Assessing Nuclear Spin Registers Coupled to Defects for Quantum Memories

Filippos Dakis

Economou & Barnes Groups

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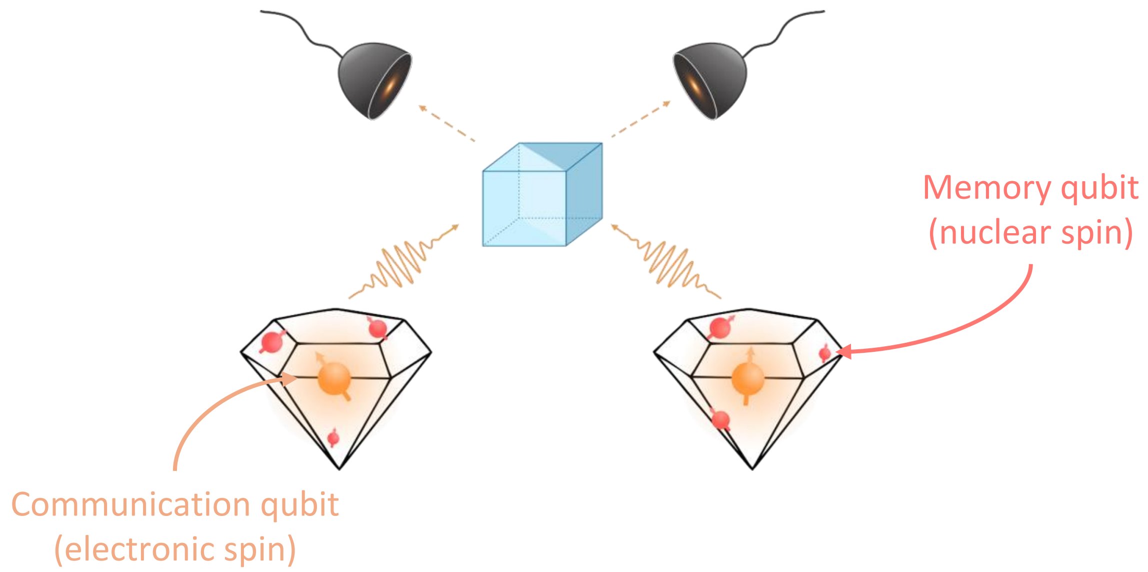
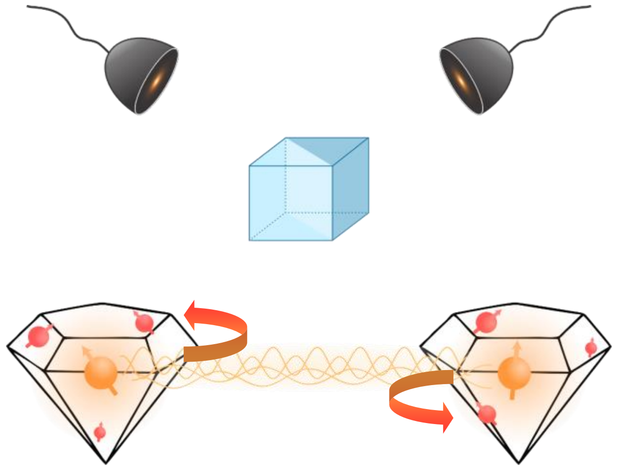
Memory-based repeaters

Generate remote entanglement



Swap into nuclear spin memories



Memory-based repeaters

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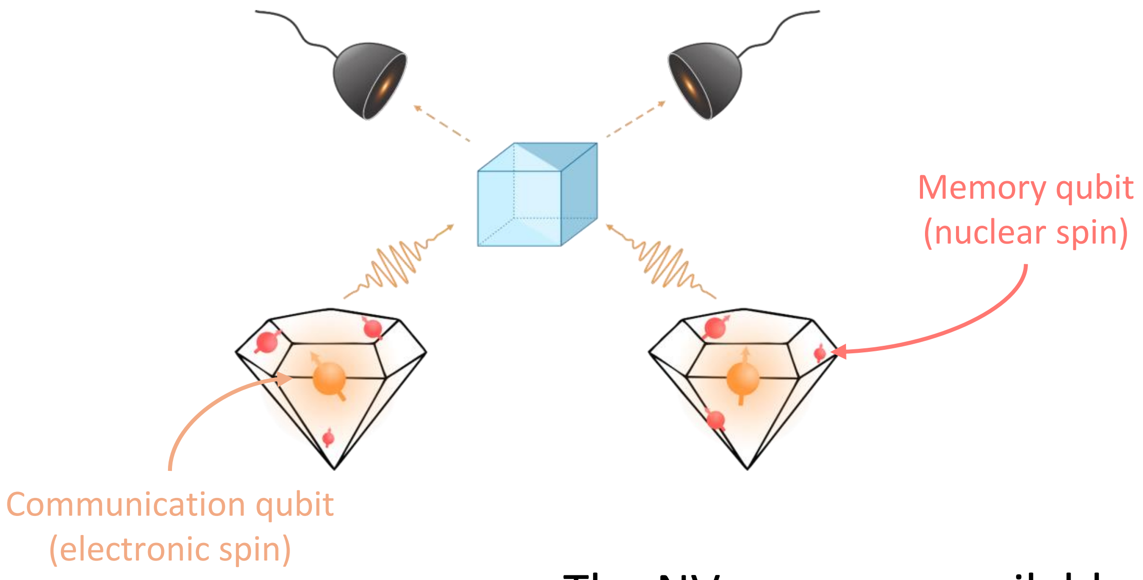

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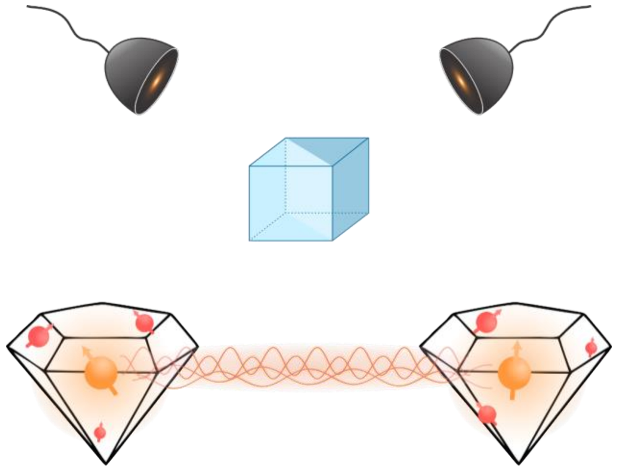
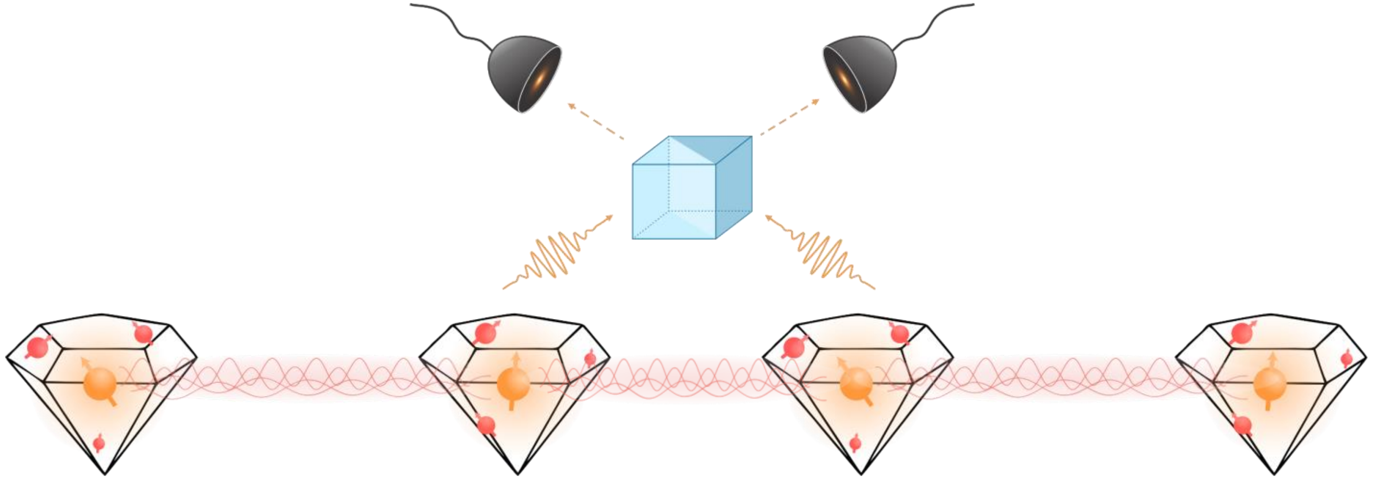


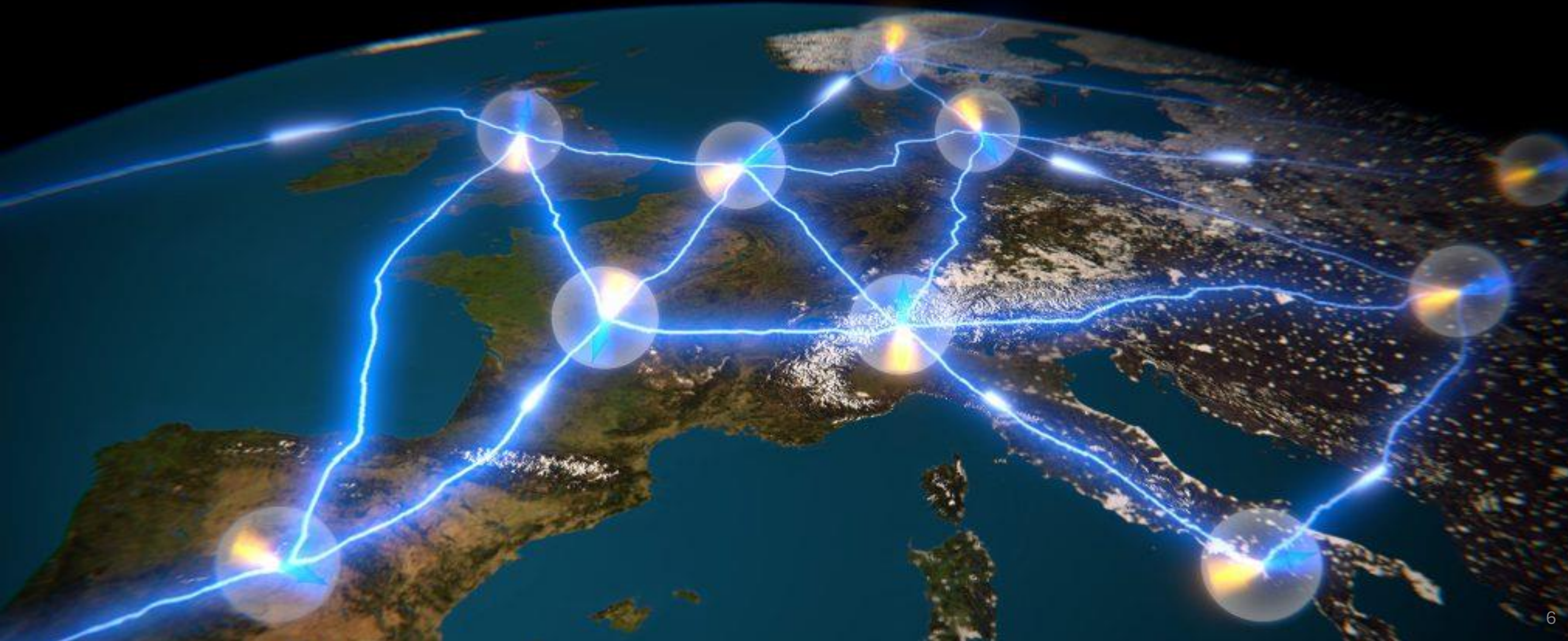
The NVs are now available for entanglement with other NVs



