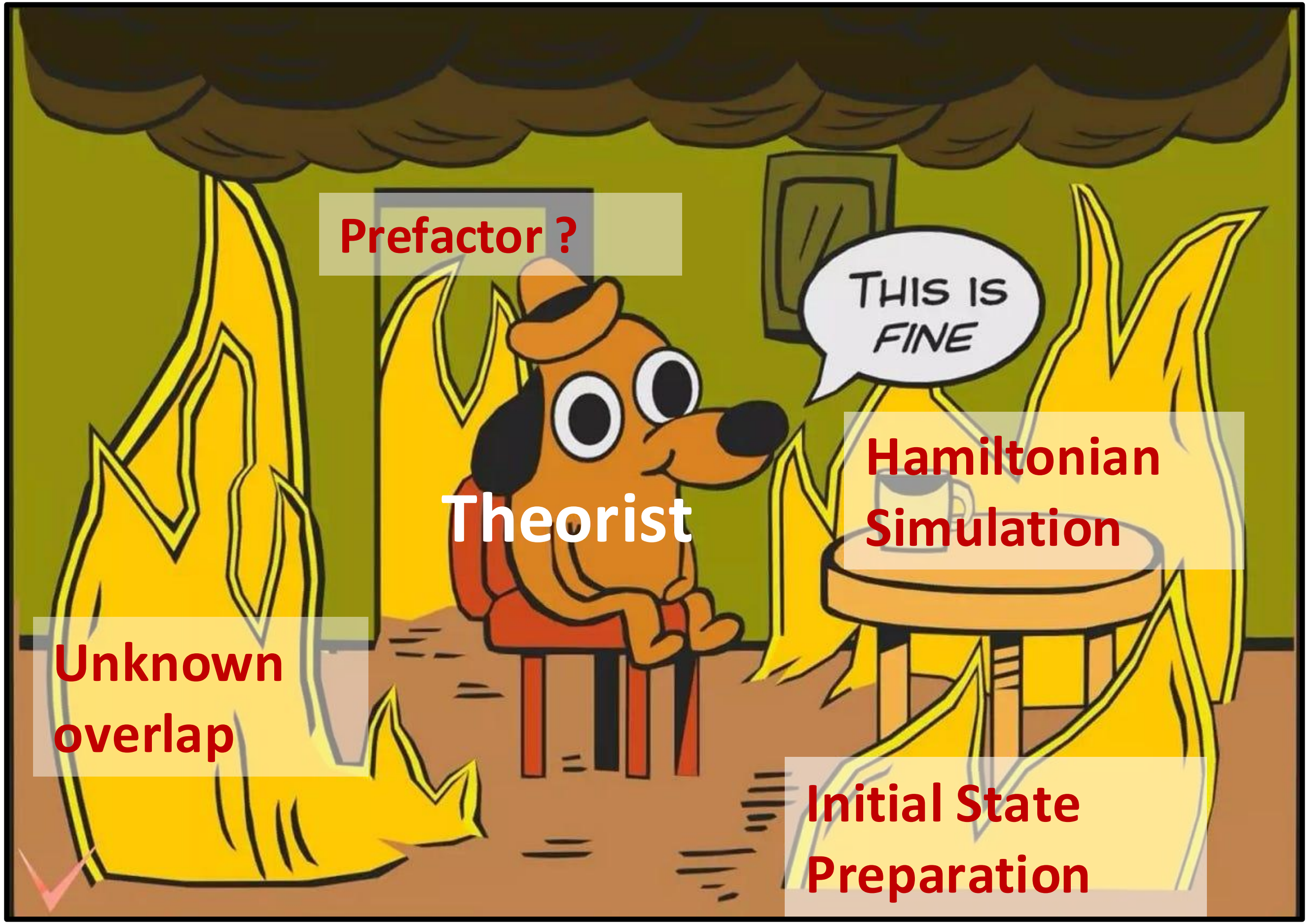
$$G(x) = \frac{1}{2} + \frac{2\mathcal{F}}{M} \sum_{i=1}^{M} \Big[\mathrm{Re}[g_{k_i}(\tau)] \sin(k_i x) + \mathrm{Im}[g_{k_i}(\tau)] \cos(k_i x) \Big].$$



Prefactor ?
THIS IS FINE
Theorist
Hamiltonian Simulation
Unknown overlap
Initial State Preparation

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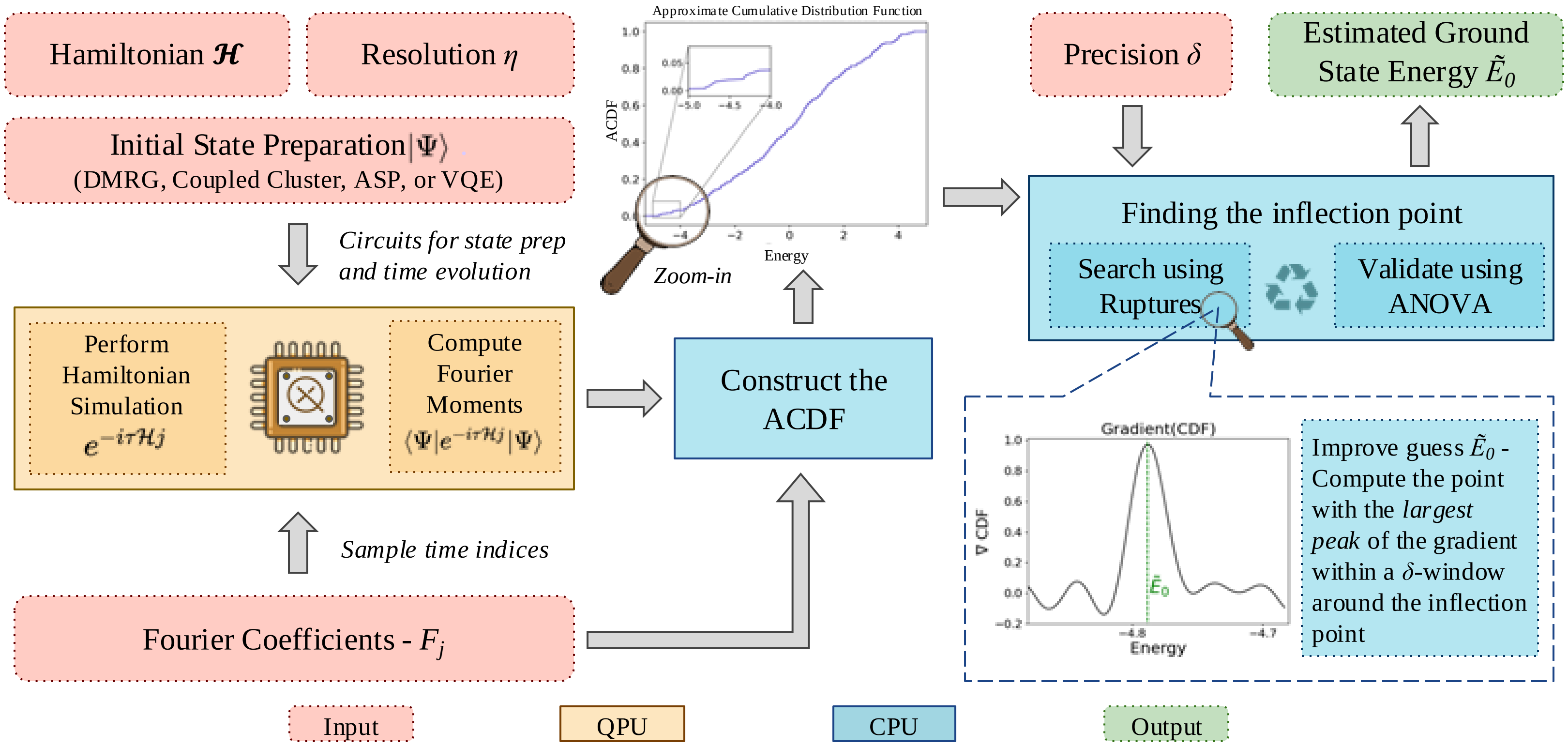
- Hamiltonian Simulation: system-specific; we use (asymptotical sub-optimal) product formula because:

