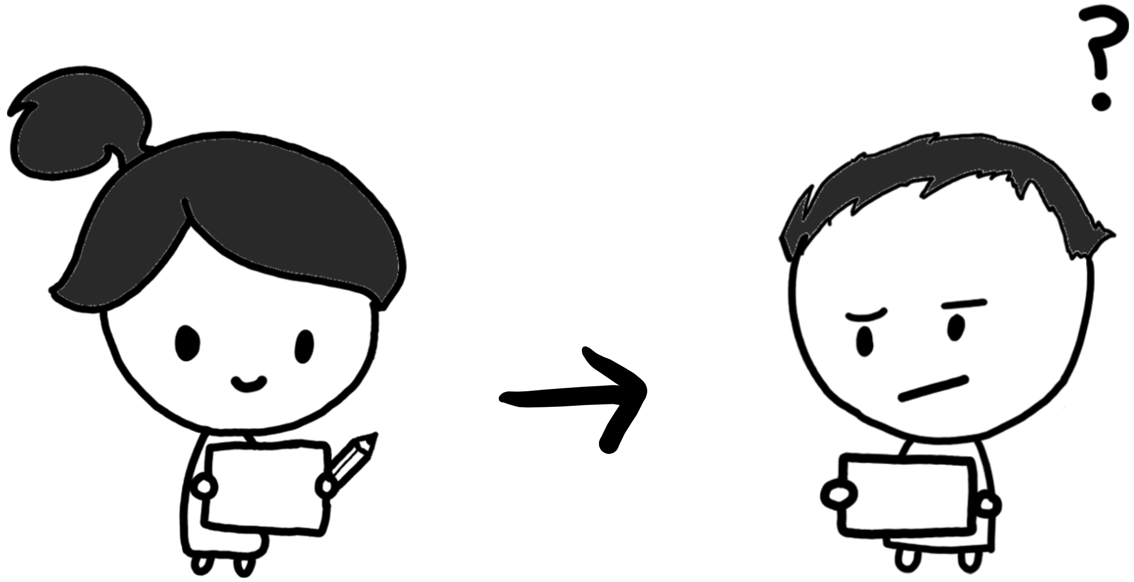
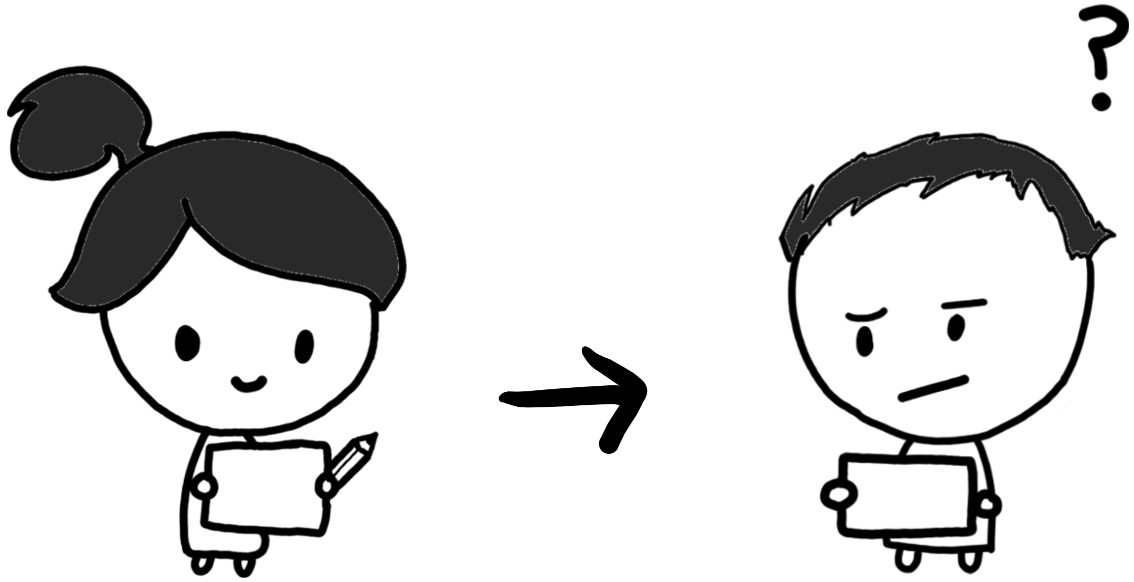
Implications of Information Causality and its Generalisations

Prabhav Jain[1], Nikolai Miklin[2], Mariami Gachechiladze[1]

[1] Technische Universität Darmstadt, [2] Technische Universität Hamburg



MOTIVATION



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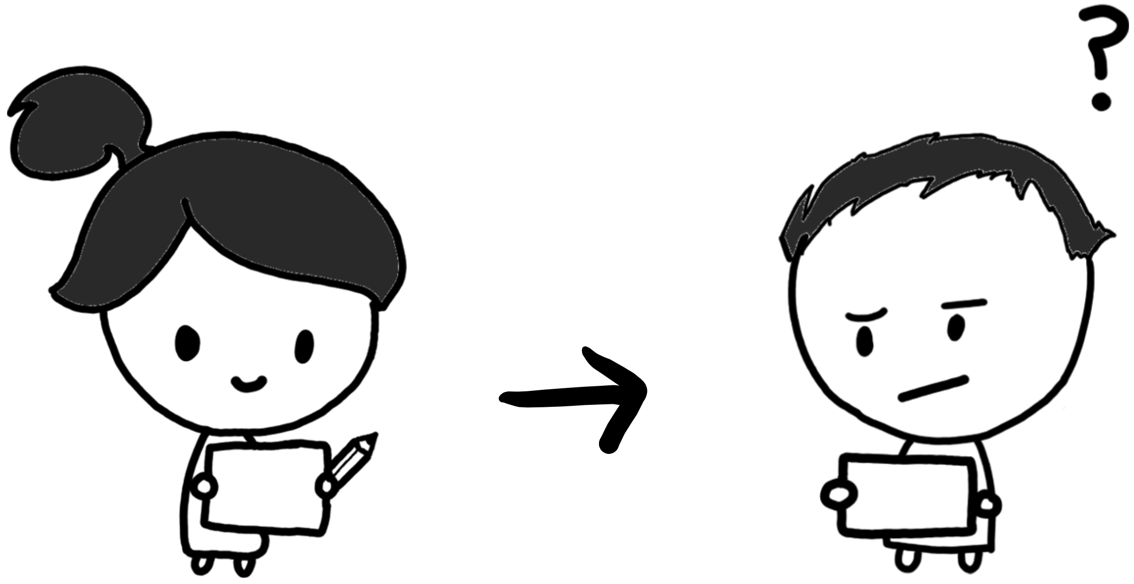