Quantum internet

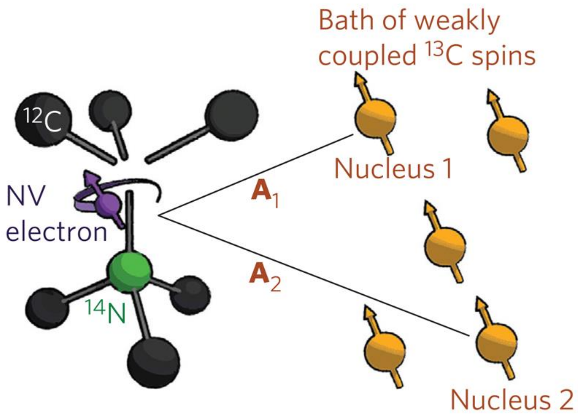
Create and distribute entanglement

Control with high precision/fast the nodes of the network (gates)

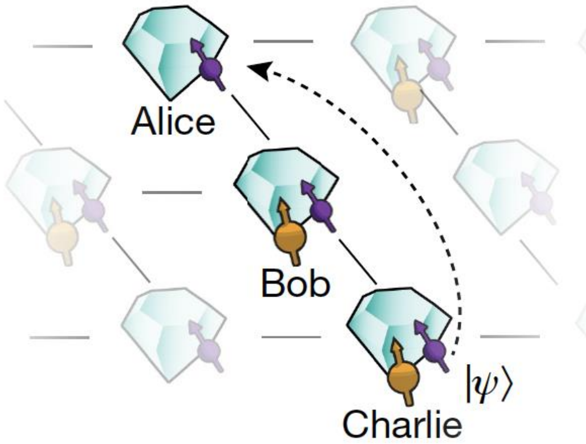


Seminal Experiments

NV (Delft):



T. H. Taminiau et al, *Nat. Nanotechnol*. **9**, 171 (2014)



S. L. N. Hermans et al**,** *Nature* **605**, 663 (2022)

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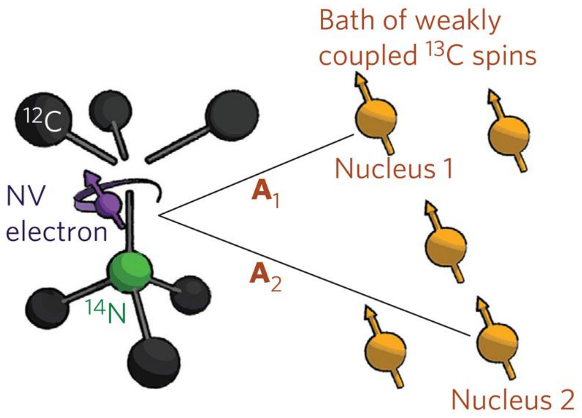

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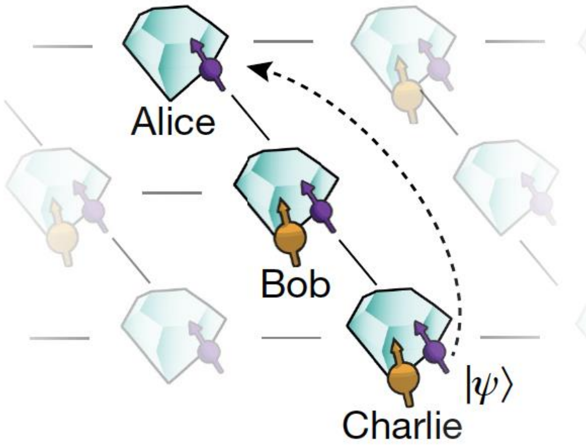
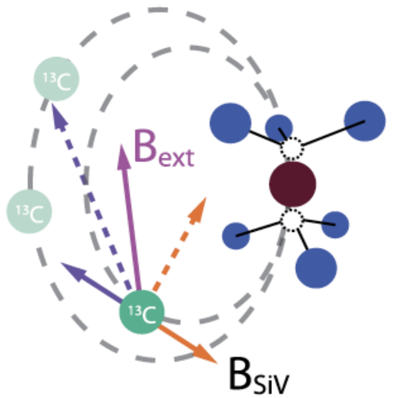


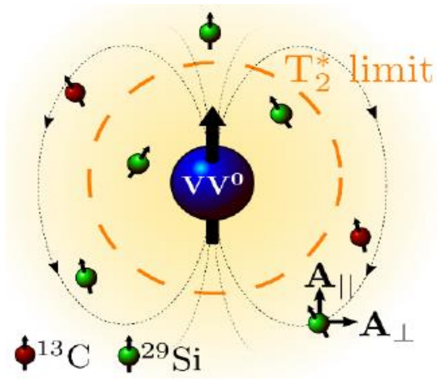
S. L. N. Hermans et al, *Nature* **605**, 663 (2022)

SiV:



C.T. Nguyen et al, *Phys. Rev. Lett.* **123**, 183602 (2019)

SiC divacancy:



A. Bourassa et al, *Nat. Mater.* **19**, 1319 (2020)

Contents

- Introduction
- Single nuclear spin
- Many nuclear spins and formalism
- Effectively assessing qubits in SiC
- Summary

Single spin coupled to defect

Hamiltonian for a nuclear spin (e.g., ^{13}C) coupled to an electronic qubit (e.g., NV center)

Larmor Frequency

Nuclear spin operators

$$H = |0\rangle\langle 0| \otimes \omega_{\mathrm{L}} I_z + |1\rangle\langle 1| \otimes \left[(\omega_{\mathrm{L}} - A_{||}) I_z - A_{\perp} I_x\right]$$

Hyperfine Interactions

$$\Rightarrow \quad H = |0\rangle\langle 0| \otimes H_0 + |1\rangle\langle 1| \otimes H_1$$

The nuclear spin experiences a different Hamiltonian conditional to the state of the electronic spin!

$$U = |0\rangle\langle 0| \otimes R_{n_0}(\phi_0) + |1\rangle\langle 1| \otimes R_{n_1}(\phi_1)$$

2x2 nuclear spin
Rotation matrix

Example

$$U = |0\rangle\langle 0| \otimes R_{n_0}(\phi_0) + |1\rangle\langle 1| \otimes R_{n_1}(\phi_1)$$

$$\phi_0 = -\phi_1 = \pi/2 \qquad \mathbf{n}_0 = \mathbf{n}_1 = \mathbf{x}$$

$$U = \sigma_{00} \otimes \mathrm{R}_x(\pi/2) + \sigma_{11} \otimes \mathrm{R}_x(-\pi/2) = \mathrm{CR}_x(\pi/2)$$

$$\mathrm{CR}_x(\pi/2)\,|{+0}\rangle = |0\rangle\,|-\rangle + |1\rangle\,|+\rangle$$

$$\mathrm{CR}_x(\pi/2) \sim \mathrm{CNOT} = \sigma_{00} \otimes \mathbb{1} + \sigma_{11} \otimes X$$

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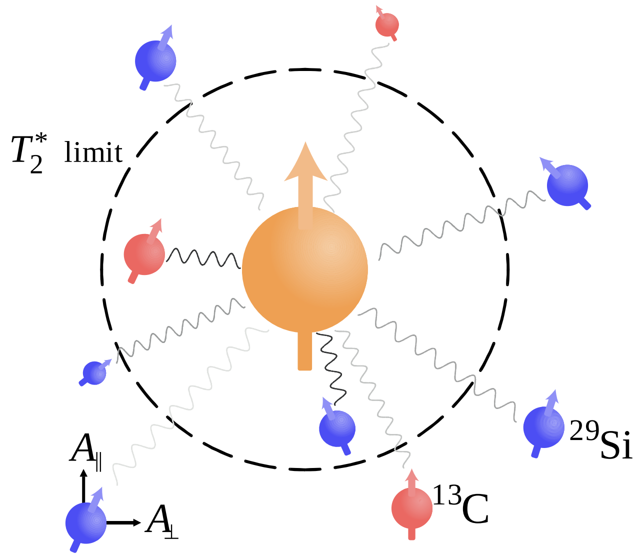
Controlling nuclear spins selectively

What happens when there are many nuclear spins?

Challenges:

- **All** nuclei are **always** coupled to defect (crosstalk)
- Experimental constraint: only defect drive available



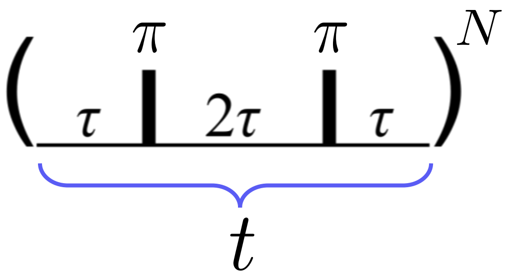
Potential:

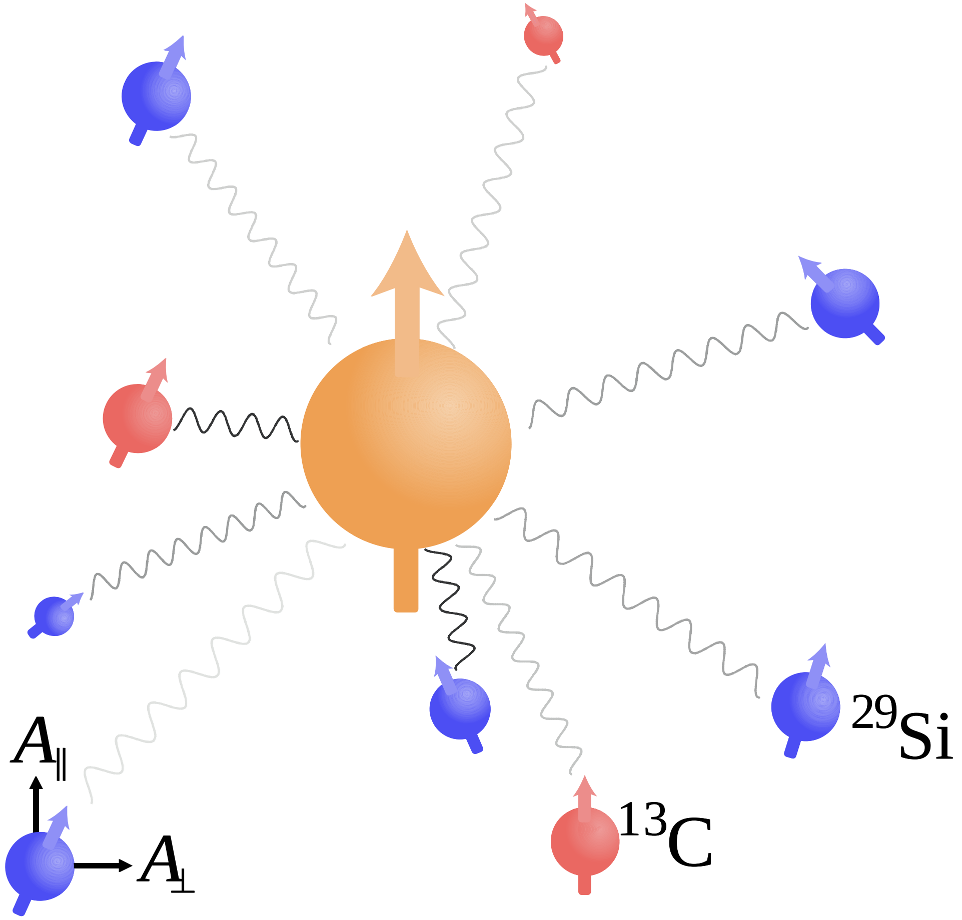
- Large entangled states (q. info, metrology, etc)
- Exploit large coherence times of nuclear spins
- High-fidelity quantum gates