1. no ancilla overhead
2. take advantage of locality
3. often much better than what is guaranteed
4. can scale better than qubitization in some regime

// Our Workflow



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CDF smoothes out with the system size (random initial state)

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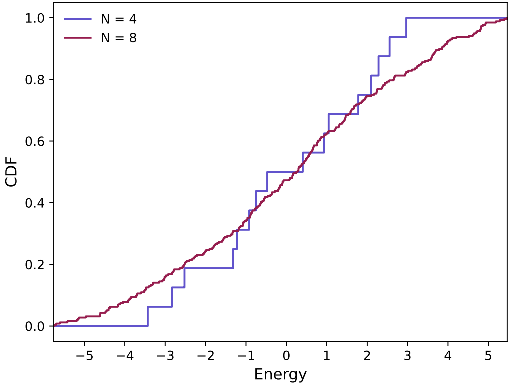

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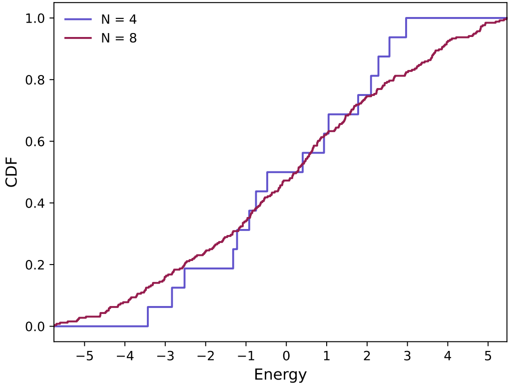

XXZ model

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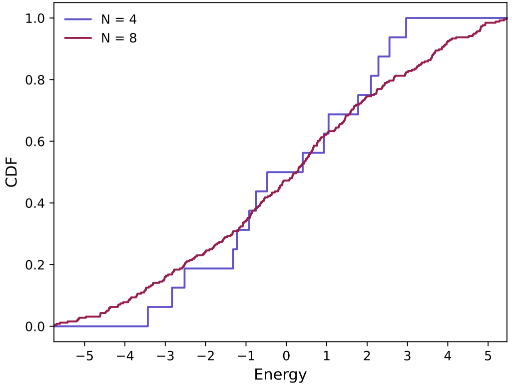

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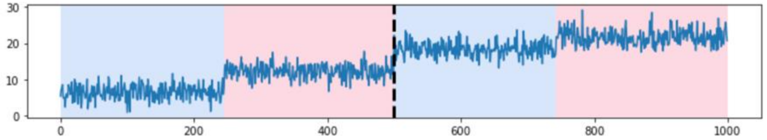

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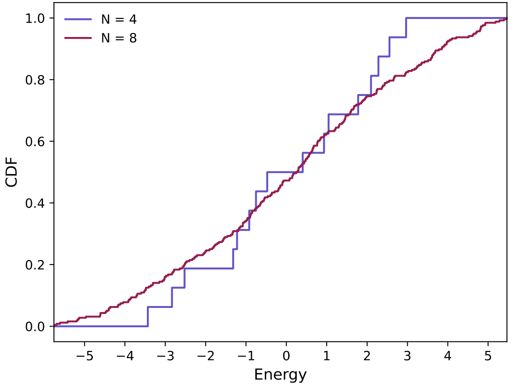

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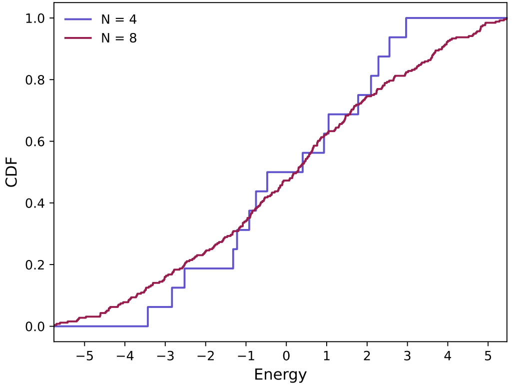

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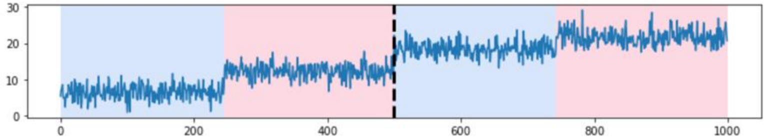

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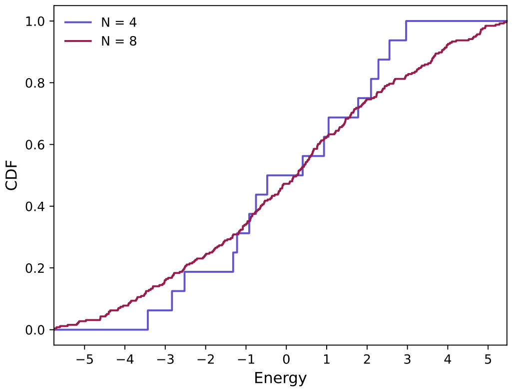

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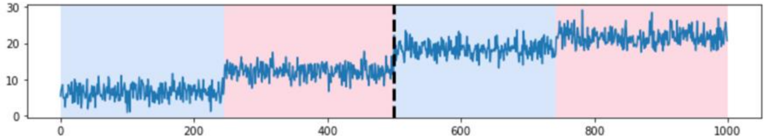

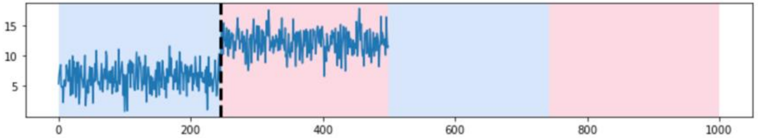

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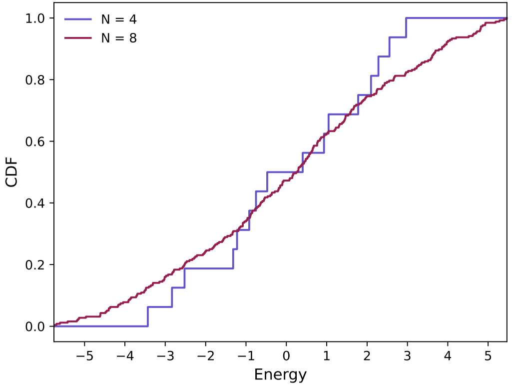

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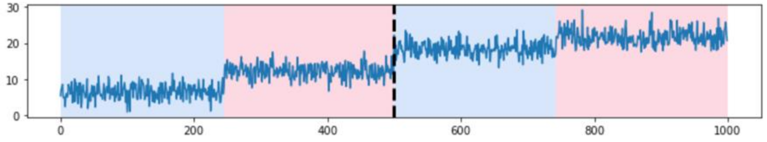

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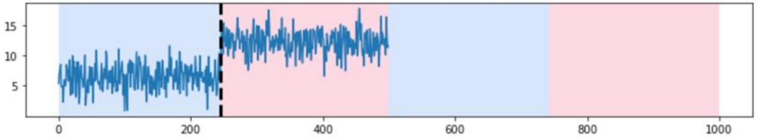

Validate the breakpoint

Use ANOVA to test the statistical significance using F-test

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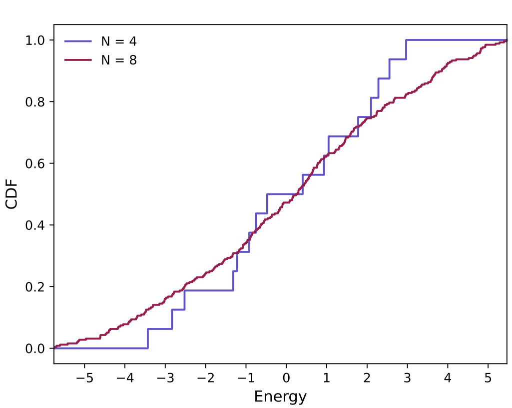