information sent by Alice $\geq$ information available to Bob

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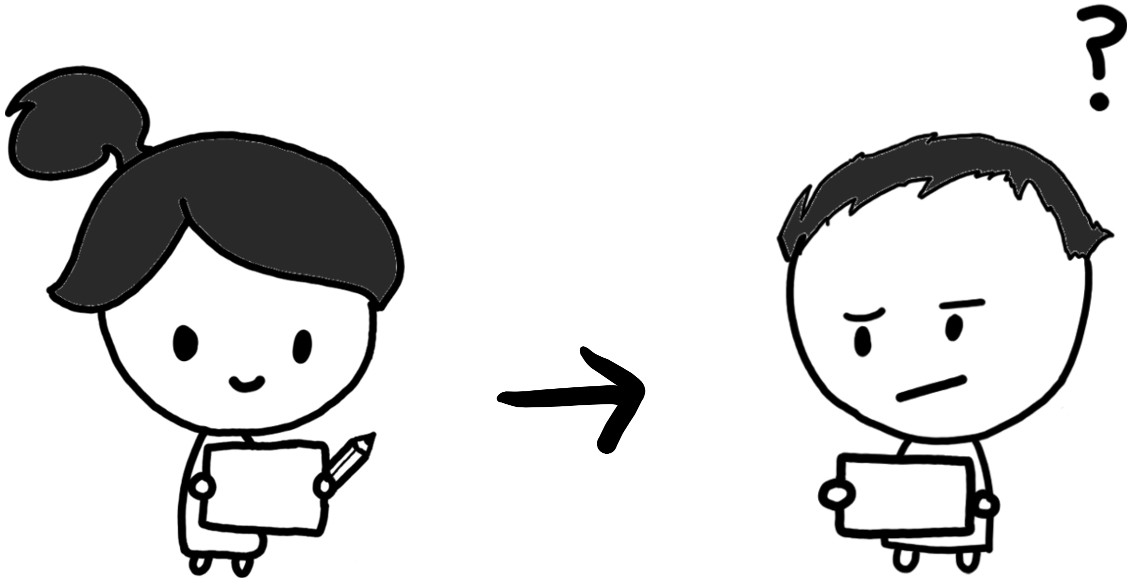
MOTIVATION

For such scenarios, one can show: $I(A;B) \leq \mathcal{C}$



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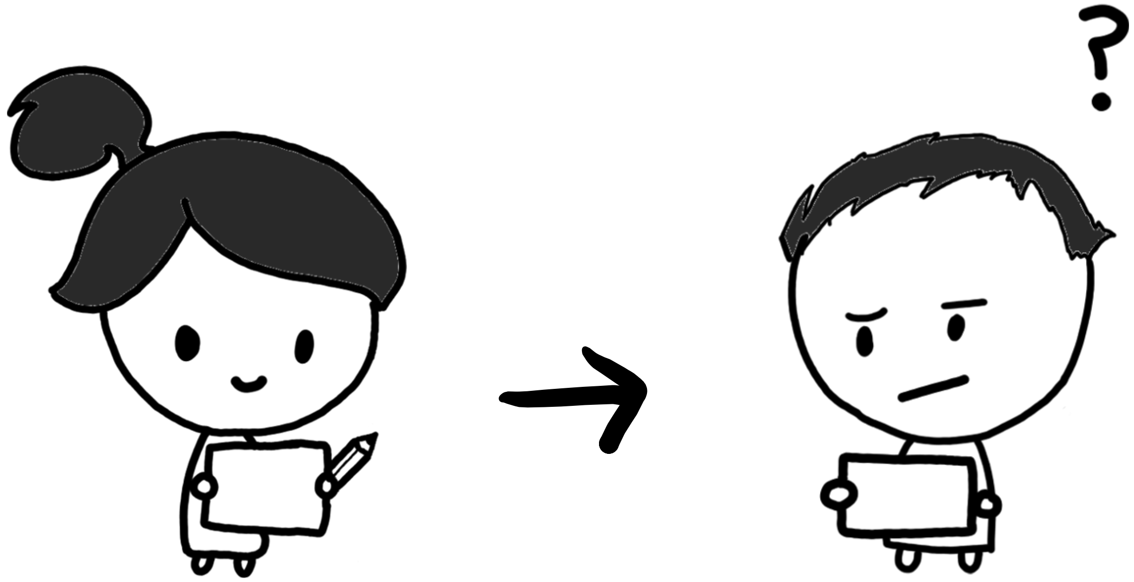
MOTIVATION

For such scenarios, one can show:

$$\sum_{i=0}^{n-1} I(g; a_i | b = i) \leq \mathcal{C}$$



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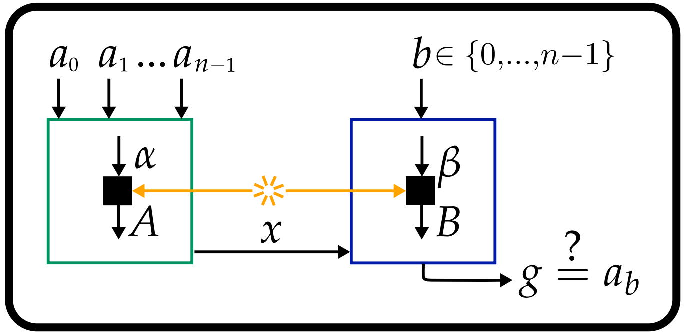

THE ALGORITHM

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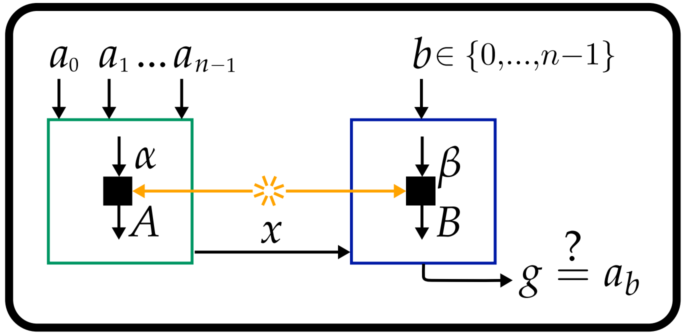

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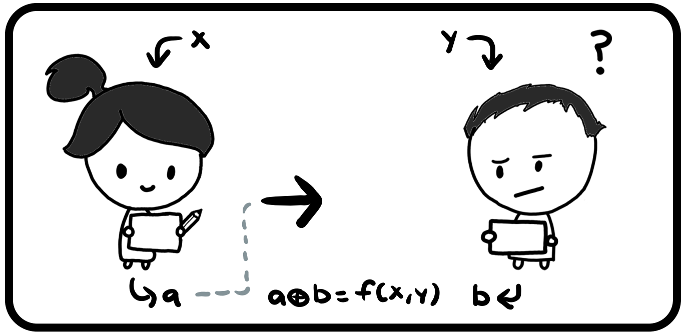

EXTENDED IC

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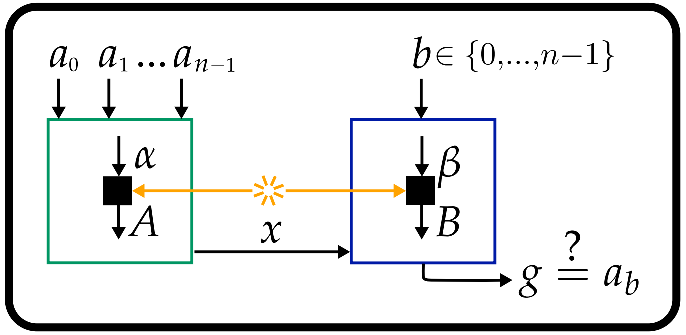

