arXiv > quant-ph > arXiv:2405.08743

Robust self-testing of Bell inequalities tilted for maximal loophole-free nonlocality

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Motivation: Maximizing loophole-free nonlocality

1. Limited detection efficiency of off-the-shelf detectors,

$$\eta_0 < 1$$

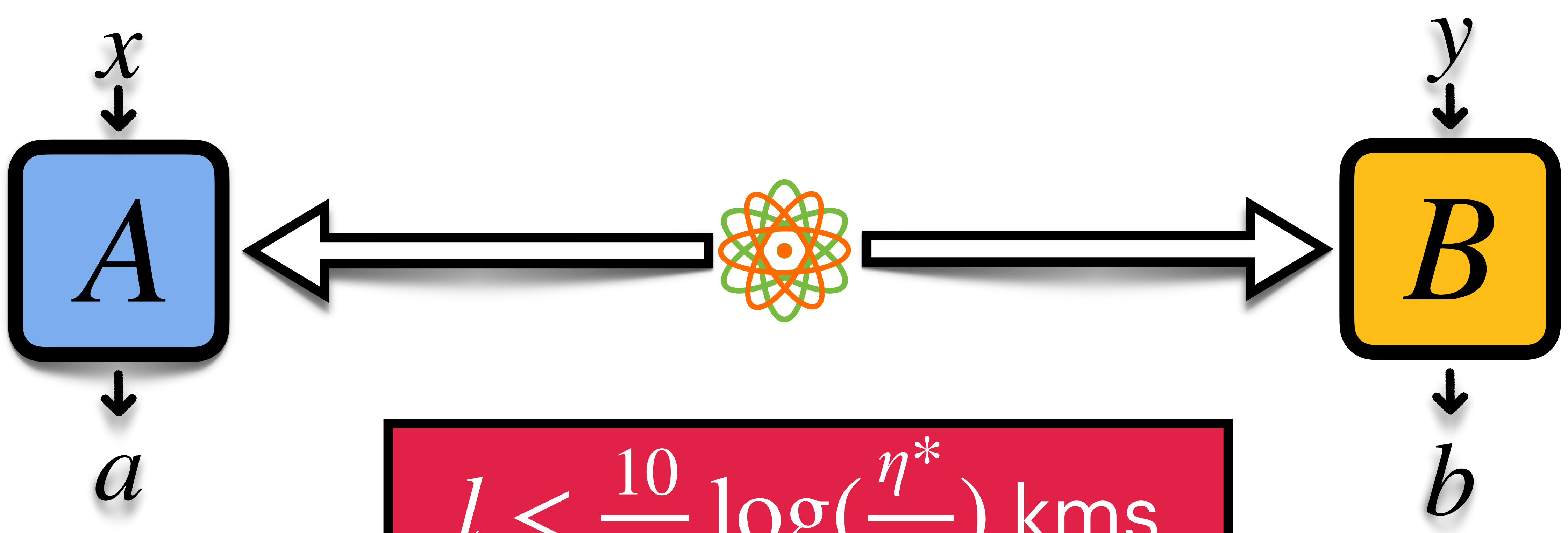
2. Exponential decay of effective detection efficiency,

$$\eta = \eta_0 10^{\frac{-\alpha l}{10}} \lll 1$$

3. High threshold critical detection efficiency,

$$\eta > \eta^*$$

Previous research focussed on minimizing η^*



$$l < \frac{10}{\alpha} \log(\frac{\eta^*}{\eta_0}) \text{ kms}$$

However, for real-world applications to be effective, mere violation of a Bell inequality is insufficient!

Instead, their efficacy requires a high degree of loophole-free nonlocality!

Overview of our findings

- We solve a largely overlooked application-oriented question:

Which quantum strategies yield the maximum loophole-free nonlocality in the presence of inefficient detectors?

Quantum strategies that maximally violate a tilted version of the Bell inequality ideally, yield the maximum loophole-free violation in the presence of inefficient detectors!