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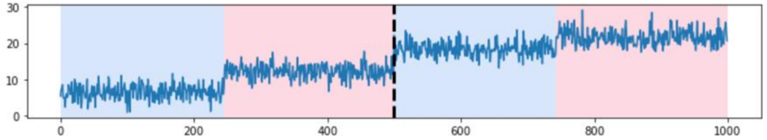

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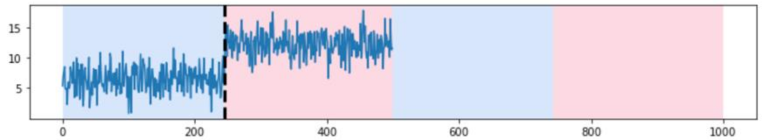

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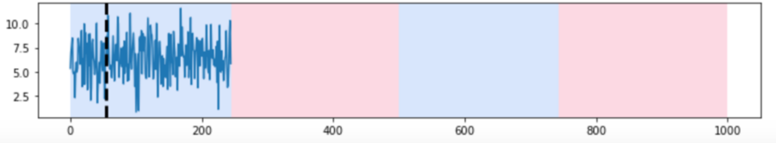

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1. Wan, Berta and Campbell, Phys. Rev. Lett. **129**,030503

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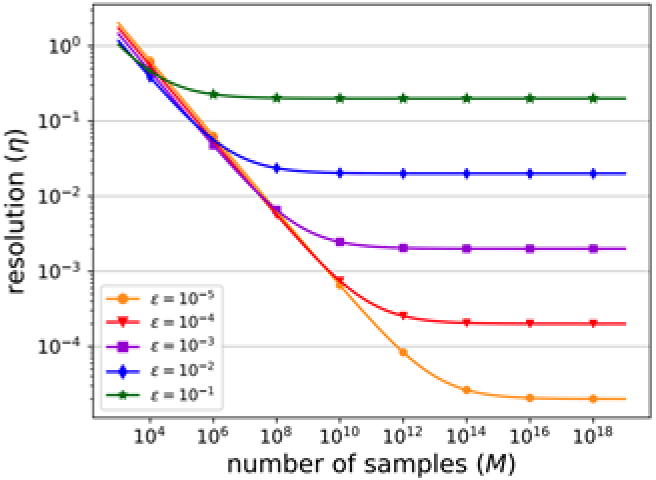

CERN XANADU

1. Wan, Berta and Campbell, Phys. Rev. Lett. **129**,030503

// Key Insights II: Quantitative Resources

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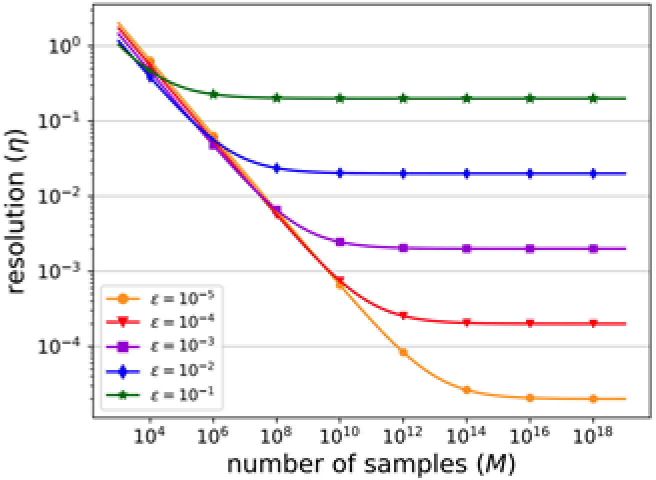

To resolve bad initial states, we require more depth (and not only samples).

1. Wan, Berta and Campbell, Phys. Rev. Lett. **129**,030503

// Overview

1. Early fault-tolerant algorithms for GSEE: review of the Lin&Tong algorithm.
2. Applying the Lin&Tong algorithm in practice.
3. **Numerical simulations.**
4. Bonus: application on NISQ devices.

// Numerical Simulations

We consider a fully-connected **Heisenberg model** with random couplings over N spins

$$\mathcal{H} = \frac{1}{N} \sum_{i<j} \sum_{a \in \{x,y,z\}} J_a^{ij} \cdot \sigma_a^i \sigma_a^j, \text{ where } J_a^{ij} \sim \mathcal{N}(0,1).$$

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Why? : universal, long-range, challenging for DMRG.

Cubitt, Montanaro, and Piddock, Universal quantum Hamiltonians, 115 (38) 9497-9502

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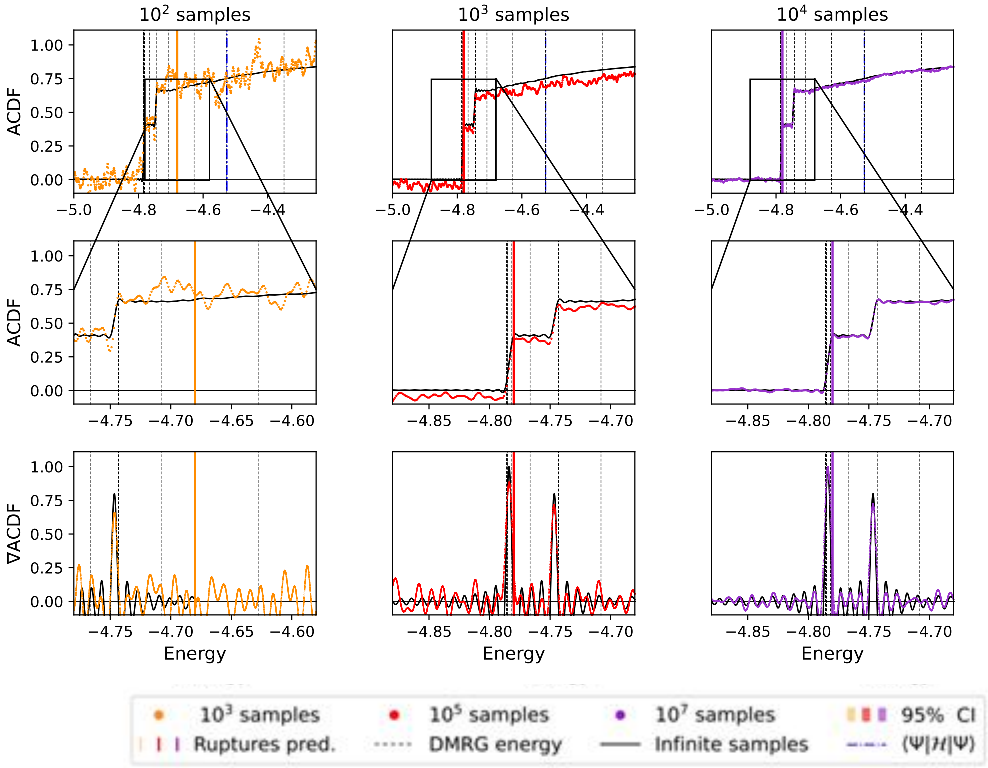
Why? : universal, long-range, challenging for DMRG.

For dynamics, we use a **second-order Trotter-Suzuki** with time step Δt = τ/8.
The circuit construction is based on SWAP networks.

We look at N=26 spins,
and use initial states prepared via DMRG (with low bond dimension).