Dynamical Decoupling pulses

Suppress the decoherence caused by the (unwanted) nuclear spins

- CPMG
- UDD-n
- XY-n





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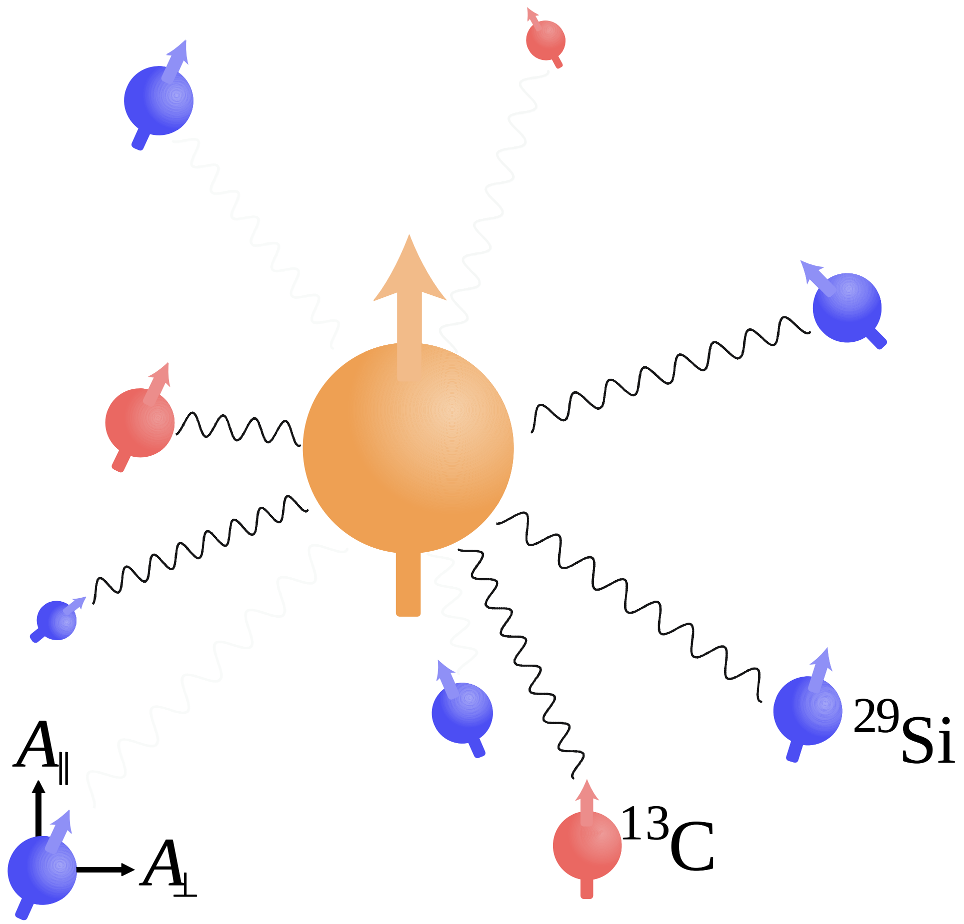
- ➢ XY-n



Also enhance interactions with specific nuclei (registers)

Control knobs:

- Unit time τ_k
- Iterations N

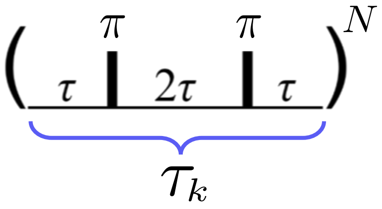
Spin distinguishabity

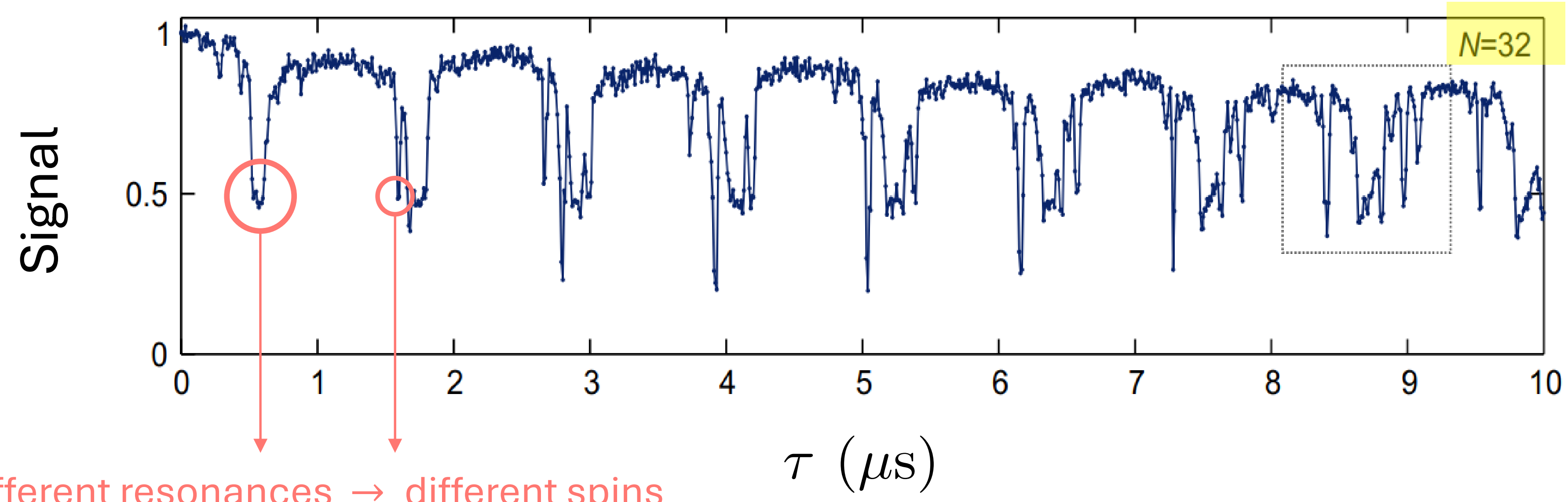
Control knobs:

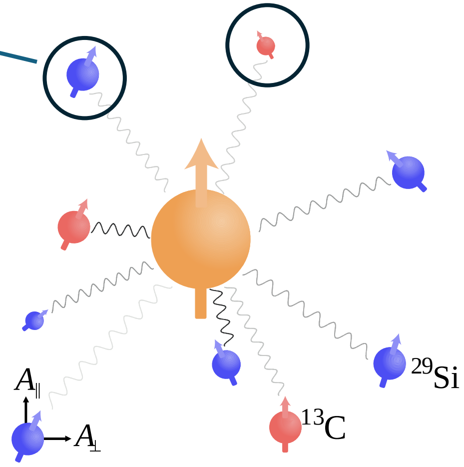
- Unit time τ
- Iterations N

Resonances times

$$\tau_k(A_{||}, A_\perp) \simeq \frac{(2k-1)\pi}{2\omega_L + A_{||}}$$







Different resonances → different spins

- Unit time τ_k
- Iterations N

$$U = \sigma_{00} \otimes \mathrm{R}_x(\pi/2) + \sigma_{11} \otimes \mathrm{R}_x(-\pi/2) = \mathrm{CR}_x(\pi/2)$$

Taminiau et al. *Phys. Rev. Lett.* **109**, 137602 (2012)

Defect coupled to many nuclear spins

$$H = \sum_{j=0}^{1} \sigma_{jj} \otimes \left(H_j^{(1)} \otimes \mathbb{1}_{2^{L-1}} \; + \; \mathbb{1} \otimes H_j^{(2)} \otimes \mathbb{1}_{2^{L-2}} \; + \; \ldots \; + \; \mathbb{1}_{2^{L-1}} \otimes H_j^{(L)} \right)$$

** Neglect nuclear-nuclear interactions*
Weaker than electron-nuclear interactions

Hamiltonian of l^{th} spin

$$H_j^{(l)} = \frac{\omega_L^{(l)} + s_j A_{\parallel}^{(l)}}{2} \sigma_z^{(l)} + \frac{s_j A_{\perp}^{(l)}}{2} \sigma_x^{(l)}$$

Multi-qubit evolution operator

$$U = \sigma_{00} \otimes_{l=1}^{L} R_{\mathbf{n}_0}^{(l)} \left(\phi_0^{(l)} \right) \; + \; \sigma_{11} \otimes_{l=1}^{L} R_{\mathbf{n}_1}^{(l)} \left(\phi_1^{(l)} \right)$$

Rotation of l^{th} spin

E. Takou, E. Barnes and S. E. Economou, *Phys. Rev. X* **13**, 011004 (2023)

One-tangles

Calculate correlations between single nuclear spin & remaining system

$(\epsilon_{\text{rest}||\text{i}^{\text{th}}\text{ spin}})$

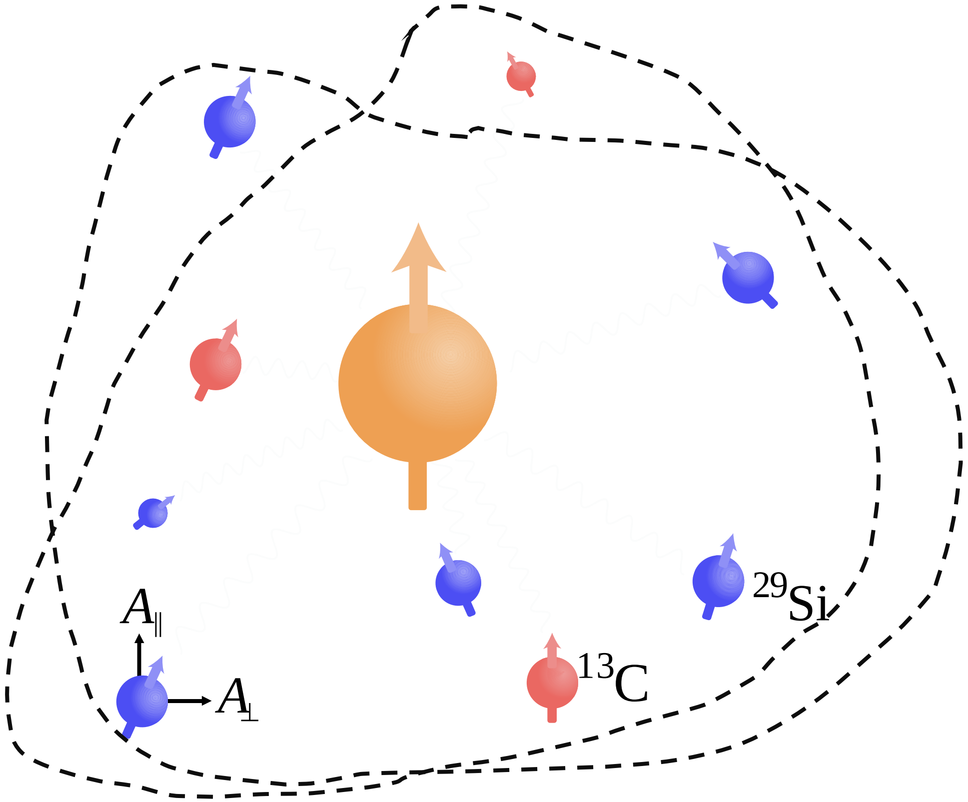
- Shown that the one-tangle for the i[th] spin is

$$\epsilon_{\text{rest}||\text{i}^{\text{th}}\text{ spin}} = 1 - G_1^{(i)}$$

- Reliable metric of nuclear spin selectivity

$\epsilon = 1$ Spin is maximally entangled with de the defect

$\epsilon = 0$ Spin is completely decoupled



* Perfect entangling gates

$(G_1, G_2) = (0, 1)$

T. Linowski et al. *Journal of Physics A* **53**, 125303 (2020)
E. Takou, E. Barnes and S. E. Economou, *Phys. Rev. X* **13**, 011004 (2023)

Quantifying entanglement

For any $\pi-$ pulse sequence

$$G_1 = \left(\cos\frac{\phi_0}{2}\cos\frac{\phi_1}{2} + \mathbf{n}_0\cdot\mathbf{n}_1 \sin\frac{\phi_0}{2}\sin\frac{\phi_1}{2} \right)^2$$

$$G_2 = 1 + \mathbf{n}_0\cdot\mathbf{n}_1 \sin\phi_0 \sin\phi_1 + 2\left(\cos^2\frac{\phi_0}{2}\cos^2\frac{\phi_1}{2} + (\mathbf{n}_0\cdot\mathbf{n}_1)^2 \sin^2\frac{\phi_0}{2}\sin^2\frac{\phi_1}{2} \right)$$