- We completely solve the CHSH scenario:

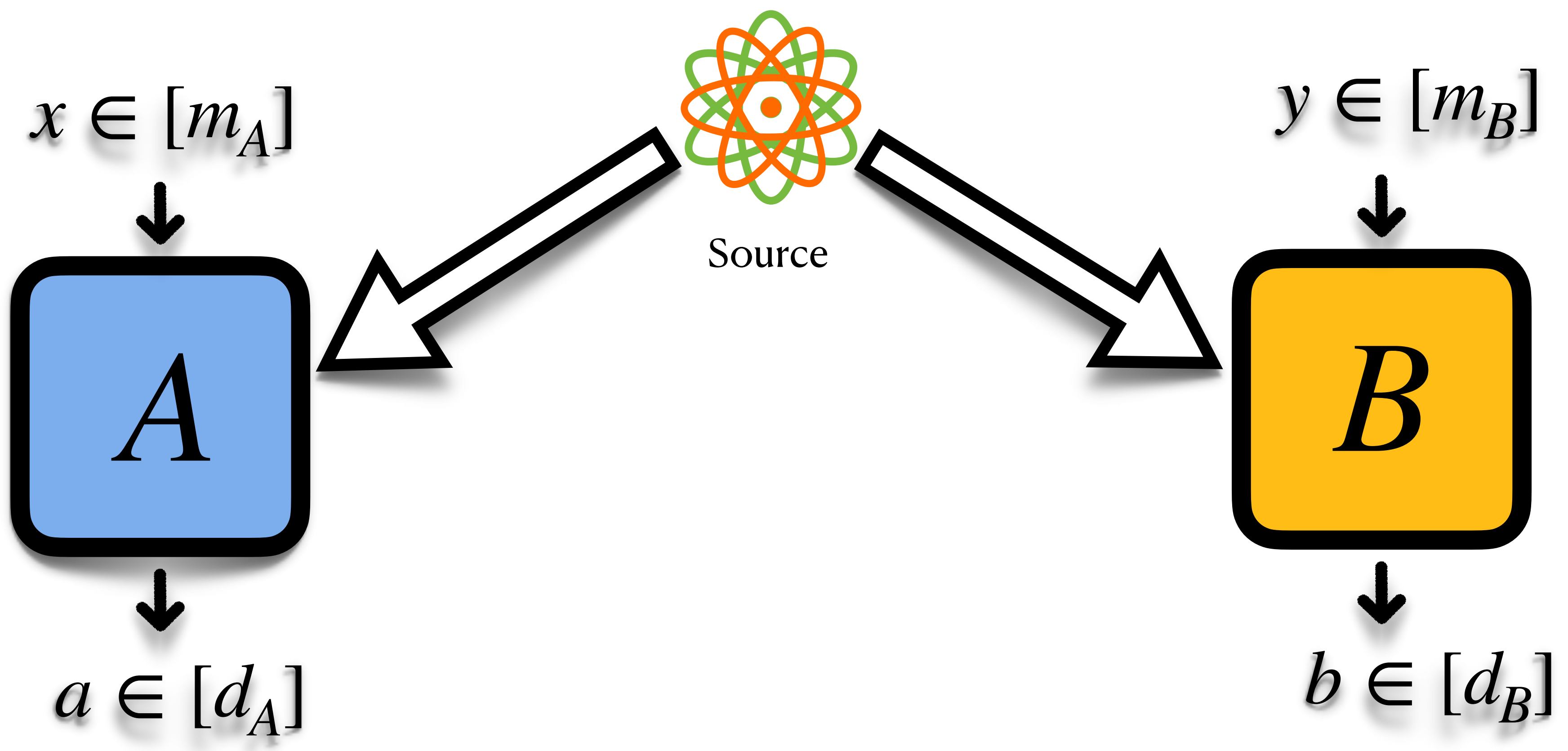
We find robust **analytical self-testing statements** for doubly titled CHSH inequalities, entailing the **unique optimal quantum strategies,** for any specification of detection efficiencies!

- As a byproduct, we uncover an intriguing phenomenon:

The **explosion of NPA levels** in the simplest Bell scenario!