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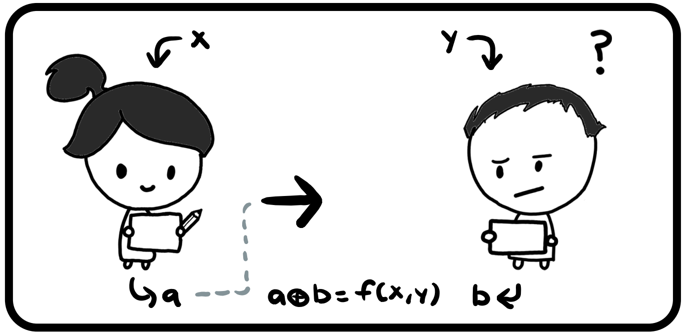

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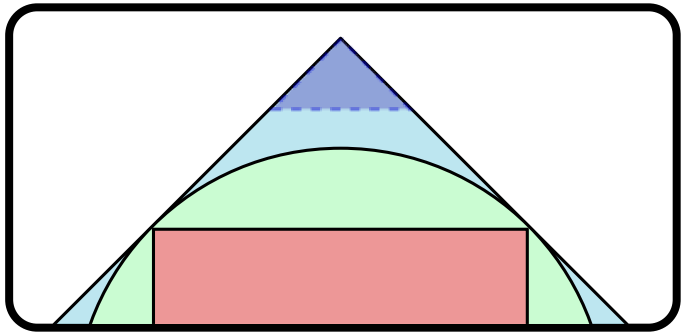

IC VS. NTCC

THE ALGORITHM

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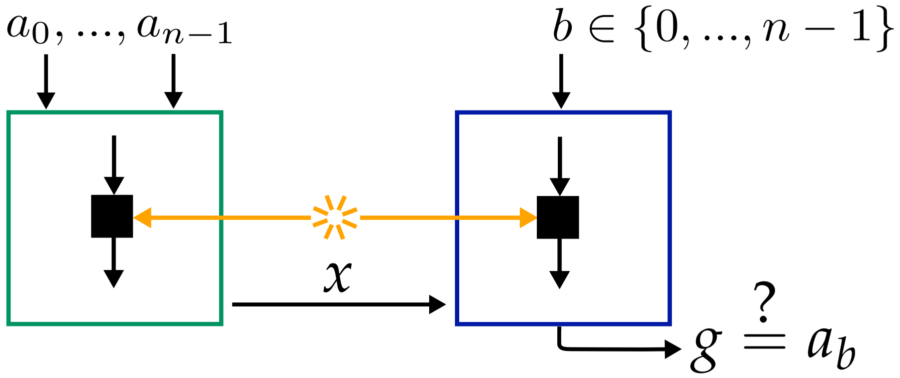

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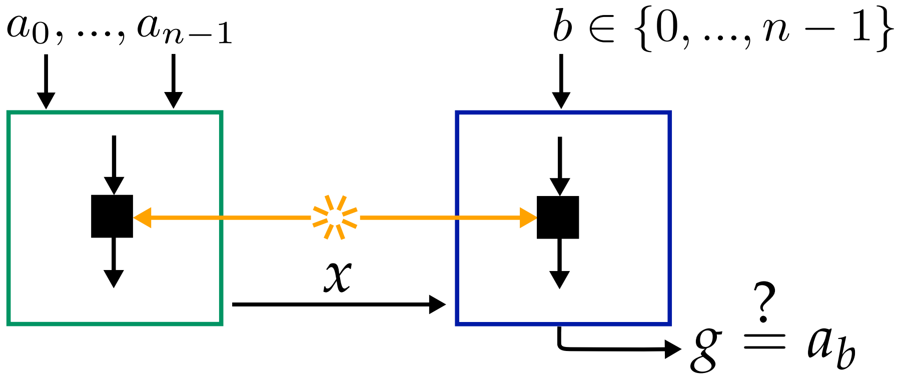

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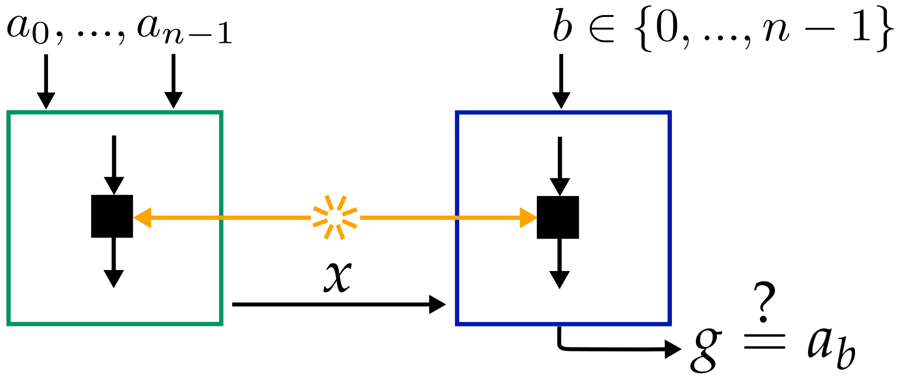

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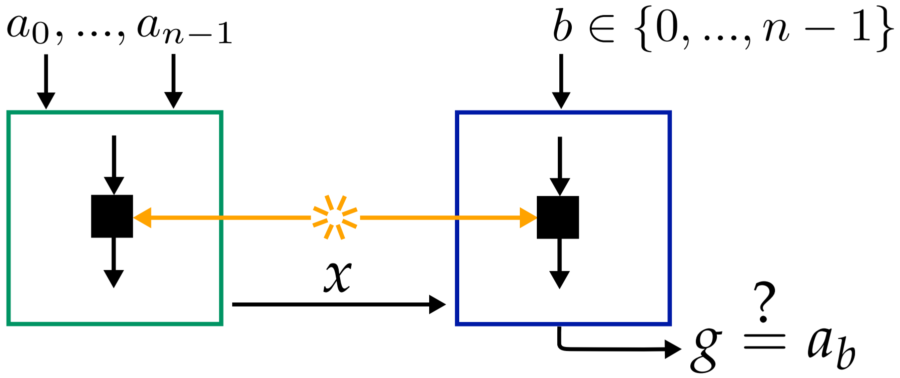

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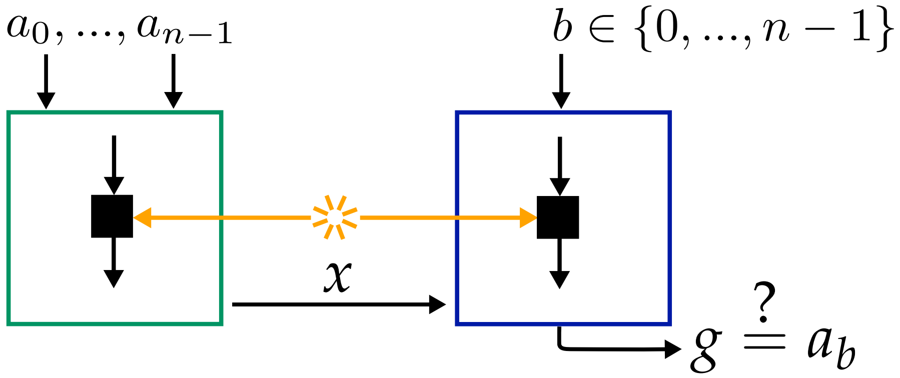

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(two rights) **(two wrongs)**