Makhlin invariants (G_1, G_2) encode the non-local part of the gate:

- Depend on nuclear rotation axes & angles
- Directly optimize the gate

* Perfect entangling gates

	Identity	CNOT	SWAP	$\sqrt{\text{SWAP}}$
G_1	1	0	−1	$i/4$
G_2	3	1	−3	0

Makhlin, QIP **1**, 243-252 (2002)
E. Takou, E. Barnes and S. E. Economou, *Phys. Rev. X* **13**, 011004 (2023)

Multipartite entanglement

Gate:

$$U = \sum_{j\in\{0,1\}} \sigma_{jj} \underbrace{\otimes_{k=1}^{K} R_{\mathbf{n}_j^{(k)}}\left(\phi_j^k\right)}_{\text{Register spins}} \underbrace{\otimes_{l=1}^{L-K} R_{\mathbf{n}_j^{(K+l)}}\left(\phi_j^{(K+l)}\right)}_{\text{Bath spins}}$$

Target gate:

$$U_0 = \sum_{j\in\{0,1\}} \sigma_{jj} \otimes_{k=1}^{K} R_{\mathbf{n}_j^{(k)}}\left(\phi_j^k\right)$$

Kraus Operators

$$E_i = \sum_{j=0,1} \underbrace{c_j^{(i)} p_j^{(i)}}_{\text{Bath}} \sigma_{jj} \underbrace{\otimes_{k=1}^{K} R_{\boldsymbol{n}_j^{(k)}}\left(\phi_j^{(k)}\right)}_{\text{Register}}$$

Fidelity:

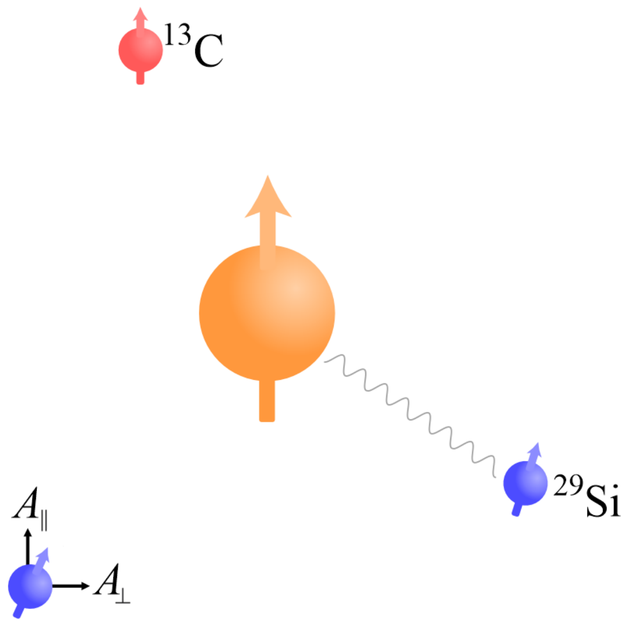
$$F = \frac{1}{m(m+1)} \sum_k \mathrm{Tr}\left[(U_0^\dagger E_k)^\dagger U_0^\dagger E_k\right] + \left|\mathrm{Tr}\left[U_0^\dagger E_k\right]\right|^2$$

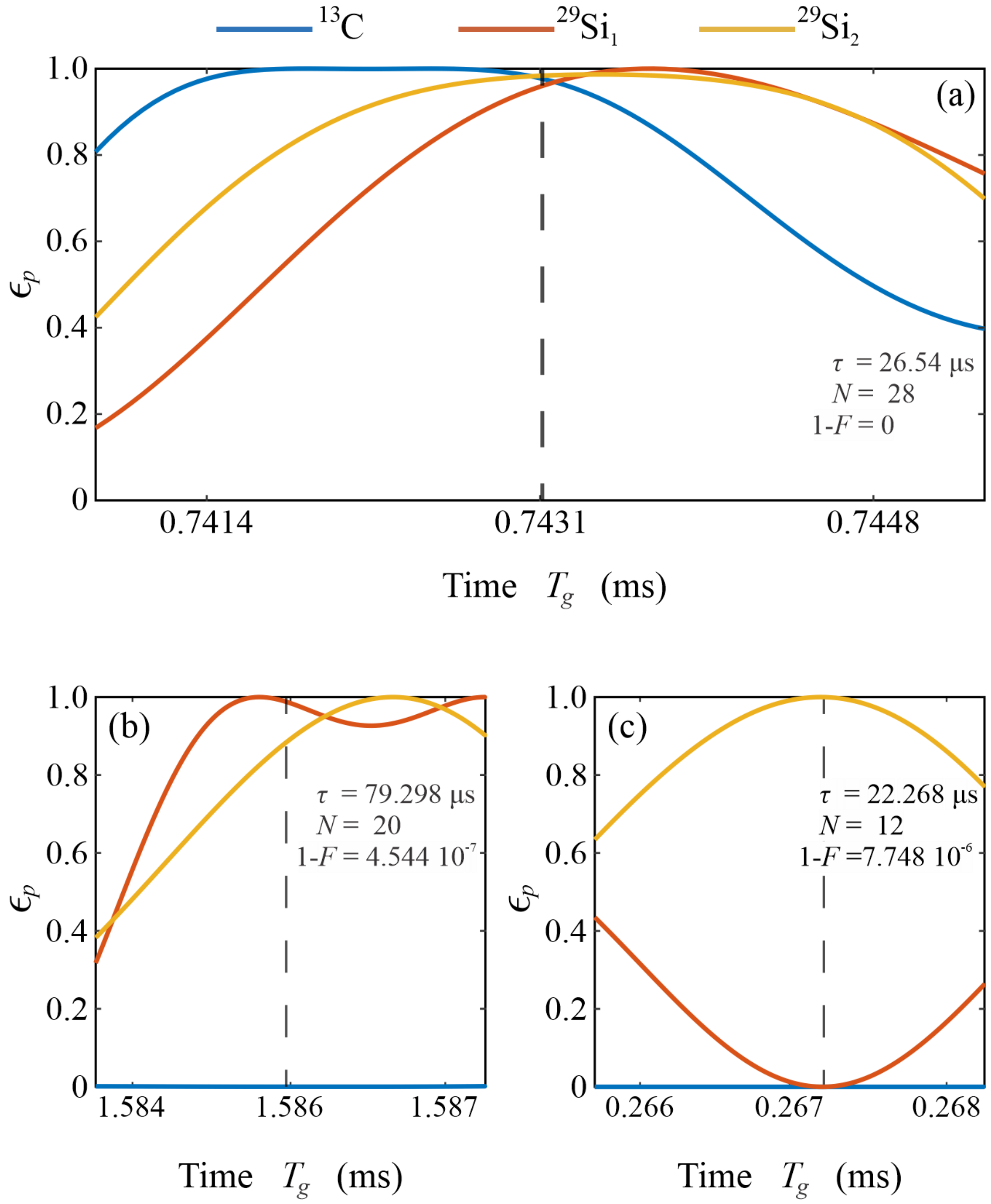
Multipartite entanglement with single shot operation

Chose N^* and τ such that:

- ϵ_p is maximized for selected spins
- ϵ_p is minimized for unwanted spins
- Short gate time $N^*\tau = T_g < T_2^*$

Example





Questions we want to answer

- What DD sequences perform best for various cases?
- How does Fidelity deteriorate with increasing register and bath sizes?
- What are the protocols for achieving entangling states for (realistic) SiC defects?

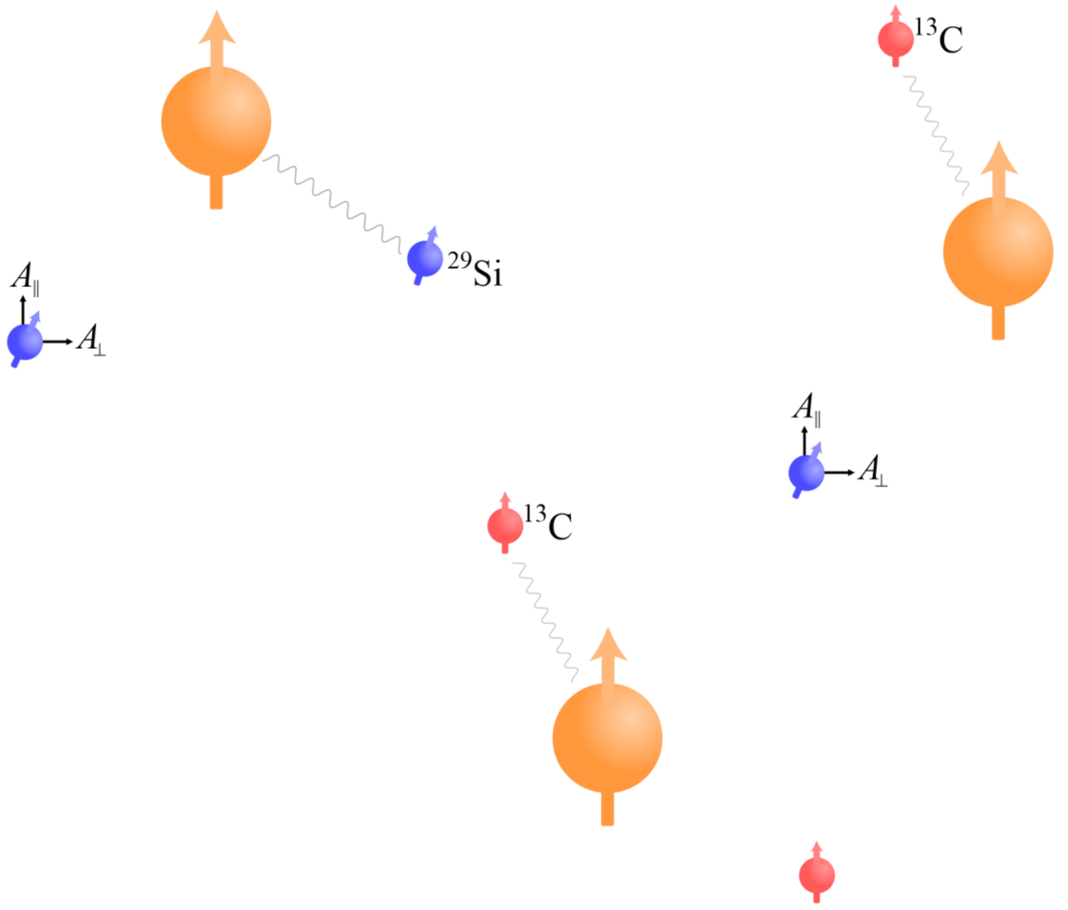
Help experimental physicists conduct experiments!