Preliminaries

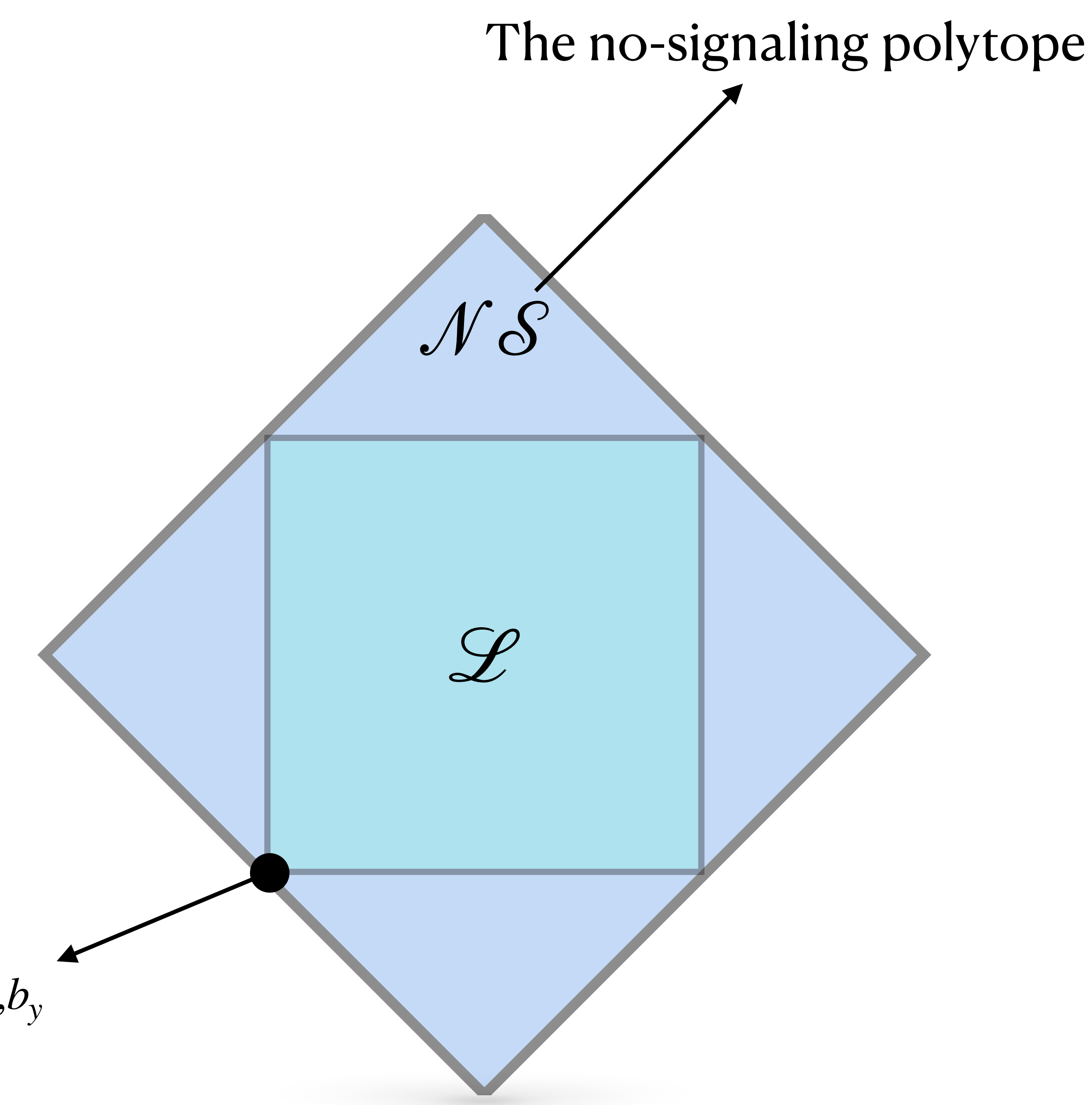
- Bipartite Bell experiments and experimental behavior



$$\mathbf{p} \equiv \{p(ab|xy)\}$$

- Local causal behavior and the local polytope $\mathscr{L}$

$$\mathbf{p} \equiv \{p(ab|xy) = \int_\Lambda d\lambda p(\lambda) p(a|x\lambda) p(b|y\lambda)\} \in \mathscr{L}$$