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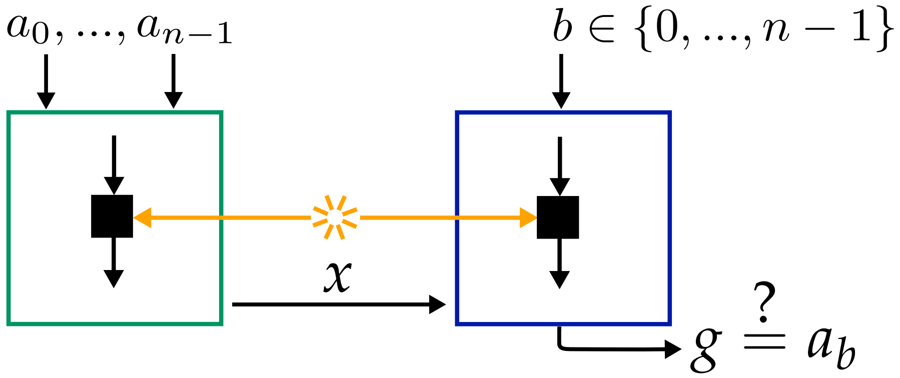

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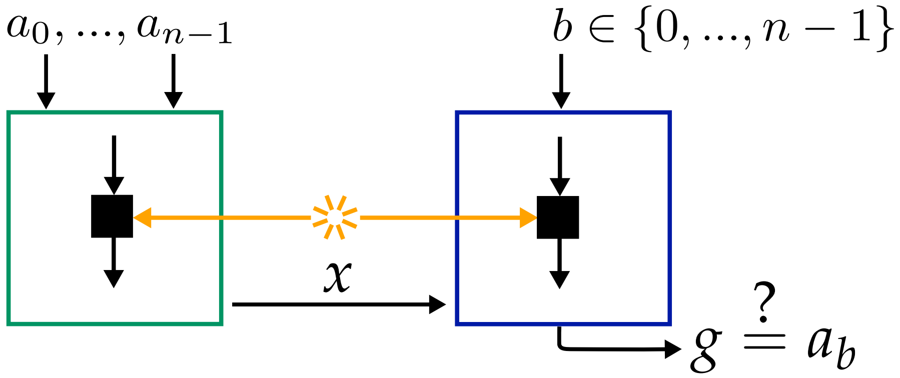

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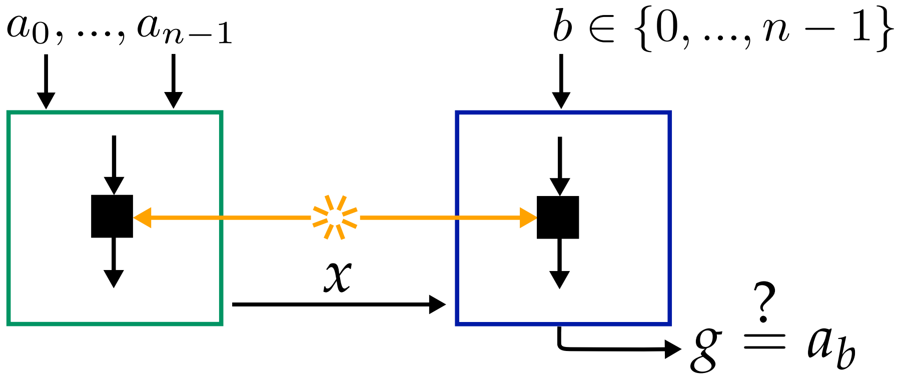

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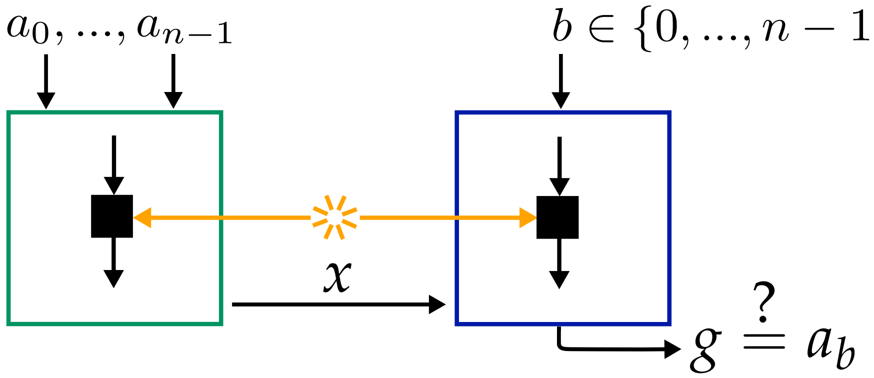

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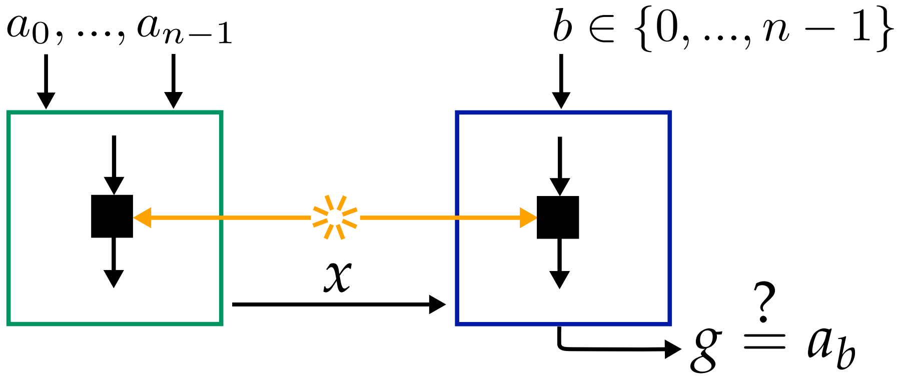

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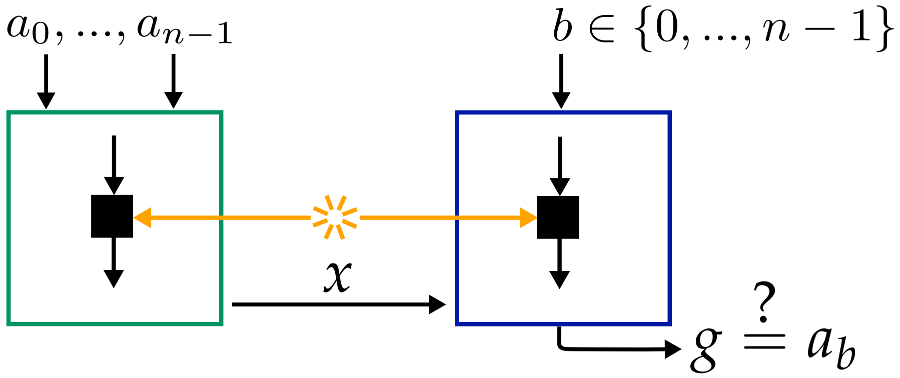