Thessaloniki, Greece 2019

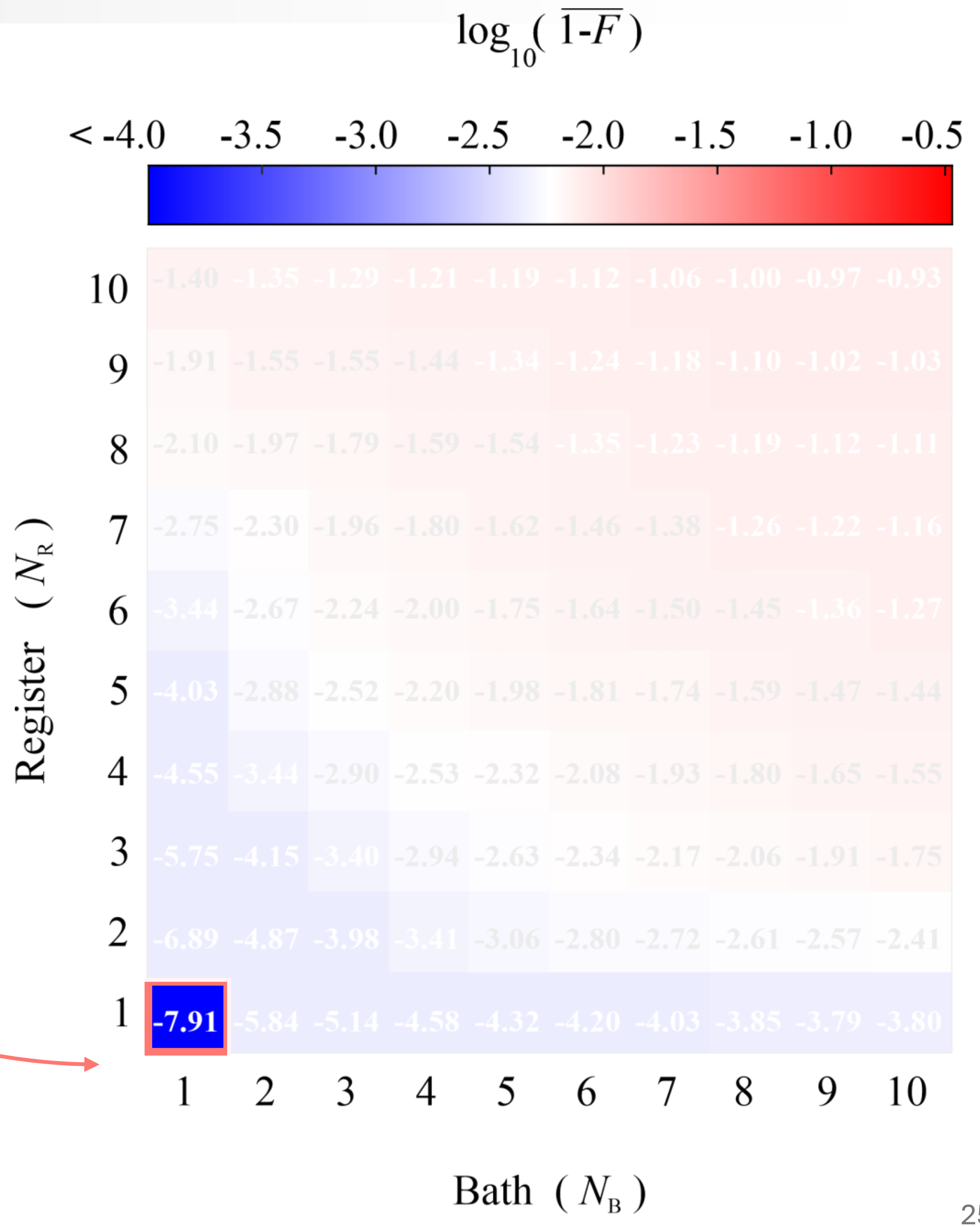
Contents

Monovacancy in SiC

Defect: **S** = 3/2
Nuclear spins: ^{13}C, ^{29}Si
Proportion of spins: $^{29}Si/^{13}C = 4.27$
Gyromagnetic ratio: $\gamma_C > 0$, $\gamma_{Si} < 0$



Average over 200 realizations



F. Dakis *et al.* arxiv.org/abs/2405.10778, *Phys. Rev. Applied, to appear*

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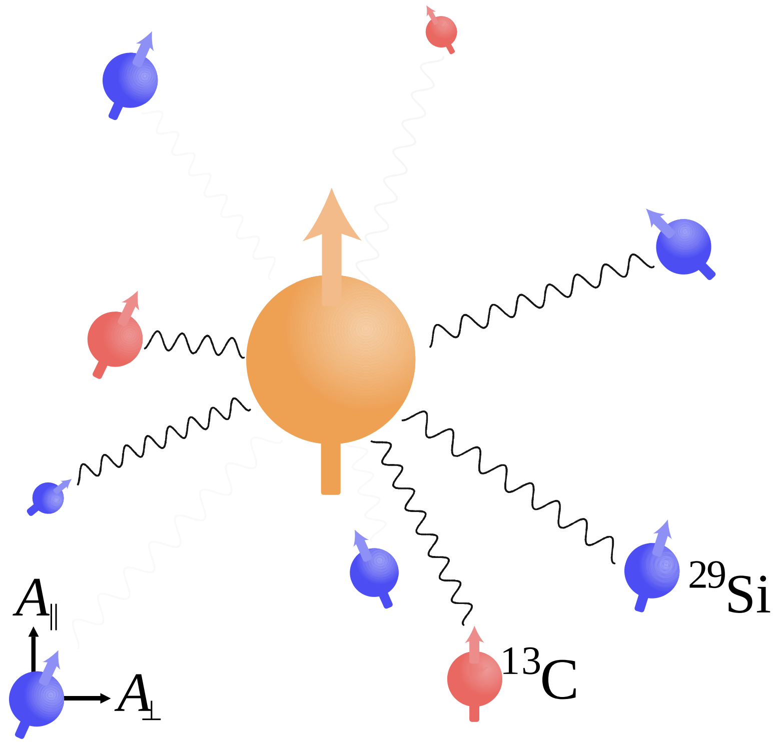

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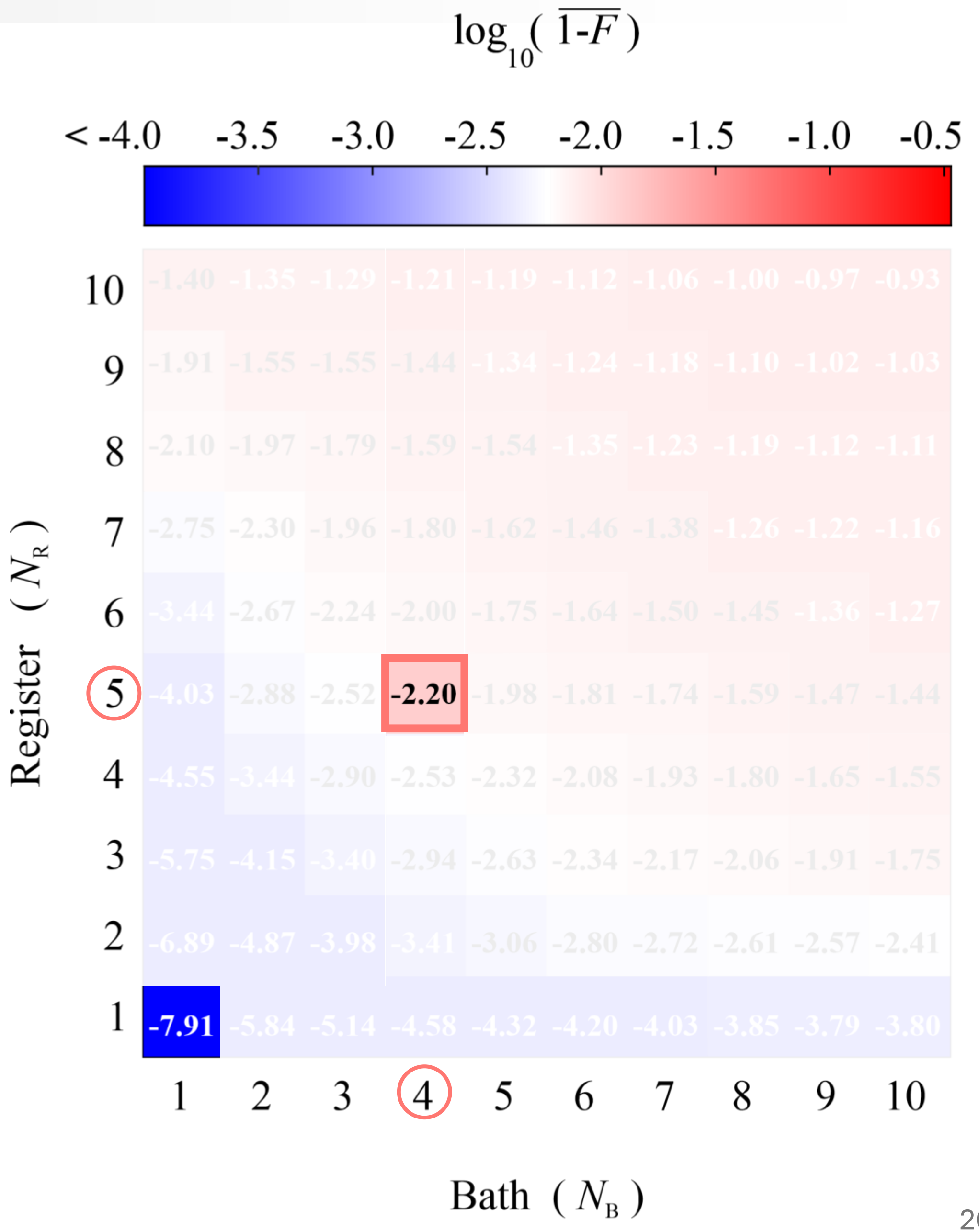

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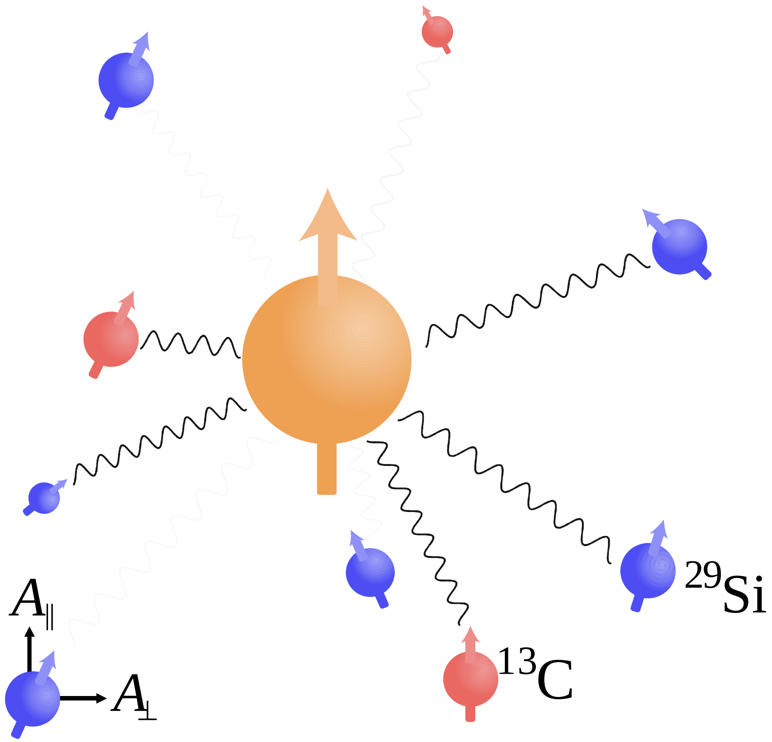