Preliminaries

- Bell inequalities

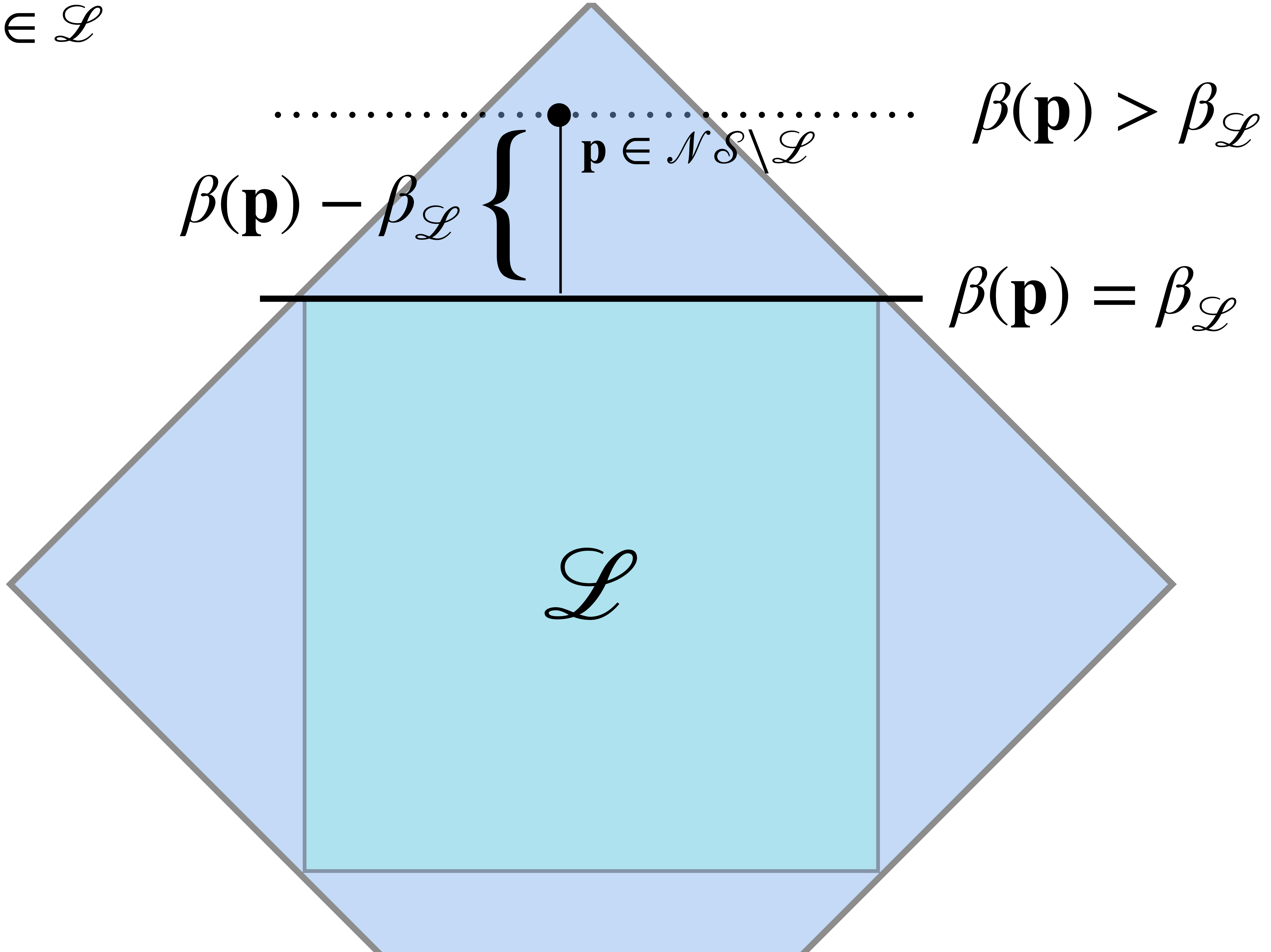
$$\beta(\mathbf{p}) := \sum_{a,b,x,y} c_{ab}^{xy} p(ab\,|\,xy) \leq \beta_{\mathscr{L}}, \ \ \forall \mathbf{p} \in \mathscr{L}$$

- Nonlocal behaviors

$$\mathbf{p} \in \mathcal{N}\mathcal{S} \backslash \mathscr{L} \implies \beta(\mathbf{p}) > \beta_{\mathscr{L}}$$