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vanishes when $e_c \to 0$

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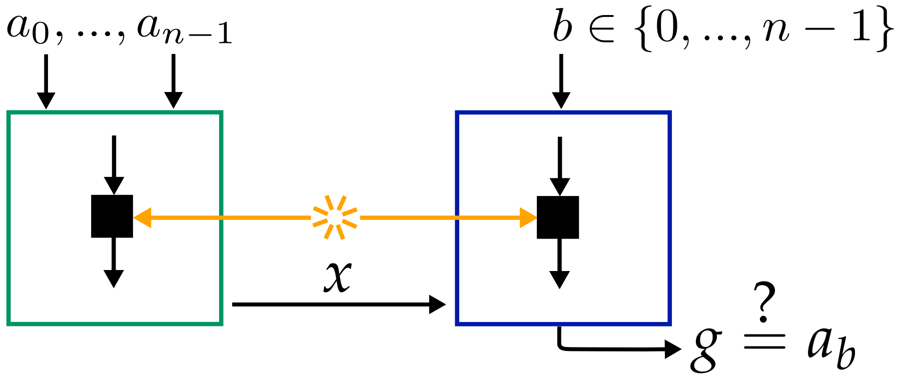

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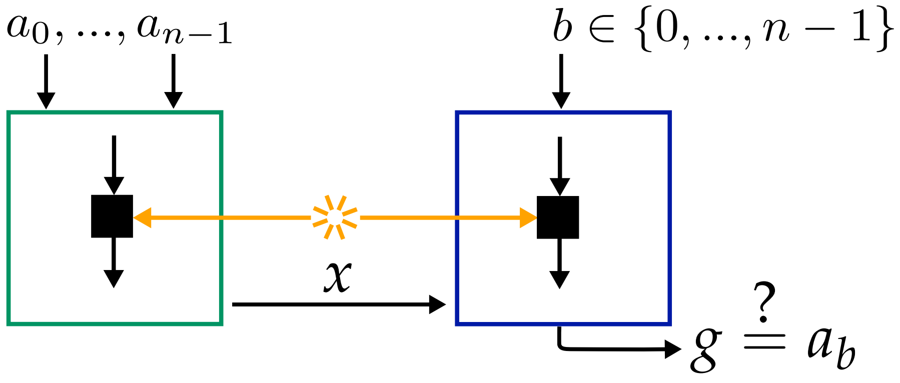

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vanishes when $e_c \rightarrow 0$ **also vanishes when $e_c \rightarrow 0$**

since both LHS and RHS vanish, we can use L'Hôpital's rule and differentiate twice to get rid of the logarithms

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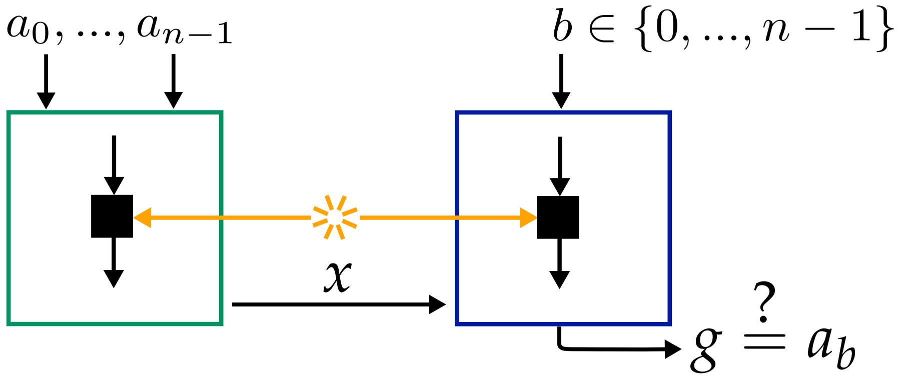

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$$n\,(1 - h(p)) \leq 1 - h(p_c) \quad \Longrightarrow \quad e^2 \leq \frac{1}{n}$$

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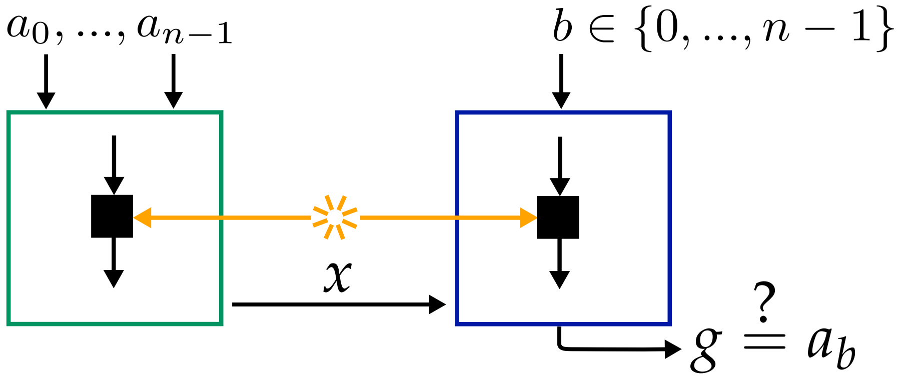

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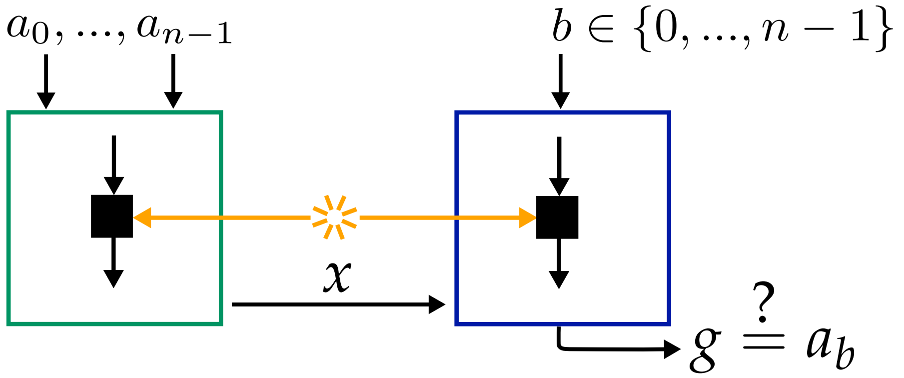

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can be easily generalised to higher dimensions

$$p \leq \frac{1}{d}\left(1 + \frac{d-1}{\sqrt{n}}\right)$$

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Getting Inequalities

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If we actually compute p, it works the same but you get inequalities