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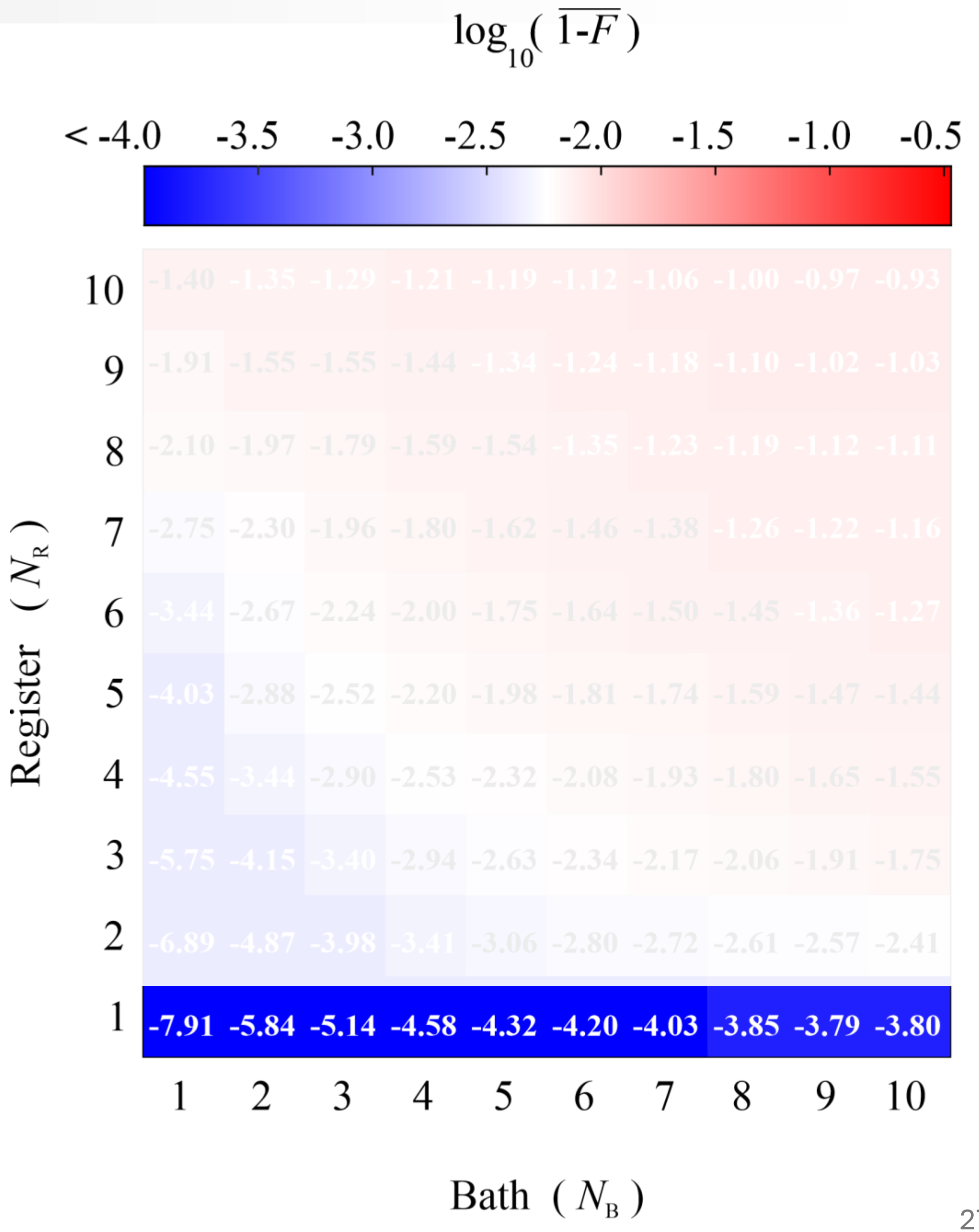

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- Use to guide experiments/assess viability

Average over
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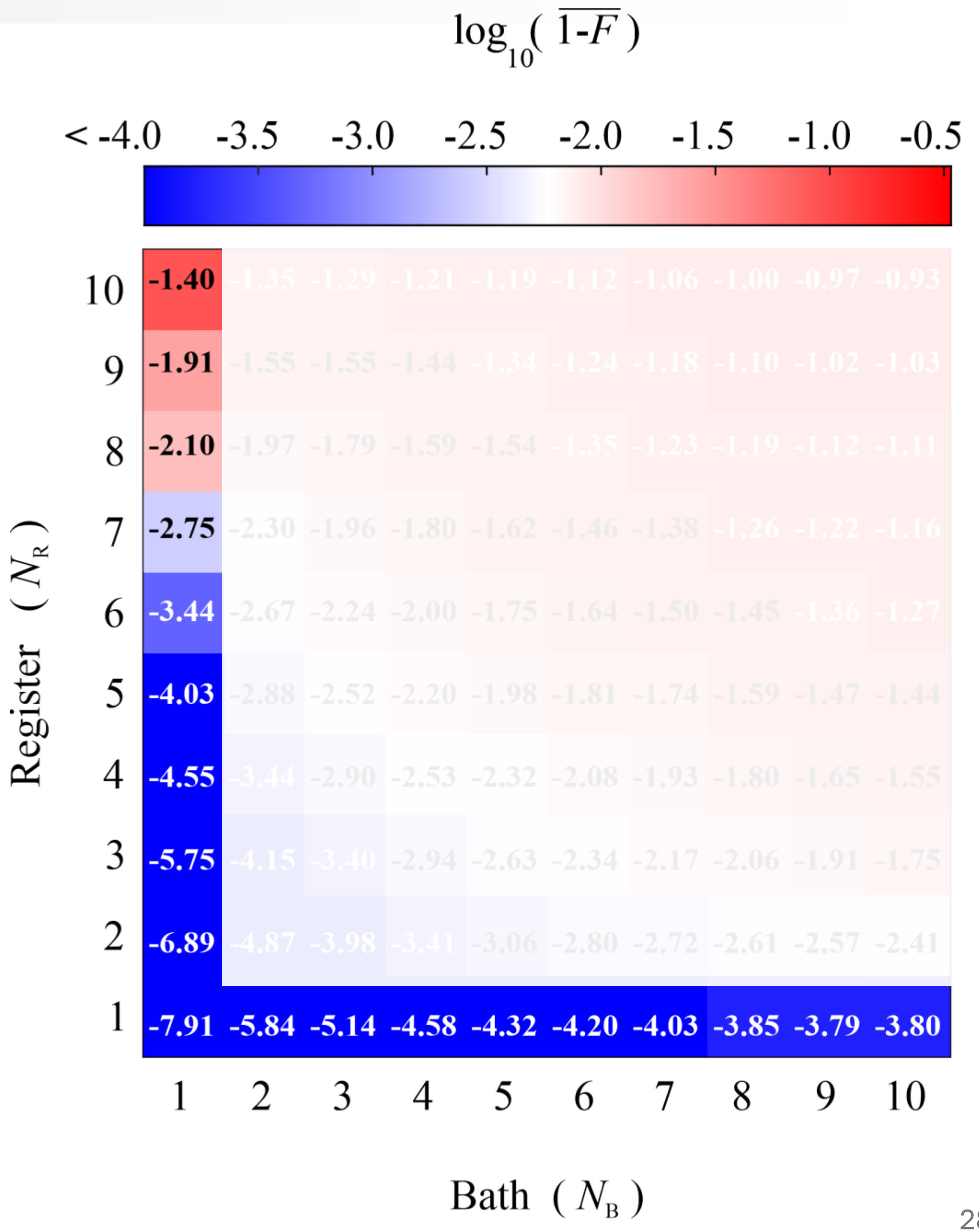

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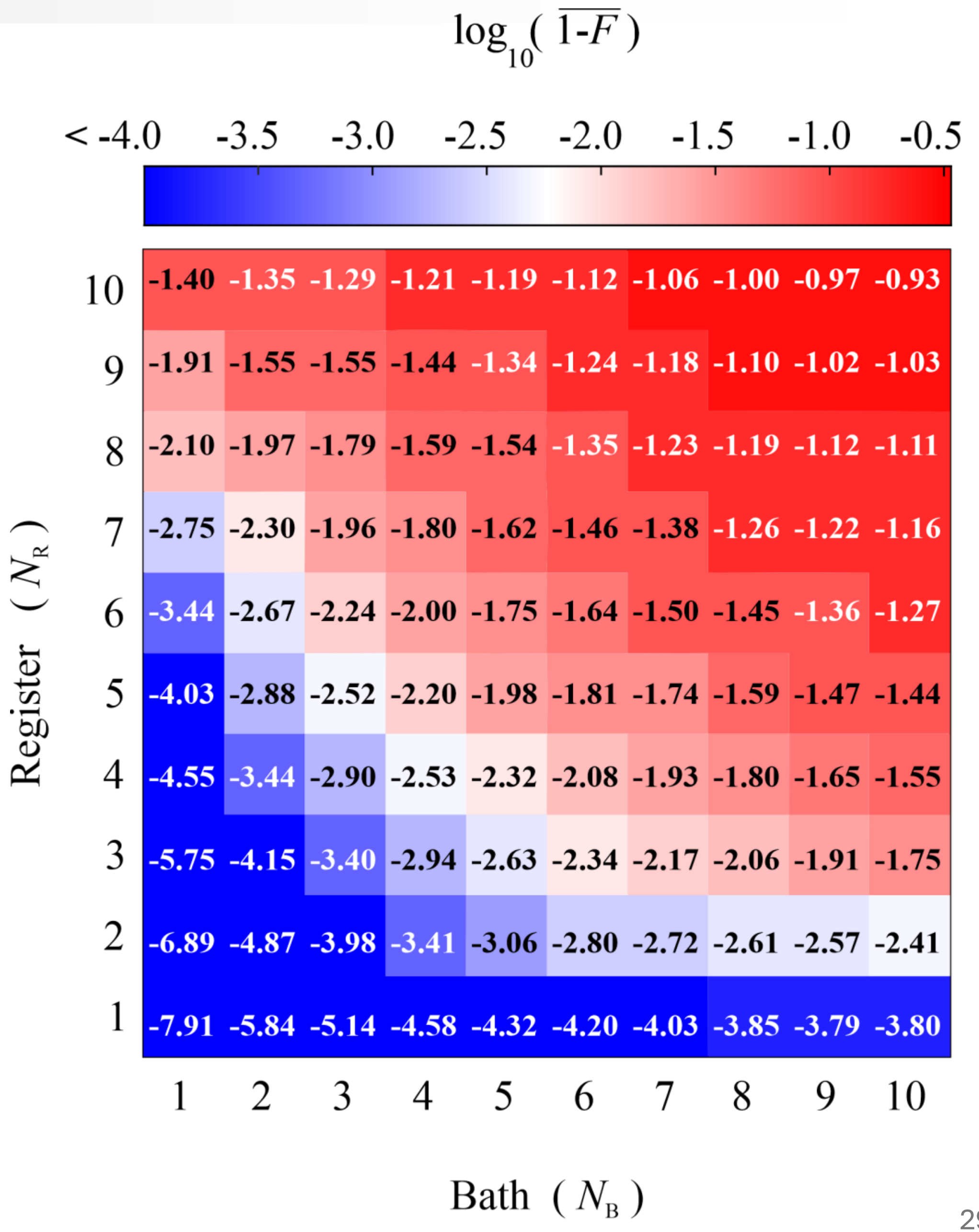

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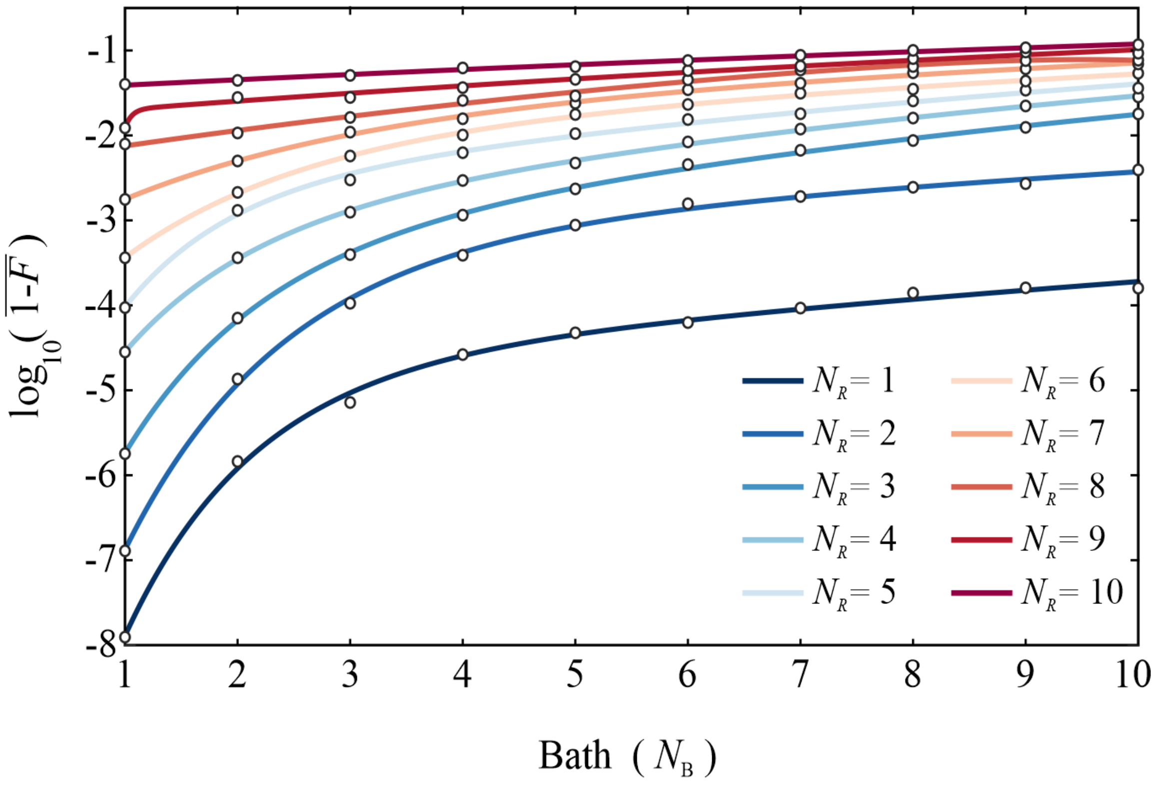