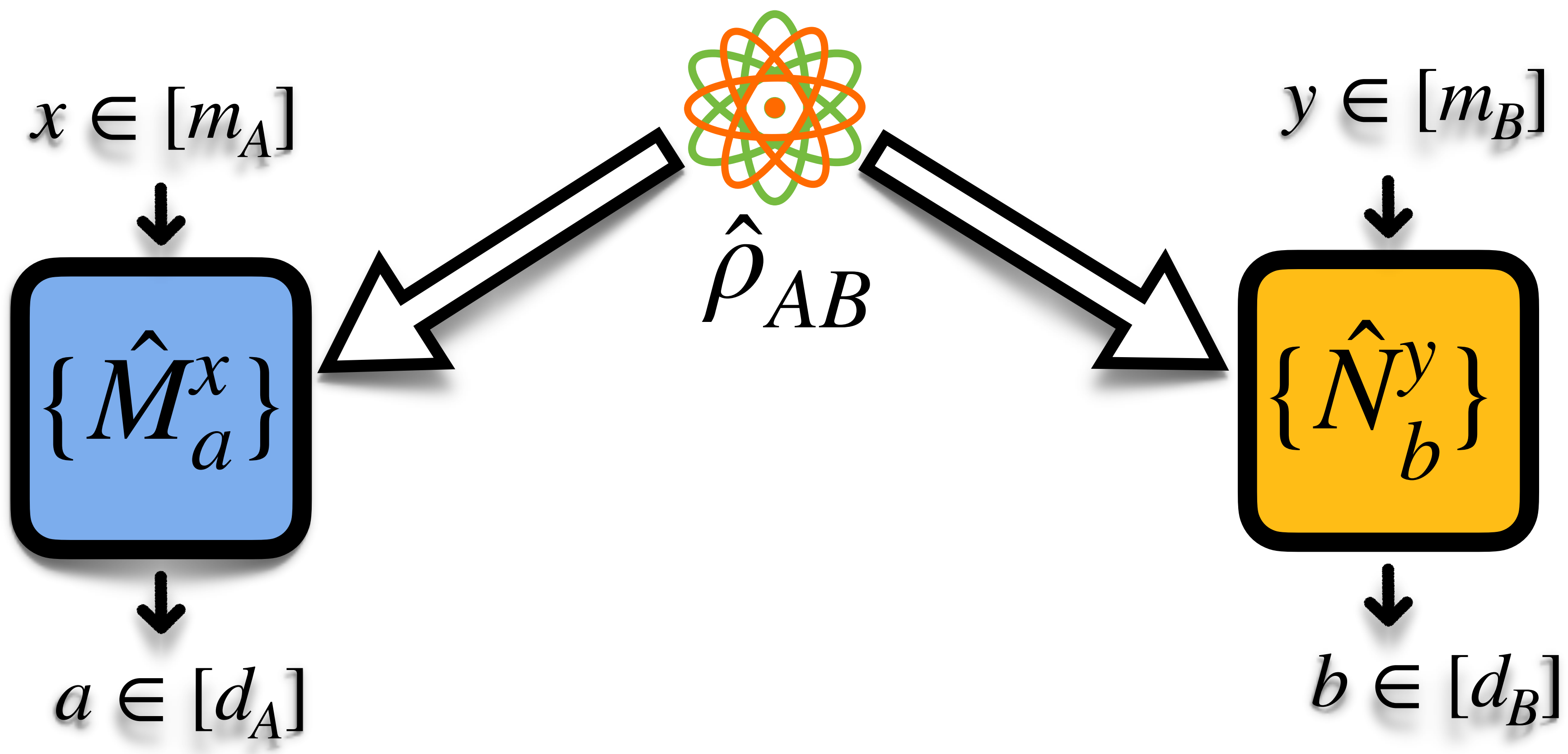
- Measure of nonlocality

$$\beta(\mathbf{p}) - \beta_{\mathscr{L}}$$

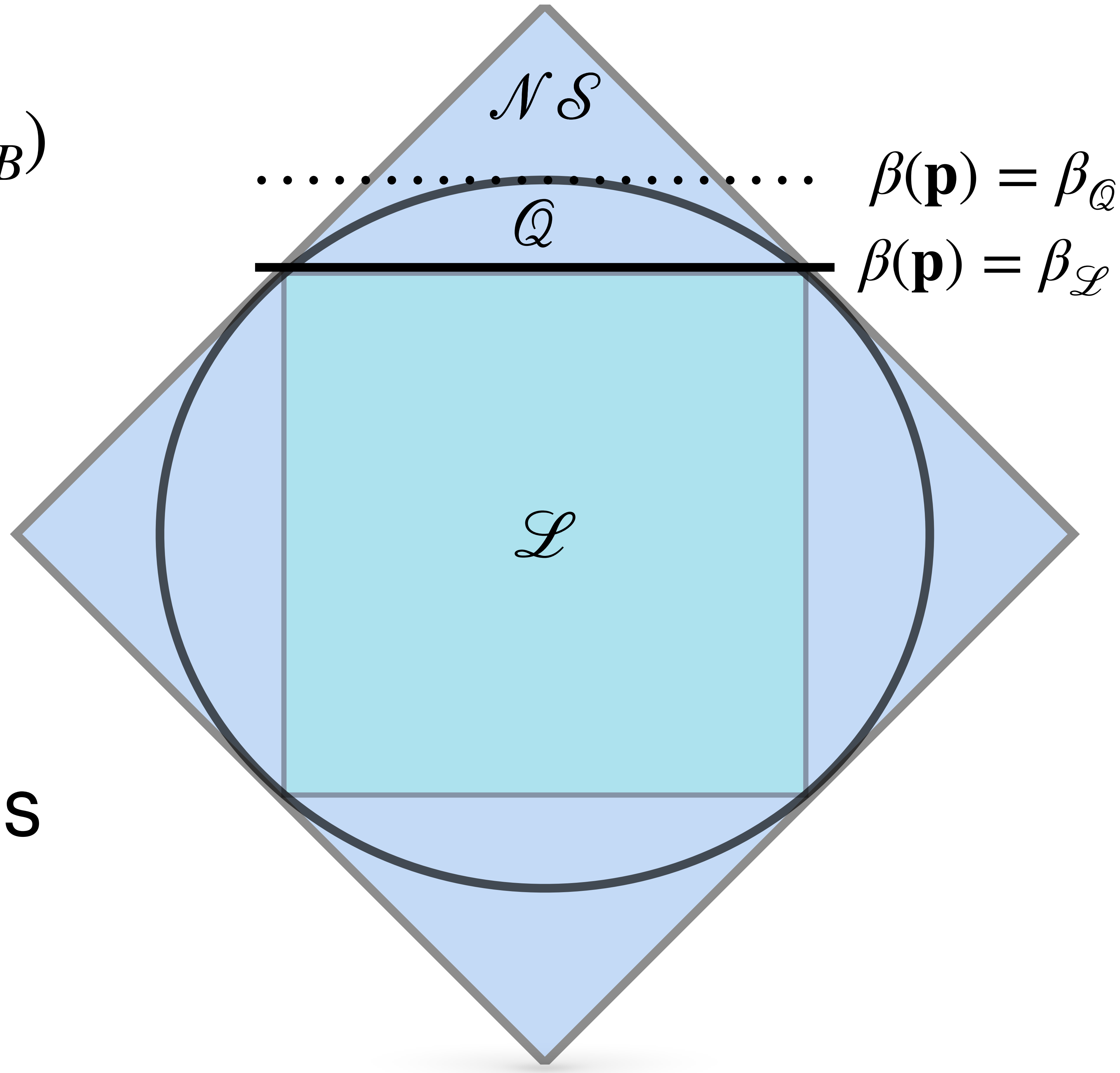


Preliminaries

- Quantum strategies



$$(\{\hat{M}_a^x\}, \{\hat{N}_b^y\}, \rho_{AB})$$



- Quantum behaviors and the quantum set

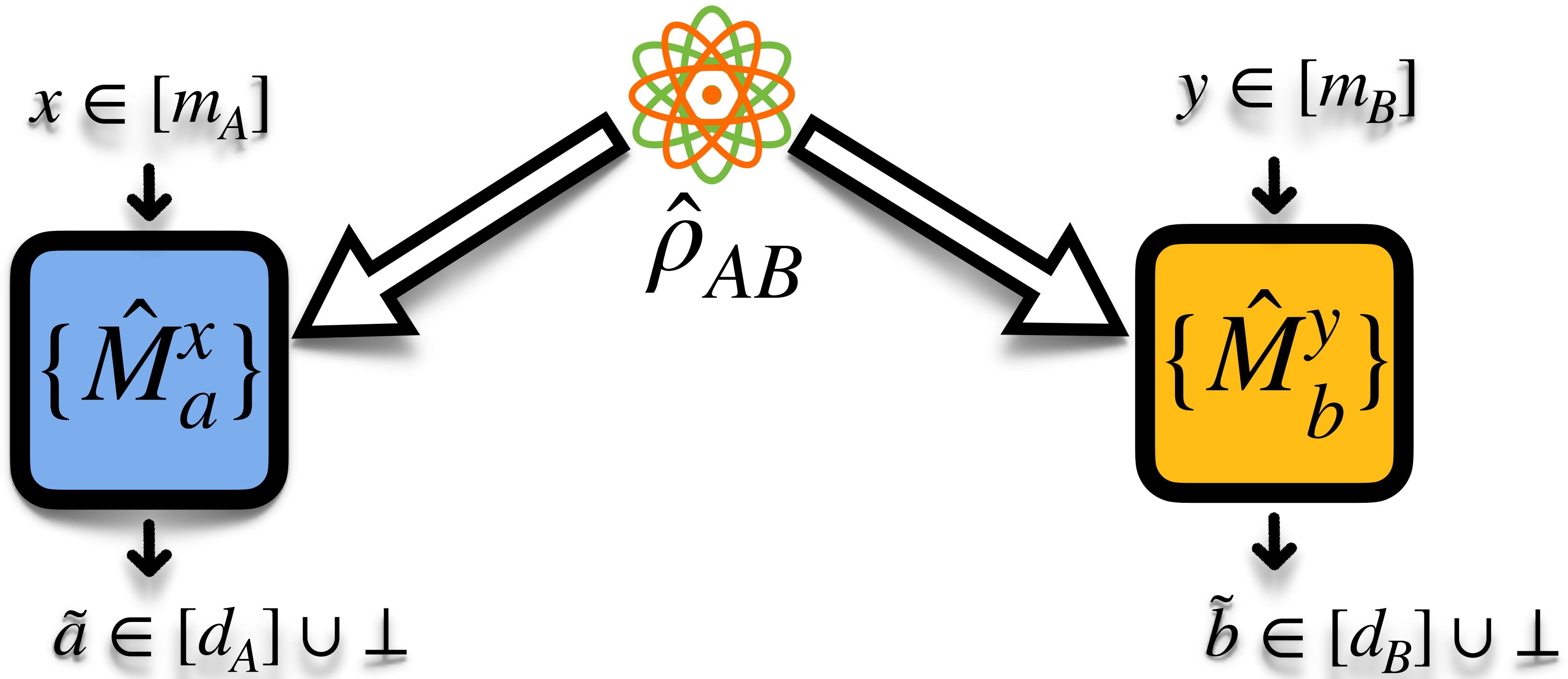
$$\mathbf{p} \equiv \{p(ab\,|\,xy) = Tr(\hat{\rho}_{AB}\,\hat{M}_a^x \otimes \hat{N}_b^y)\} \in \mathcal{Q}$$

- Maximum quantum violation of the Bell inequalities

$$\max_{\mathbf{p}\in\mathcal{Q}}\{\beta(\mathbf{p})\} = \beta_{\mathcal{Q}}$$

Effect of imperfect detectors

- The detectors sometimes fail to click, which results in the occurrence of a "no-click" event, $\perp$



- Detection efficiencies $\eta_A, \eta_B \in [0,1]$
- Treat $\perp$ as an additional outcome

$$p^{(\perp)}(\tilde{a}, \tilde{b} | x, y) = \begin{cases} \eta_A \eta_B p(a = \tilde{a}, b = \tilde{b} | x, y) \text{ if } \tilde{a} \in [d_A], \tilde{b} \in [d_B], \\ (1 - \eta_A)\eta_B p^{(B)}(b = \tilde{b} | y) \text{ if } \tilde{a} = \perp, \tilde{b} \in [d_B], \\ \eta_A(1 - \eta_B) p^{(A)}(a = \tilde{a} | x) \text{ if } \tilde{a} \in [d_A], \tilde{b} = \perp, \\ (1 - \eta_A)(1 - \eta_B), \text{ else,} \end{cases}$$