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YOU DO NOT WANT TO SEE

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turns this

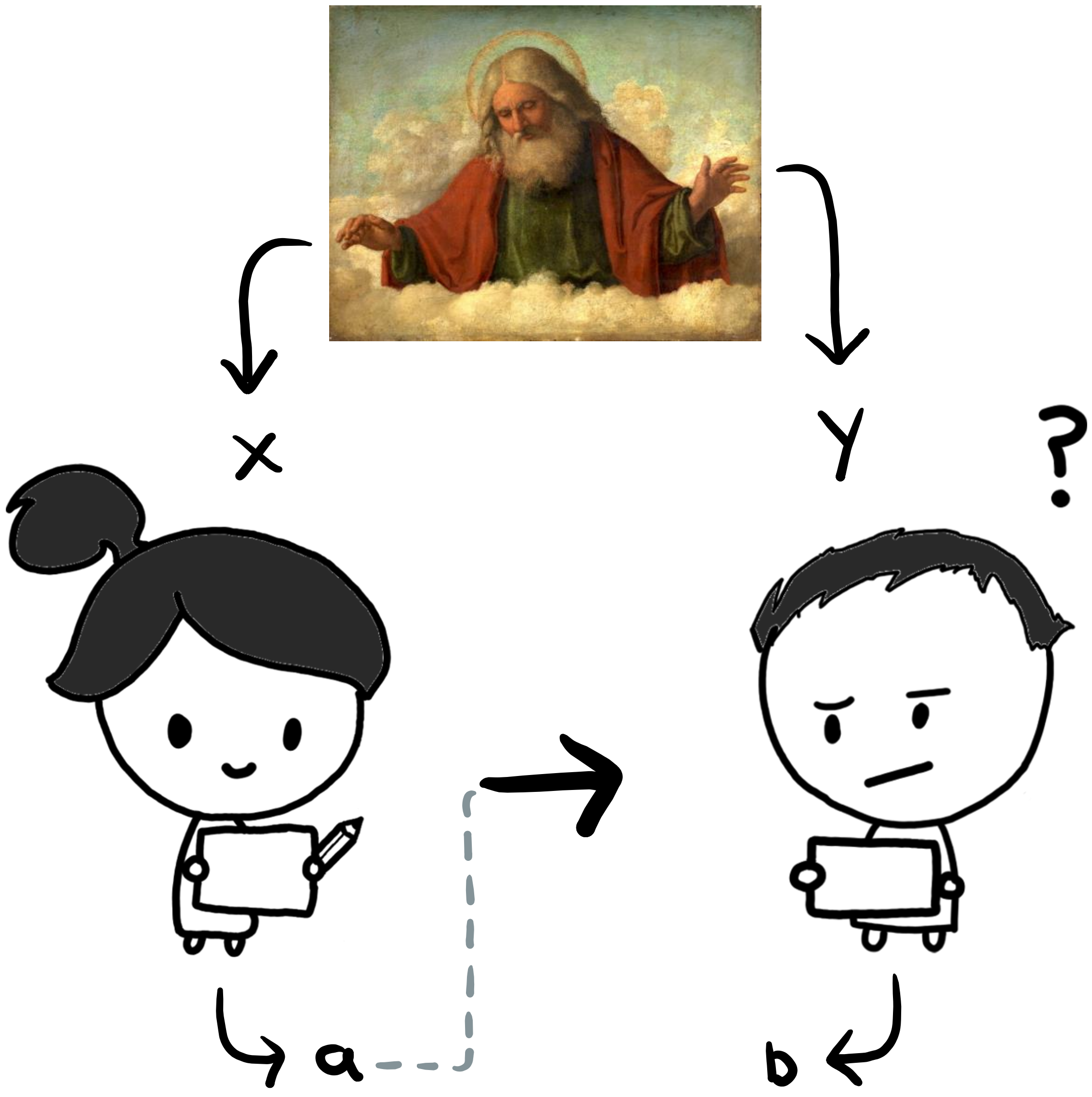
$$(e_{00}+e_{10})^2+(e_{01}-e_{11})^2\leq 4$$

into this

(inequalities bounding the quantum set)

P. Jain, M. Gachechiladze, and N. Miklin, Information causality as a tool for bounding the set of quantum correlations, Phys. Rev. Lett. 133, 160201 (2024)

EXTENDED IC



$$f(x,y) = a \oplus b$$

EIC Warm Up: Index Function

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For such a scenario, one can show:

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(condition over all ‘previous’ values)

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$$\sum_{i=0}^{n-1} I(g; x_y | y = i, \{x_0, \cdots, x_{y-1}\}) \leq \mathcal{C}$$

Reduces to original IC statement!

EIC Main Course: Inner Product Function

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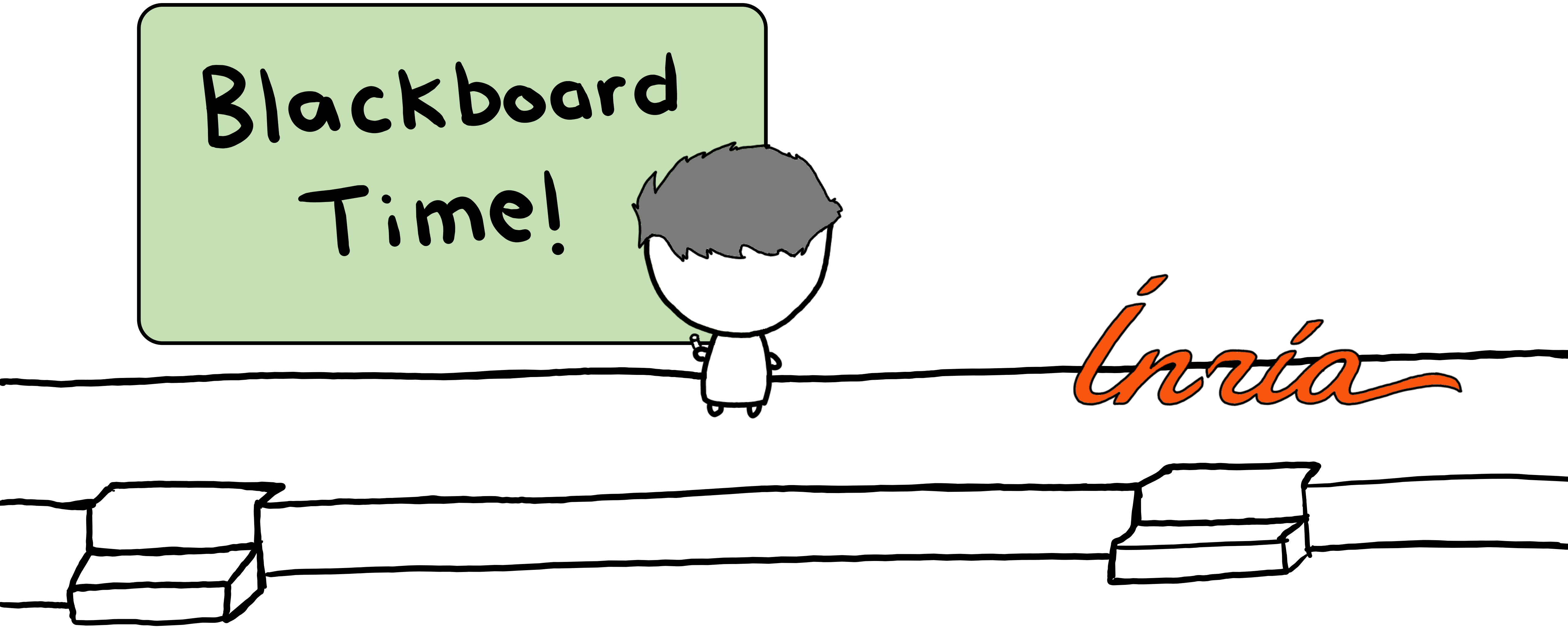


EIC Main Course: Inner Product Function

$$\mathrm{IP}_2 : f(x,y) = x \cdot y \mod 2 \equiv x_0 \cdot y_0 \oplus x_1 \cdot y_1$$

$$\begin{aligned} & I(g; f(x,00)|y=00) \\ +& I(g; f(x,01)|y=01, f(x,00)) \\ +& I(g; f(x,10)|y=10, f(x,00), f(x,01)) \\ +& I(g; f(x,11)|y=11, f(x,00), f(x,01), f(x,10)) \leq \mathcal{C} \end{aligned}$$

$$\begin{aligned} & I(g; 0|y=00) \\ +& I(g; x_1|y=01, 0) \\ +& I(g; x_0|y=10, 0, x_1) \\ +& I(g; x_0 \oplus x_1)|y=11, 0, x_1, x_0) \leq \mathcal{C} \end{aligned}$$

$$I(g;0|y=00) + I(g;x_1|y=01,0) + I(g;x_0|y=10,0,x_1) + I(g;x_0 \oplus x_1)|y=11,0,x_1,x_0) \leq \mathcal{C}$$

$$I(g;x_1|y=01) + I(g;x_0|y=10,x_1) \leq \mathcal{C} \qquad I(g;a_0|b=0) + I(g;a_1|b=1,a_0) \leq \mathcal{C}$$

IT IS EQUIVALENT TO INDEX!