Cubitt, Montanaro, and Piddock, Universal quantum Hamiltonians, 115 (38) 9497-9502

// Key Insights III: DMRG(bd = 10) Initial State



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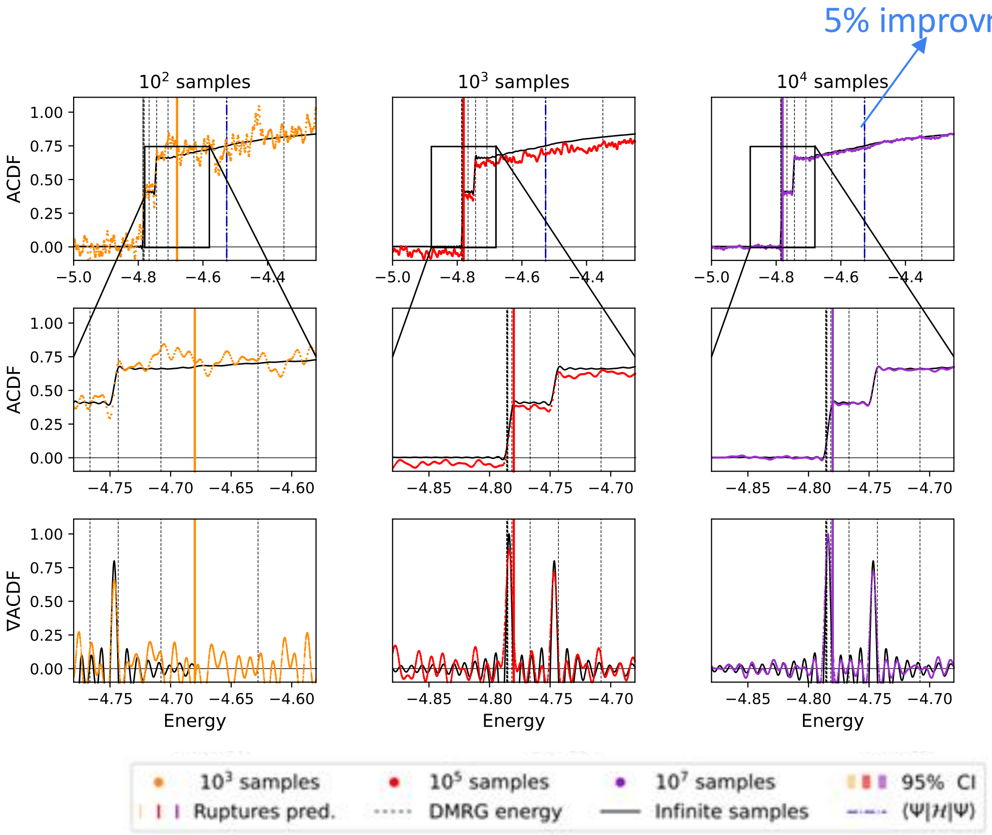
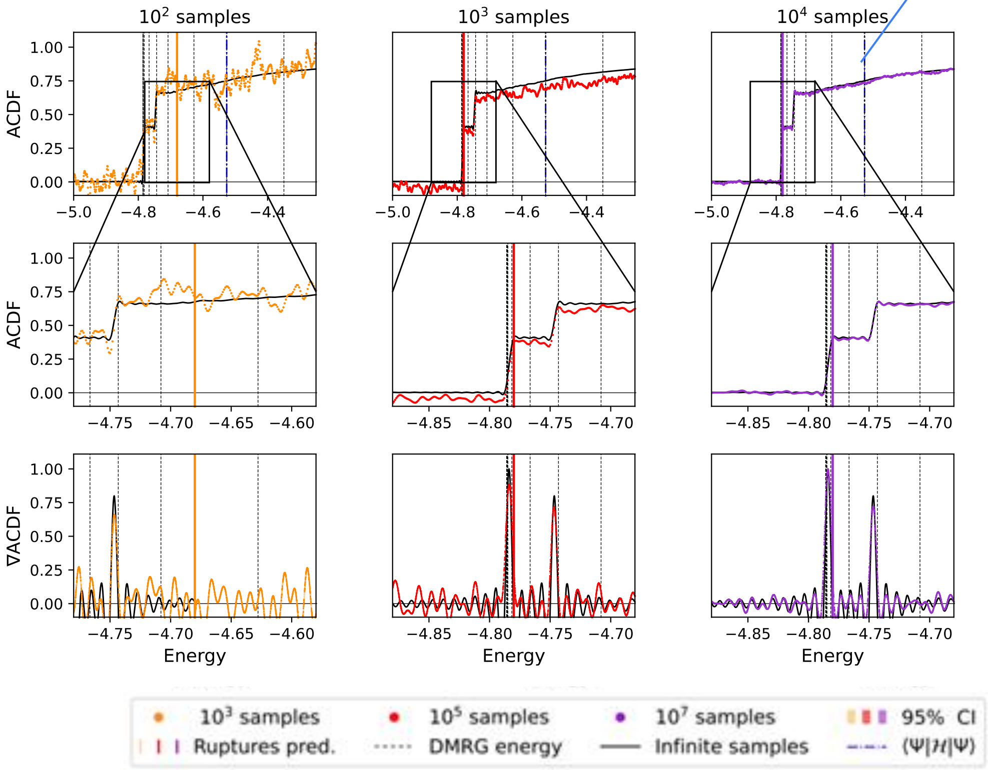
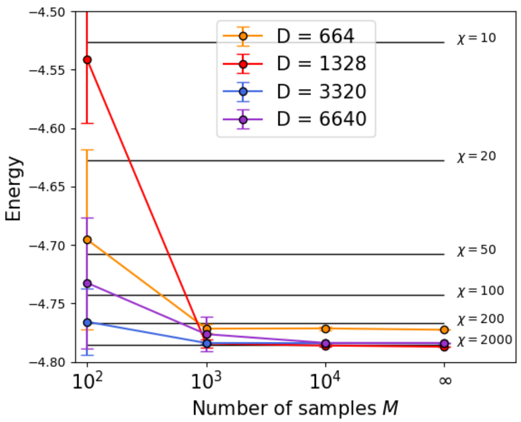


// Key Insights III: DMRG(bd = 10) Initial State

5% improvment

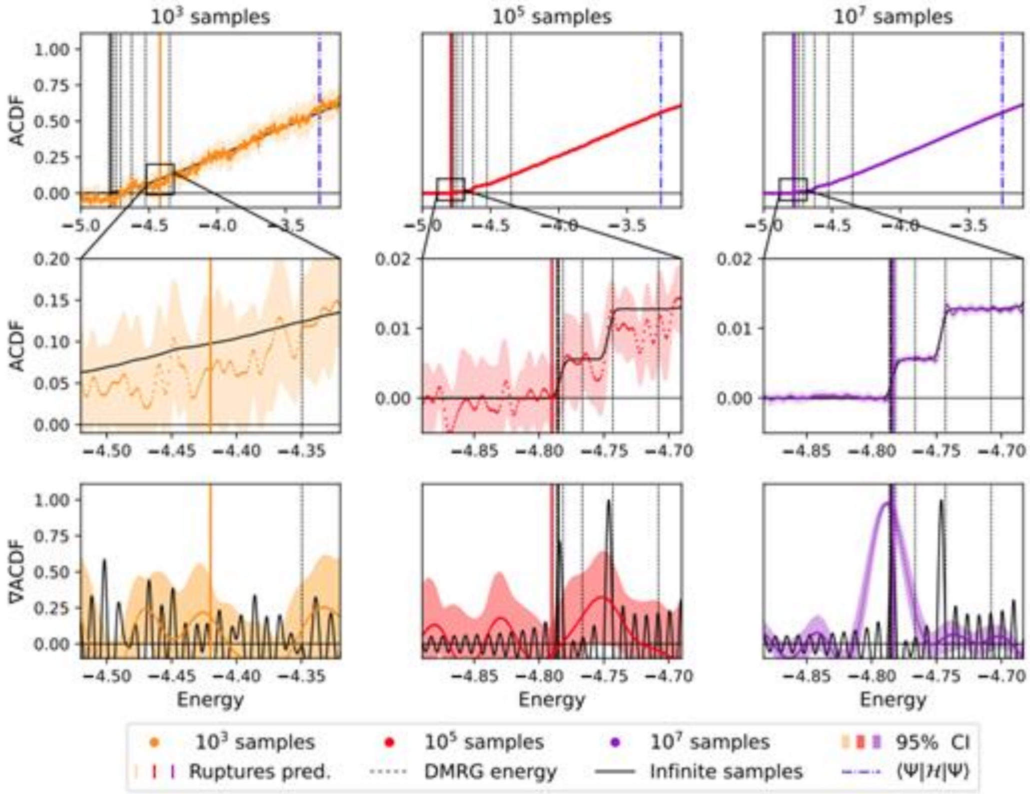




Depth matters most!