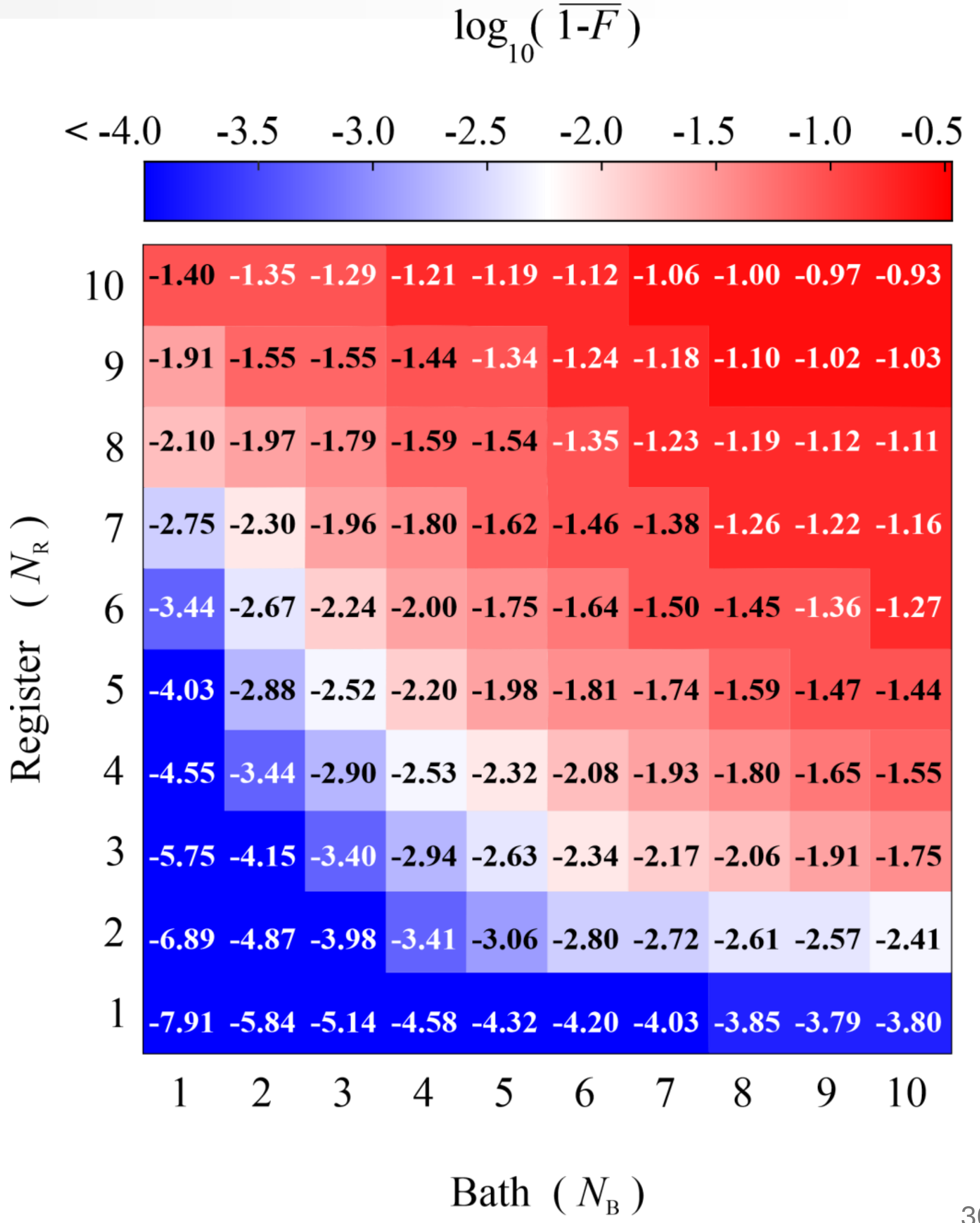

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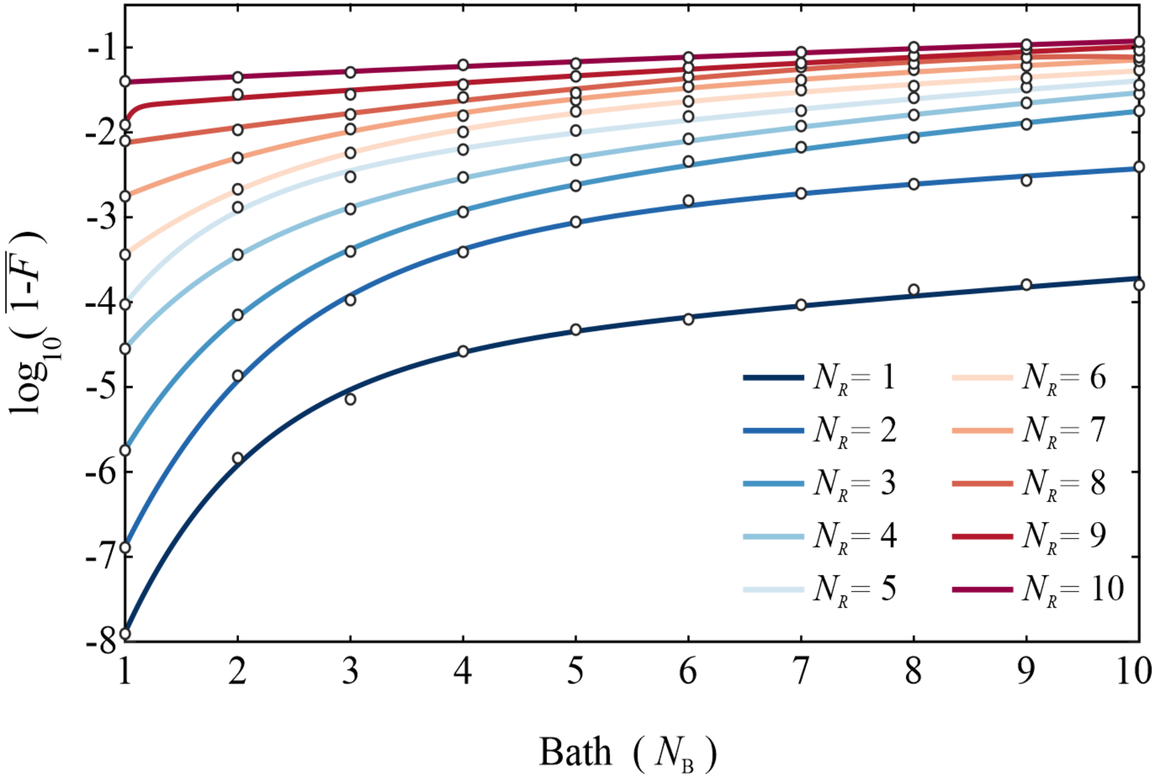

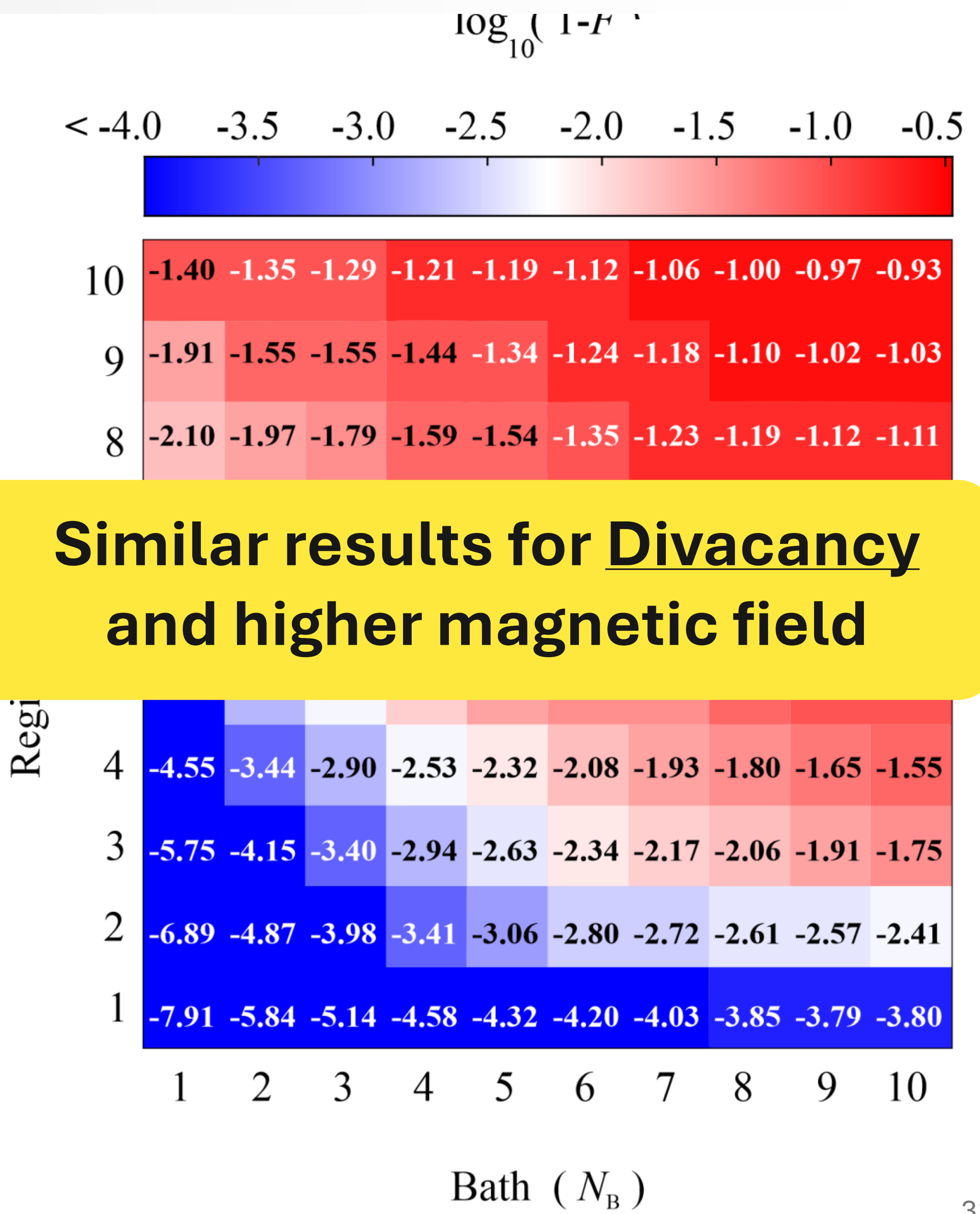

Similar results for Divacancy and higher magnetic field

F. Dakis *et al.* arxiv.org/abs/2405.10778, *Phys. Rev. Applied, to appear*

What DD pulse sequece operates better?