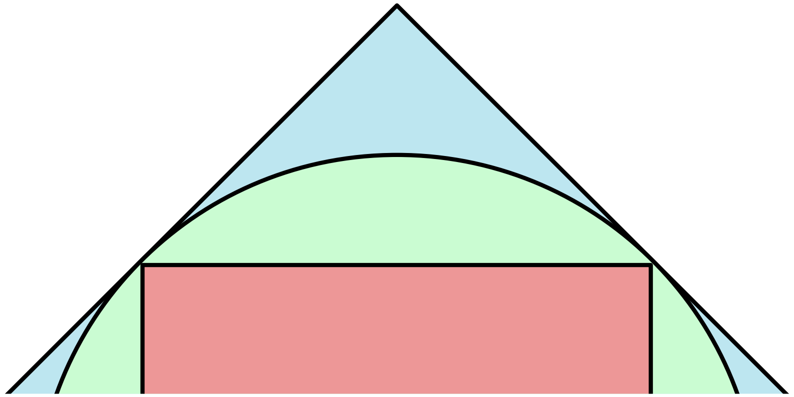
this holds true for any n, and surprisingly a similar reduction follows for $\mathrm{DISJOINT}(x,y)$

IC vs NTCC

What is NTCC?

Non-Trivial Communication Complexity (NTCC)

In a communication scenario, there are functions which are 'hard' to compute i.e. their communication cost/complexity increases with the input size.



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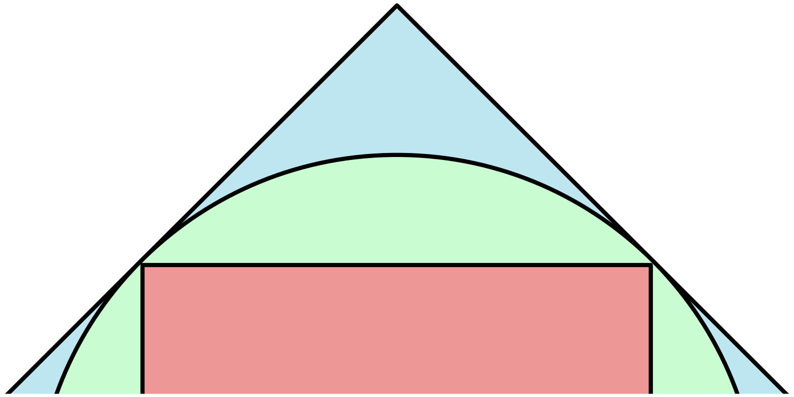

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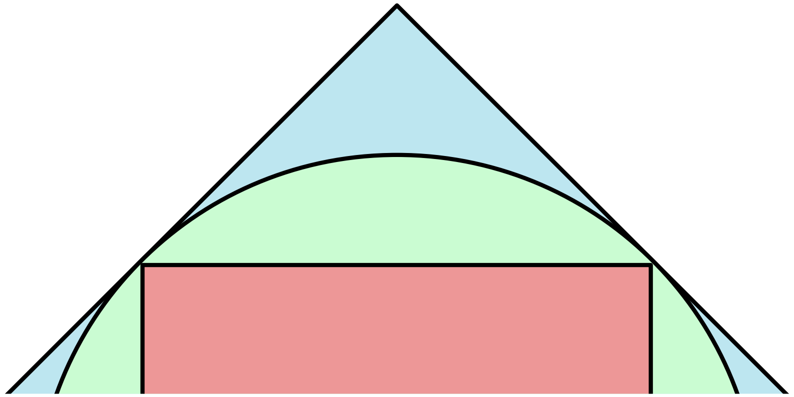

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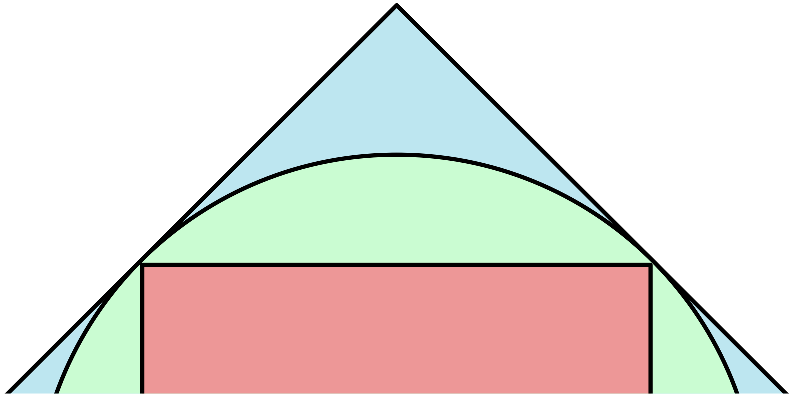

In the randomised version we ask:
for a given protocol, the winning probability should be strictly greater that 1/2

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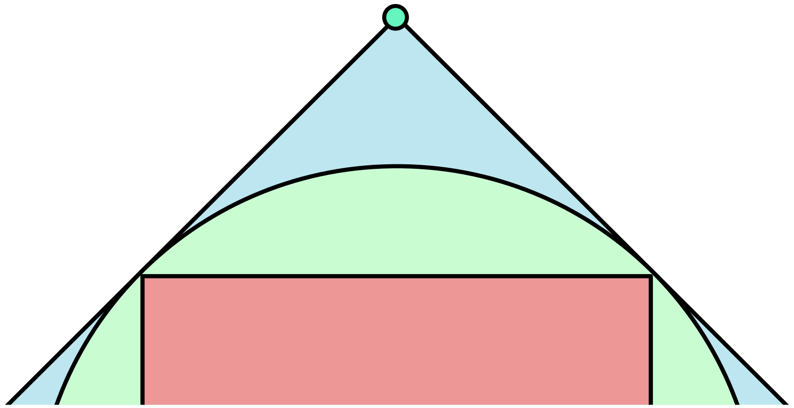
PR Boxes ‘collapse’ communication complexity

With enough PR boxes, ANY function can be decided using a single bit



Wim van Dam. Implausible consequences of superstrong nonlocality. Natural Computing, 12(1): 9–12, (2013).