$p_0 = 7x\ 10^{-6}$ (overlap)
$M = 10^{13}$ samples

// Key Insights III: Sparsified DMRG



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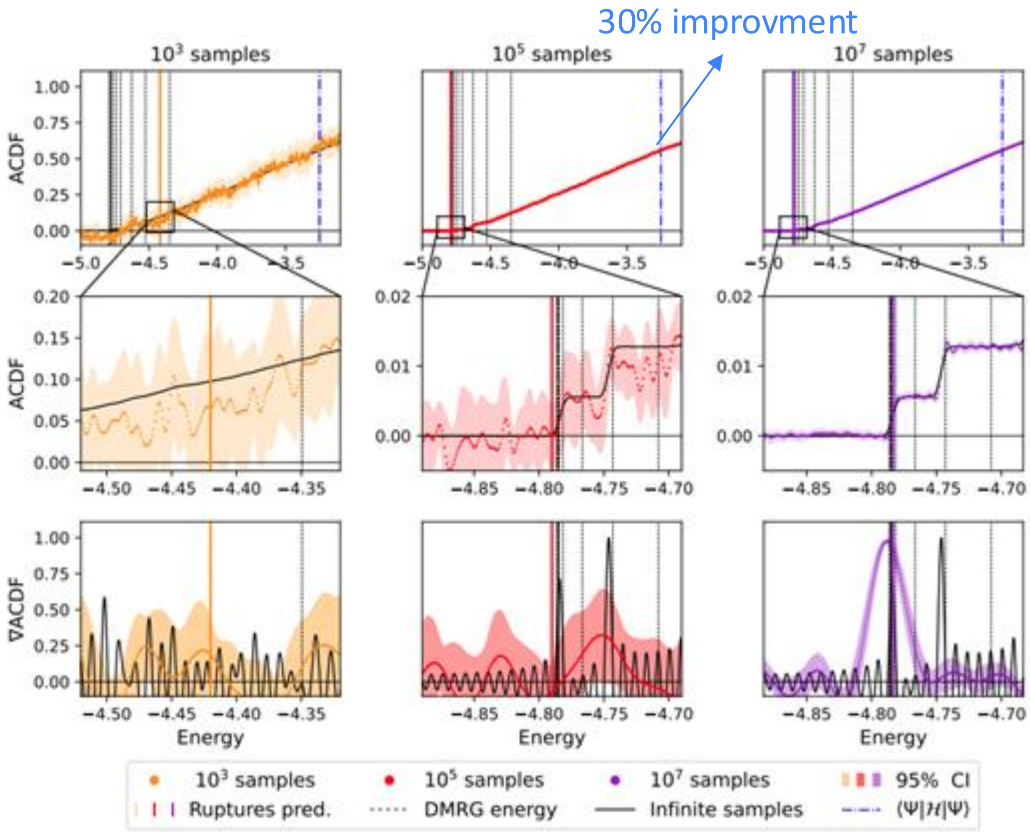
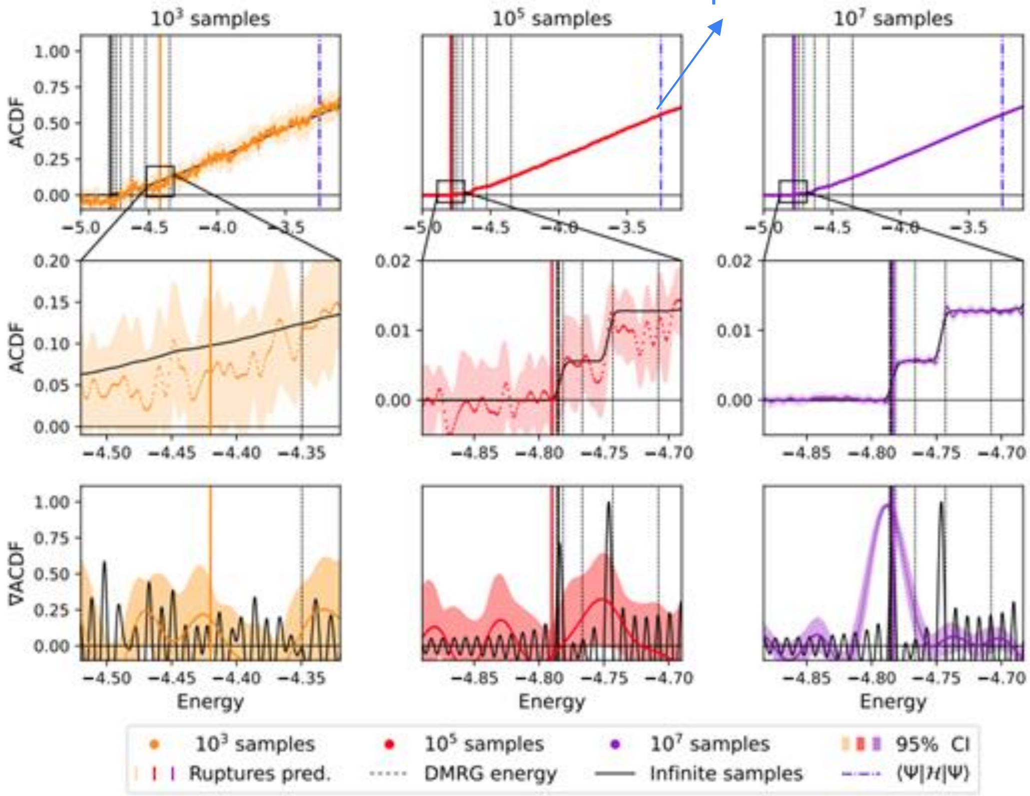
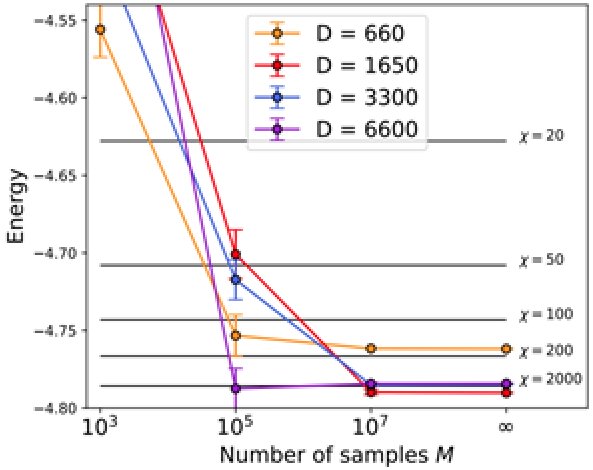