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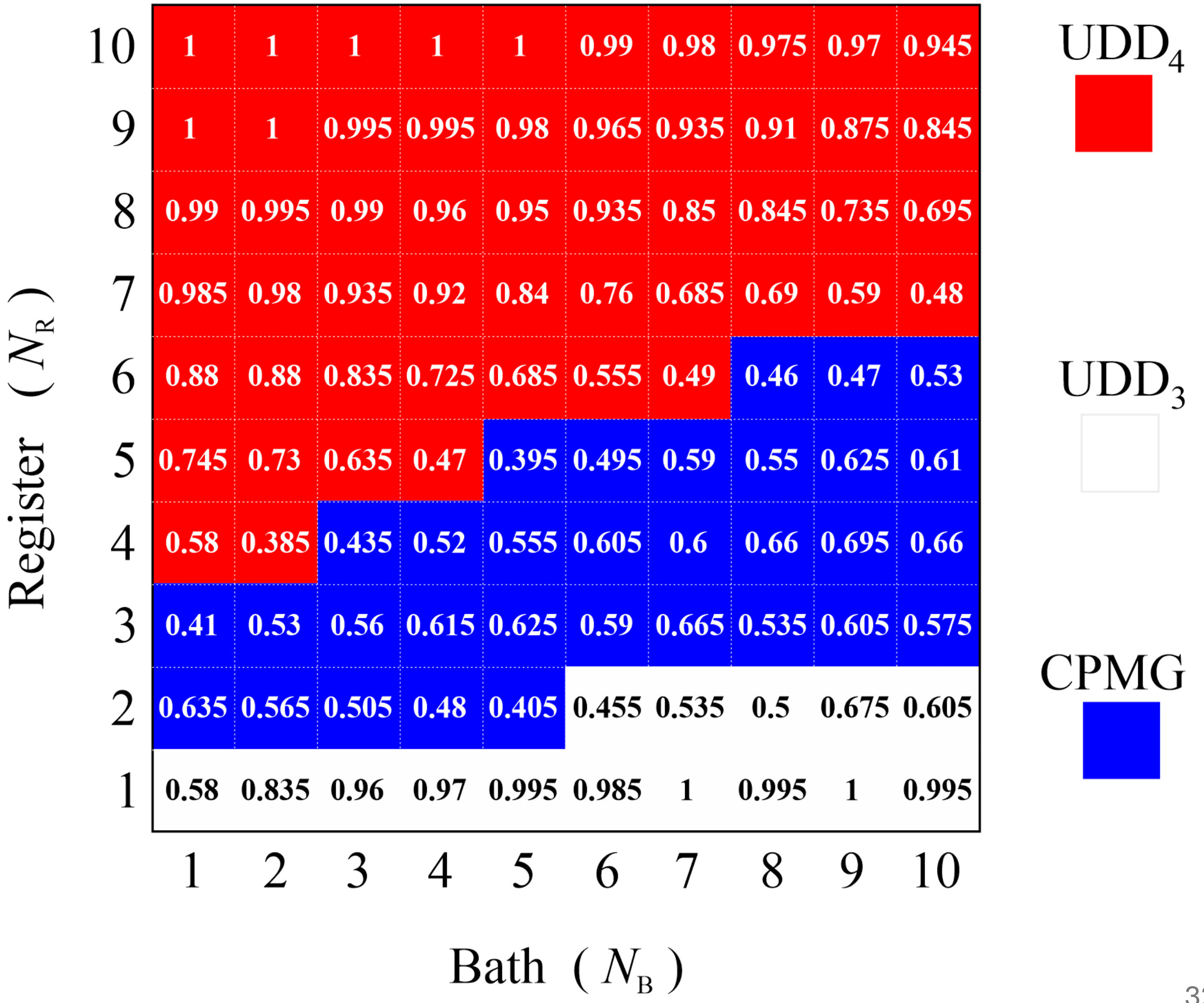

N_R \ N_B	1	2	3	4	5	6	7	8	9	10
10	1	1	1	1	1	0.99	0.98	0.975	0.97	0.945
9	1	1	0.995	0.995	0.98	0.965	0.935	0.91	0.875	0.845
8	0.99	0.995	0.99	0.96	0.95	0.935	0.85	0.845	0.735	0.695
7	0.985	0.98	0.935	0.92	0.84	0.76	0.685	0.69	0.59	0.48
6	0.88	0.88	0.835	0.725	0.685	0.555	0.49	0.46	0.47	0.53
5	0.745	0.73	0.635	0.47	0.395	0.495	0.59	0.55	0.625	0.61
4	0.58	0.385	0.435	0.52	0.555	0.605	0.6	0.66	0.695	0.66
3	0.41	0.53	0.56	0.615	0.625	0.59	0.665	0.535	0.605	0.575
2	0.635	0.565	0.505	0.48	0.405	0.455	0.535	0.5	0.675	0.605
1	0.58	0.835	0.96	0.97	0.995	0.985	1	0.995	1	0.995

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Sign difference in gyrgomagnetic ratios

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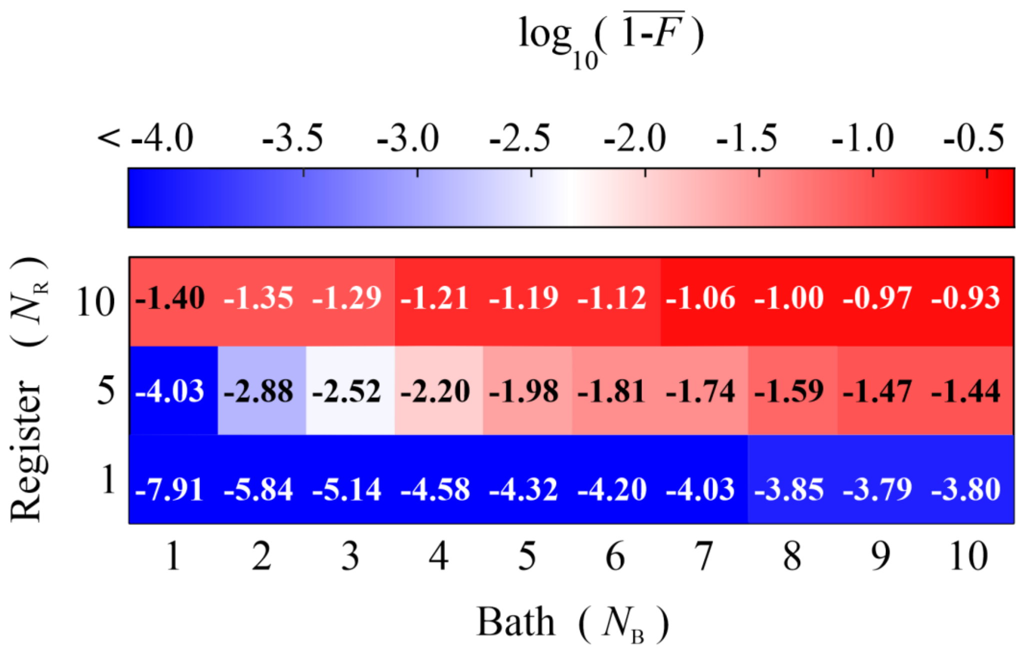
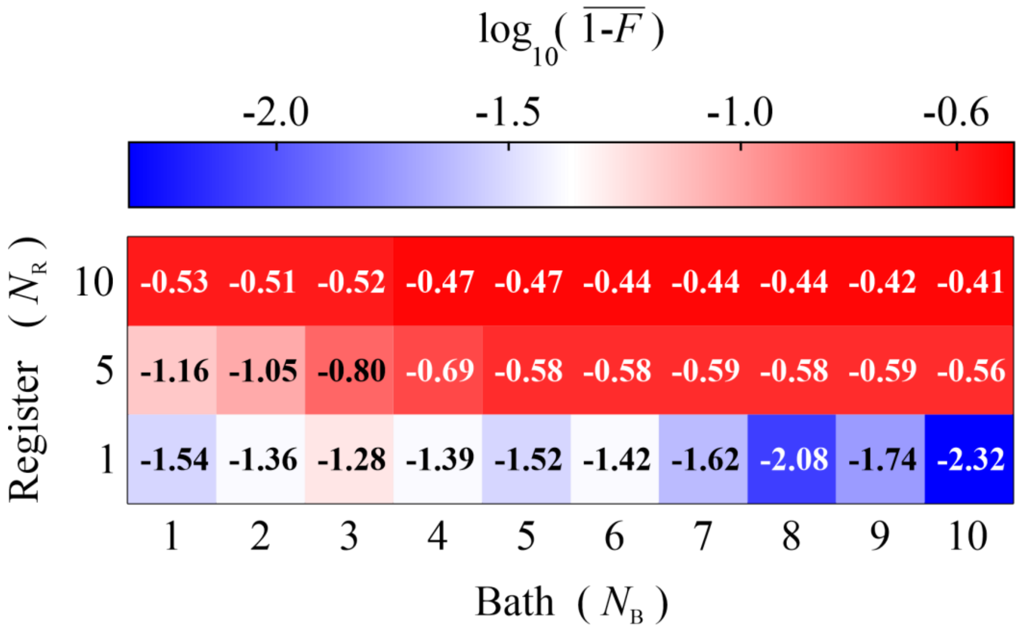
Proportion of spins: $^{29}Si/^{13}C = 4.27$

$\gamma_C > 0, \quad \gamma_{Si} < 0$



$\gamma_C > 0, \quad \gamma_{Si} < 0$



F. Dakis *et al.* arxiv.org/abs/2405.10778, *Phys. Rev. Applied, to appear*

Contents

- Introduction
- Single nuclear spin
- Many nuclear spins and formalism
- Effectively assessing qubits in SiC
- Summary

Summary

- Discussed the challenges and the potential in spins coupled to defects
- Gate fidelity deteriotates faster with size of the register
- Showed the register-bath combinations where different DD pulses operate the best
- Developed high-throughput characterization method for arbitrary defects/host materials/registers

Dakis, Takou, Barnes, Economou. arxiv.org/abs/2405.10778
Phys. Rev. Applied, to appear

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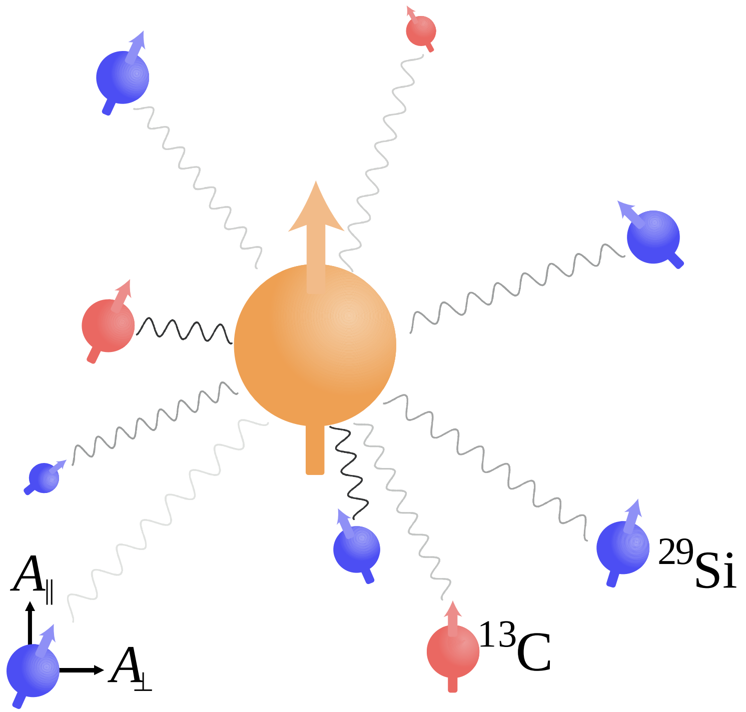

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