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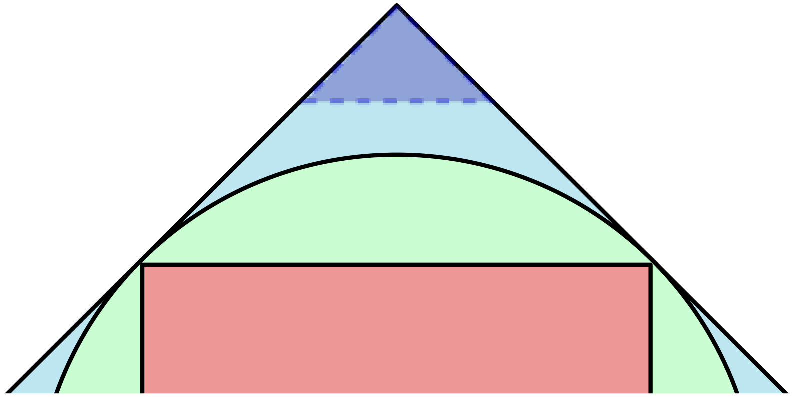
Noisy PR Boxes collapse randomised complexity

Boxes winning CHSH game with $p \geq \frac{3+\sqrt{6}}{6}$ collapse CC



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G. Brassard, H. Buhrman, N. Linden, A. A. Méthot, A. Tapp, and F. Unger, Limit on nonlocality in any world in which communication complexity is not trivial, Phys. Rev. Lett. 96, 250401 (2006).

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What does that tell us? **The capacity must be growing!**

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$\mathrm{P}(ab|xy)$ satsifies IC $\Longrightarrow$ $\mathrm{P}(ab|xy)$ satsifies NTCC

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in a theory independent manner!

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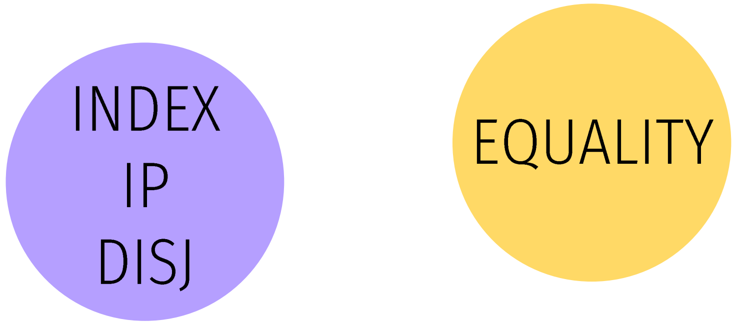

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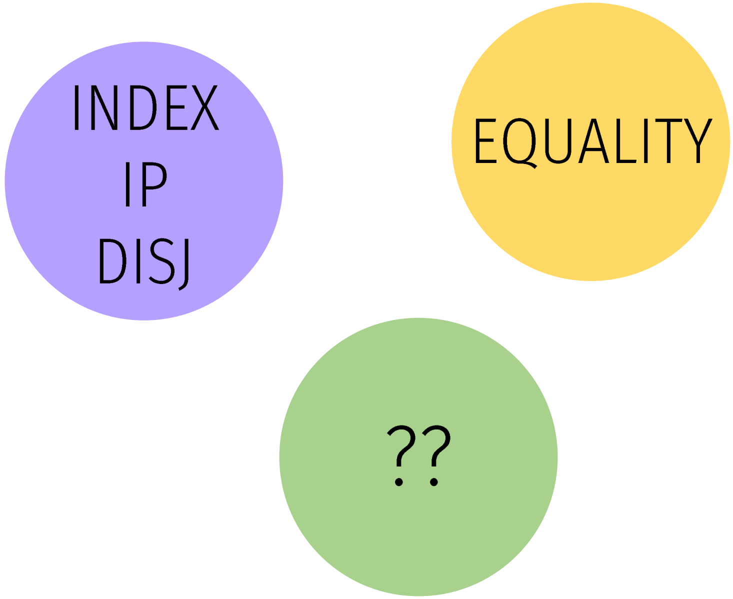

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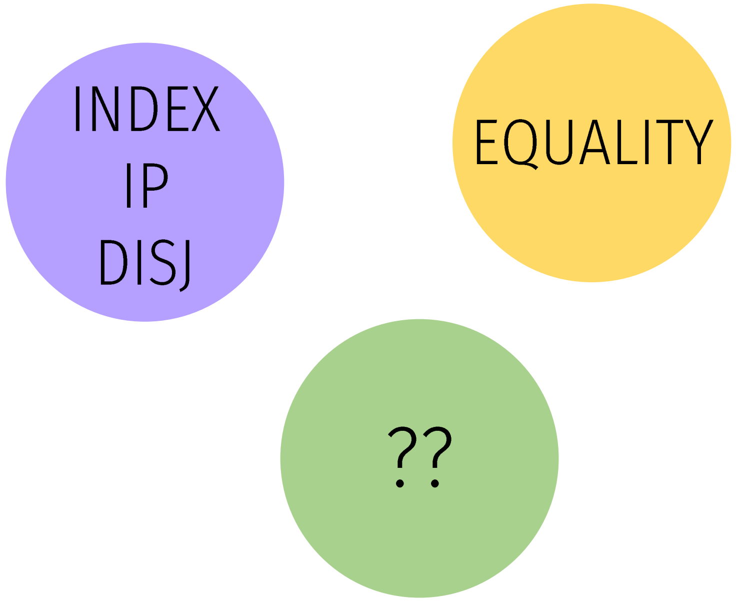

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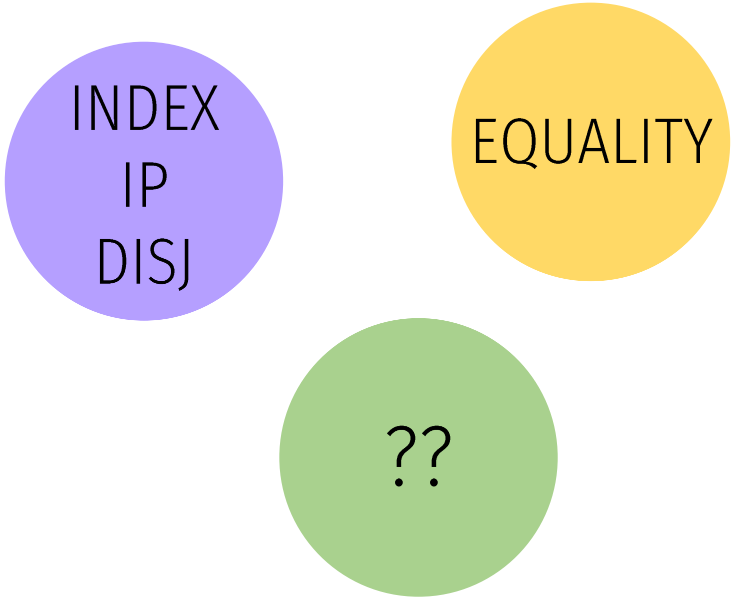

THANK YOU!

Appendix: for the nosy ones

- If two function have identical IC statements, the one which requires less number of PR boxes to decide has more bounding power. Why?

$$P(g = i|x = j, y = k) = \frac{1 + e_c(\,)\ldots(\,)}{d}$$

In the convolution, each bias being smaller than 1 causes the probability and hence the mutual information to be smaller in magnitude since the effective bias is smaller.

- The argument for bounding does not work for the randomised case because if p is probabilistic, it is some complicated function of n, as we see below

$$n(1 - h(p_n)) \leq m(n), \quad p_n = \frac{1}{2} + \sum_i q_i(n)$$

Hence, extracting the dependence of the message size on n by eliminating p_n is not straightforward and you have to consider the protocol in detail to invert the relation, optimise over the best ones and then get a bound.