// Key Insights III: Sparsified DMRG

30% improvment





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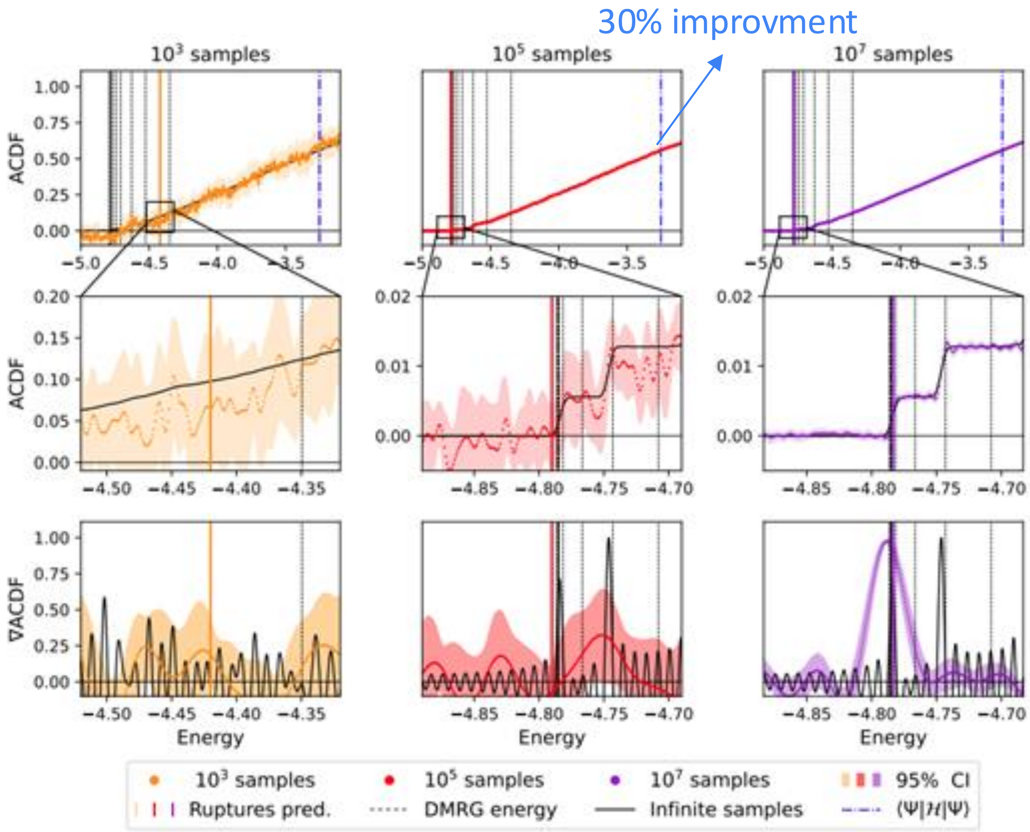

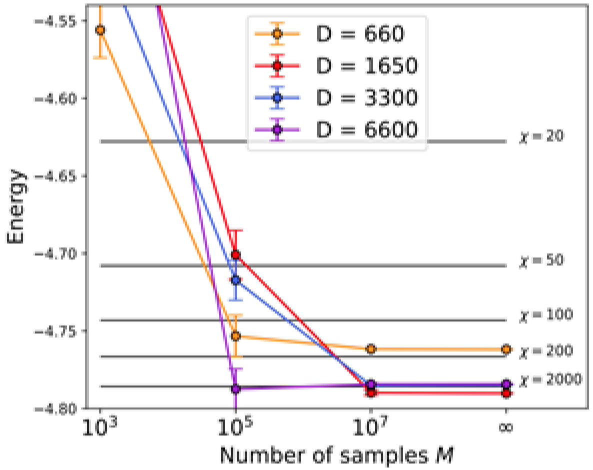

Fidelity with initial state $p_0=2x10^{-5}$

// To Conclude

1.

2.

3.

4.

// To Conclude

1. LT algorithm (and friends) bridges the gap between the NISQ and FTQC eras.
2.
3.
4.

// To Conclude

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2. Instead of aiming at the true ground-state energy, one can concentrate on a finding the inflection point of the spectral CDF.

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// To Conclude

1. LT algorithm (and friends) bridges the gap between the NISQ and FTQC eras.
2. Instead of aiming at the true ground-state energy, one can concentrate on a finding the inflection point of the spectral CDF.
3. LT algorithms are able to improve on classical solutions, using only limited quantum resources (~10^5 samples).
4. If arbitrary precision is required, use quantum phase estimation. However, If a good approximation is enough, LT is a robust algorithm, which can be run in practice.


Simulating
Physics
Classical
Quantum

// Overview

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2. Applying the Lin&Tong algorithm in practice.
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4. **Bonus: application on NISQ devices.**

// Bonus: what can we do on NISQ?

- Blunt et al: variational dynamics + ZNE on a H_3 molecule (6 qub
- We focus on extracting Fourier moments of a nuclear EFT (4 qubits).

$$\langle\Psi_0|\,\hat{O}(\vec{q})^\dagger e^{-iHj\tau}\hat{O}(\vec{q})\,|\Psi_0\rangle$$

- We use **purified echo verification** + various error suppresion techniques (twirling, dynamical decoupling, pulse efficient calibration, readout calibration).

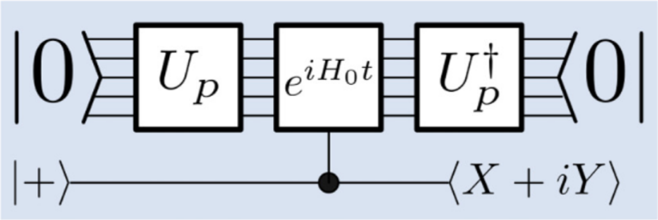
Blunt et al, PRX Quantum 4, 040341 (2023)

// Purified echo verification

// Purified echo verification

Verification

Unprepare the state and verify that it is the zero state



// Purified echo verification

Verification

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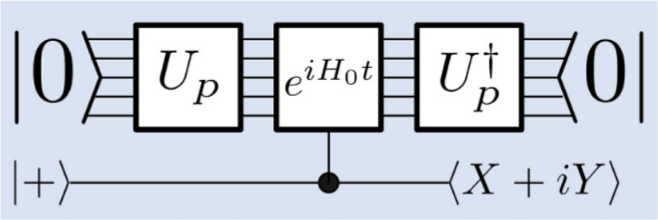

Verification passed -> state contributes +/- 1 to the expectation value.

Otherwise: -> garbage

 O'Brien, et al., PRX Quantum 2, 020317 (2021),

// Purified echo verification

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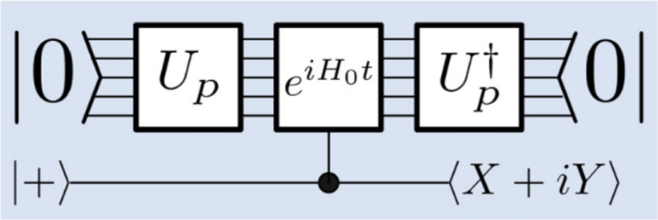

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Purification of the ancilla

$$\rho = \lambda|\psi\rangle\langle\psi| + (1-\lambda)\sum_{k=2}^{2^N} p_k|\psi_k\rangle\langle\psi_k|.$$

Pure main component

Noisy components

// Purified echo verification

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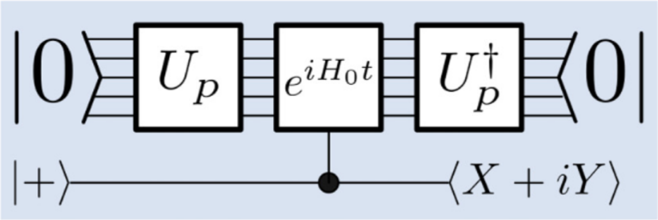

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Pure main component

Noisy components

Without noise: the ancilla is pure after post-selection.

With noise: It is not. Extract the closest pure state from measurements.

// Results (IBMQ)

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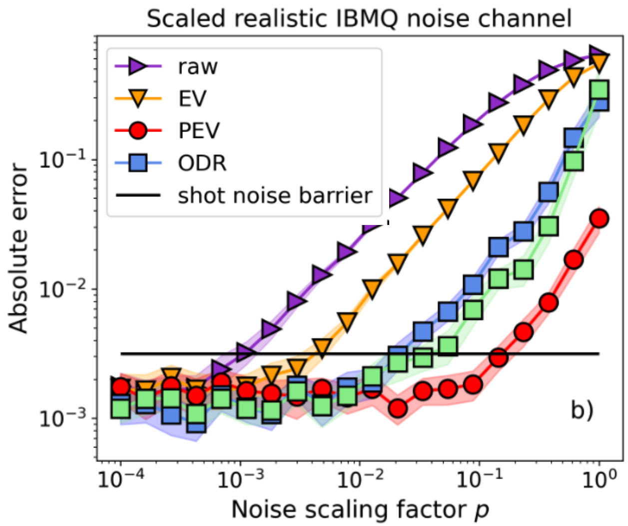
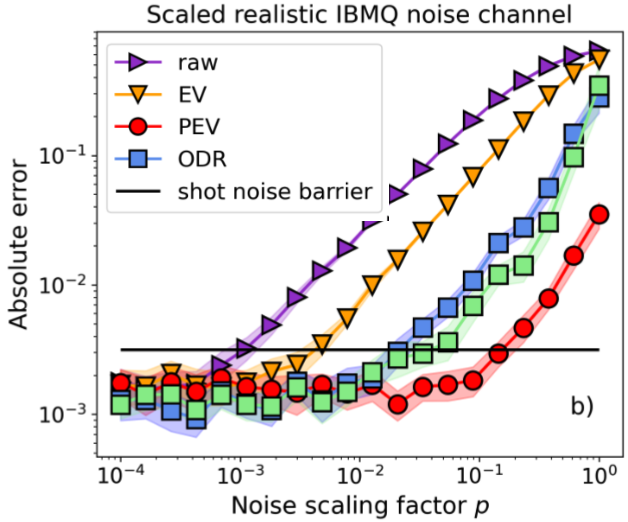
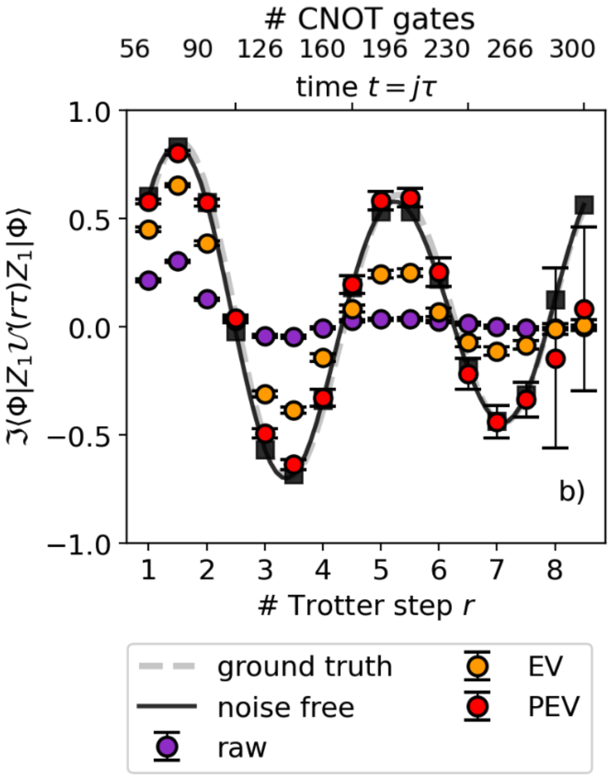


x100 error reduction

// Results (IBMQ)





x100 error reduction

Kiss et al , arXiv:2401.13048

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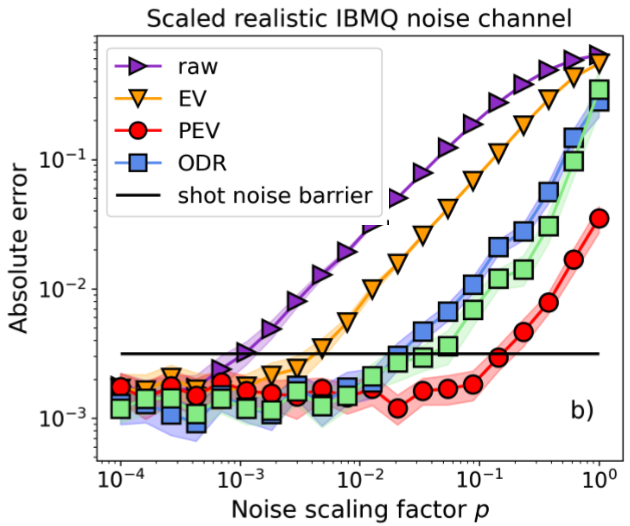

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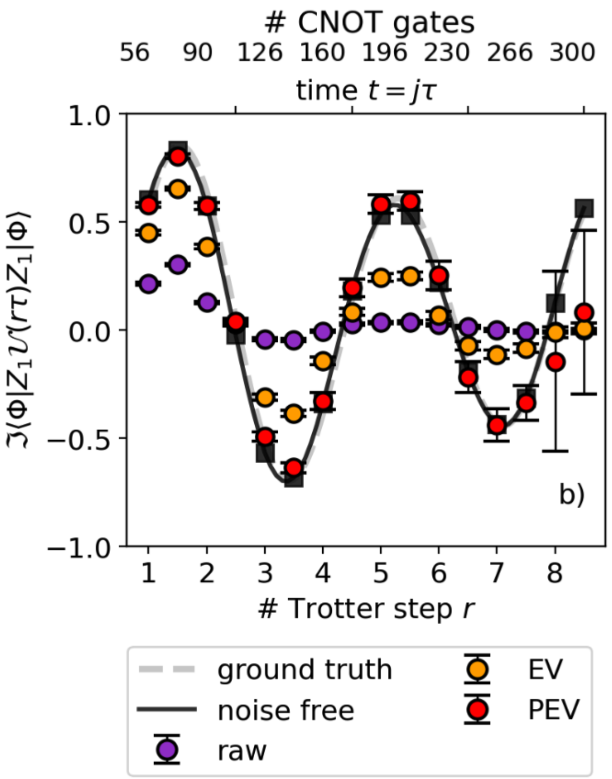
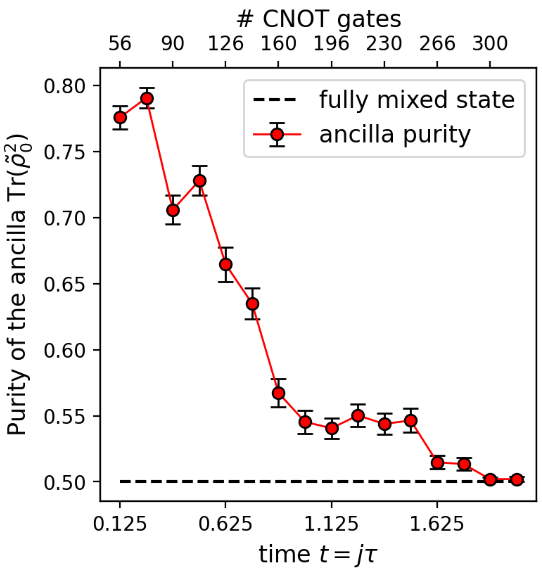


Purity of the ancilla



We know the breakpoint

Kiss et al , arXiv:2401.13048

Thank You!

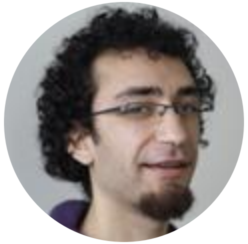
// Team

Utkarsh Azad

Borja Requena

David Wakeham

Michele Grossi



Alessandro Roggero

Juan Miguel Arrazola

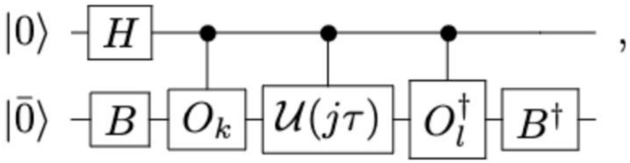
arXiv: 2405.03754

arXiv: 2401.13048

contact: oriel.kiss@cern.ch

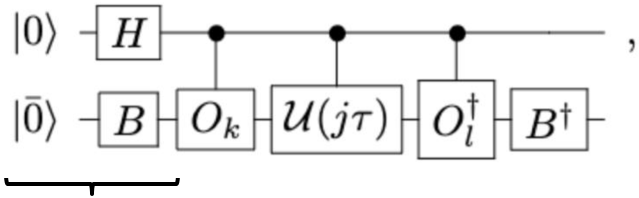
twitter: @oriel_kiss

// Purified echo verification (details)



// Purified echo verification (details)

Hadamard test



Initial state

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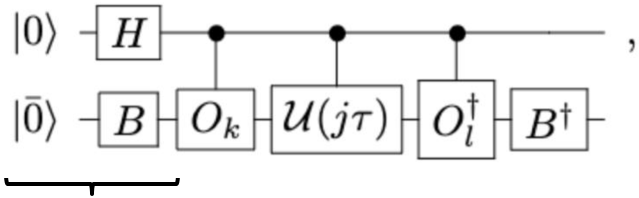

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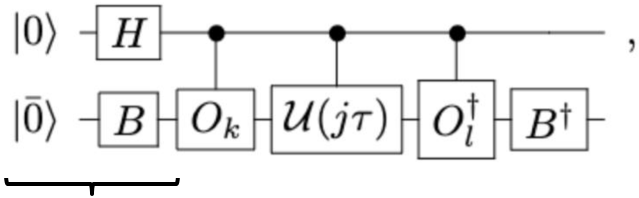

Initial state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|\bar{0}\rangle \otimes |0\rangle + B^\dagger O_k^\dagger \mathcal{U}(j\tau) O_l B |\bar{0}\rangle \otimes |1\rangle\right)$$

$$\equiv \frac{1}{\sqrt{2}}\left(|\bar{0}\rangle \otimes |0\rangle + |\phi\rangle \otimes |1\rangle\right),$$

$$|\phi\rangle = \alpha|\bar{0}\rangle + \beta|\bar{0}^\perp\rangle.$$

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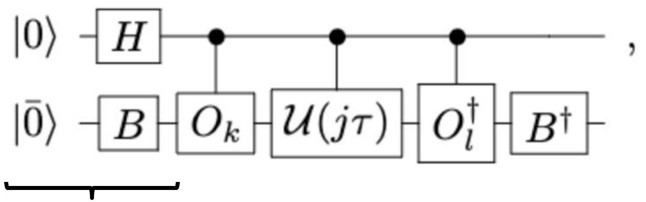

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We only care about α, so we can disregard any orthogonal states!

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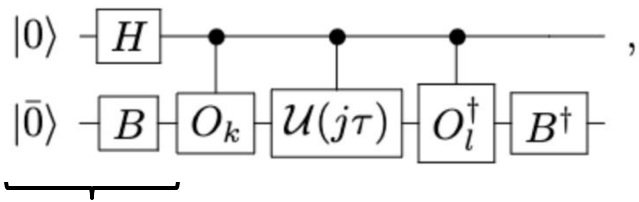

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We only care about α, so we can disregard any orthogonal states!

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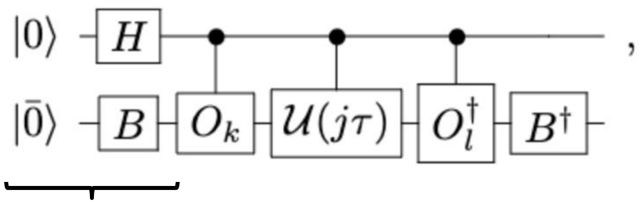
Hadamard test



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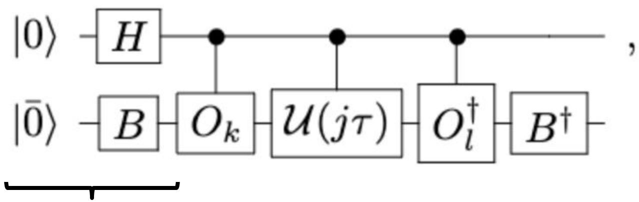
We only care about $\boldsymbol{\alpha}$, so we can disregard any orthogonal states!

2. Construct the closest compatible pure state (**purification + tomography**).

// Purified echo verification (details)

Hadamard test



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$$\mathrm{Re}\{\alpha\} = \frac{\langle X_a \rangle_0}{1 + \langle Z_a \rangle_0}, \quad \mathrm{Im}\{\alpha\} = \frac{\langle Y_a \rangle_0}{1 + \langle Z_a \rangle_0}.$$

// Sparse interpolation via compressive sensing

- Compressive sensing is an optimal technique to recover a signal from few measurements if we know a basis where the signal is sparse.

Theorem 3. *Let U be the orthogonal conversion matrix between the measurement basis and the basis where the signal is sparse, with $|U_{k,j}| \leq \mu(U)$. If the number of samples m is chosen such that*

$$m \geq C_0 \mu(U)^2 S \log d/\delta \tag{28}$$

$$m \geq C_0' \log^2 d/\delta, \tag{29}$$

for some constant C_0, C_0', then every signal of sparsity S can be recovered with probability $(1-\delta)$.

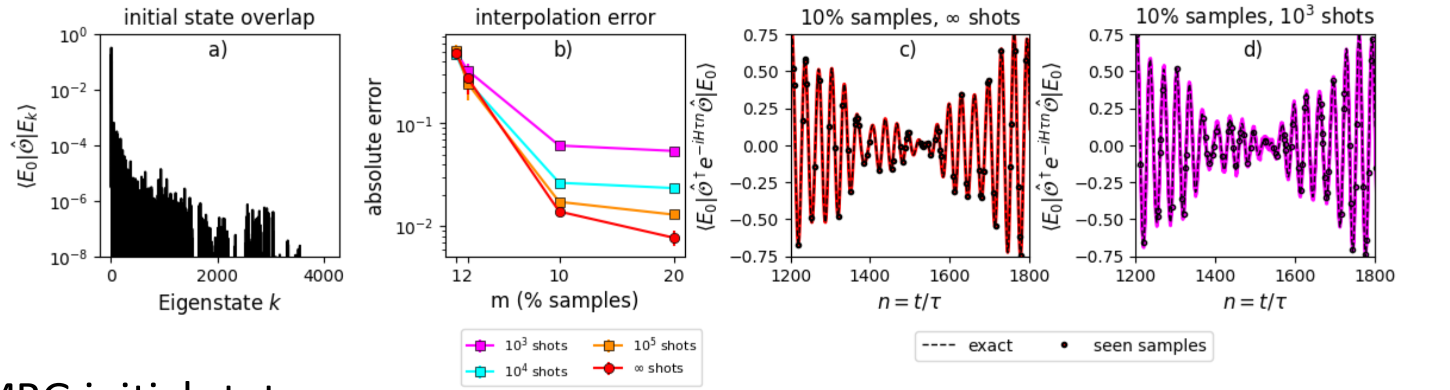
Assuming that we only have access to $m \in \Omega$ samples, and that f is S-sparse in the ψ basis, the optimal solution is given by $f^* = \Psi x^*$, where x^* is the solution of the convex optimization problem

$$\min_{\tilde{x} \in \mathbb{R}^d} \|\tilde{x}\|_1 \text{ subject to } f_k = (\Psi \tilde{x})_k \; \forall k \in \Omega. \tag{31}$$

d = number of moments

S = number of non-zero components (sparsity)

// Numerics



DMRG initial state

1. Not good if used with importance sampling.
2. Does not work for extrapolation
3. Always good if you need the whole signal (even with shot noise).