- Problem: changes the Bell scenario
- Solution: locally assign a pre-existing outcome

Local assignment strategies

- Local assignment strategy: $\mathbf{q} \equiv \{q(ab|xy) = q_A(a|x)q_B(b|y)\} \in \mathscr{L}$

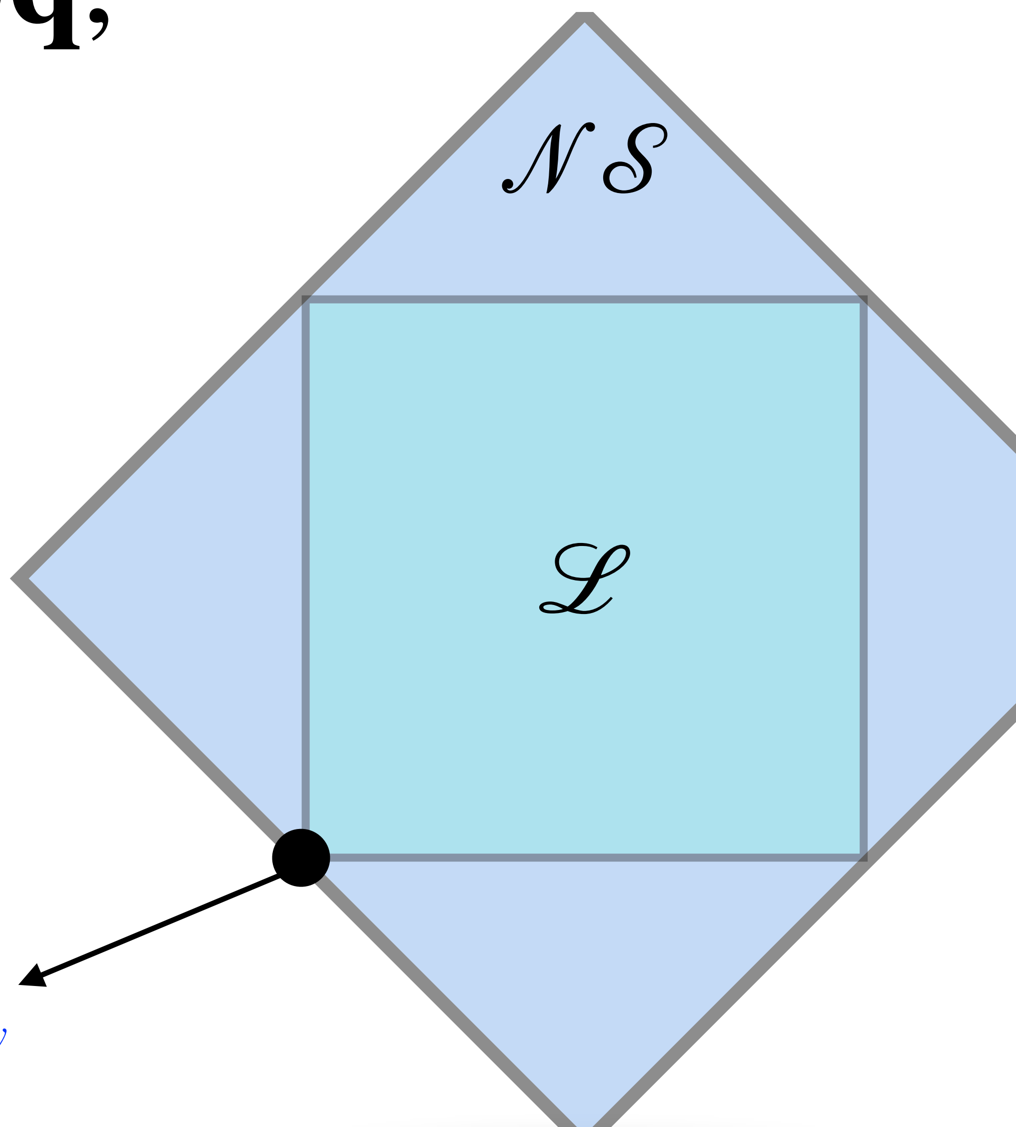
Alice assigns the outcome $a \in [d_A]$ to $\perp$ with probability $q_A(a|x)$

Bob assigns the outcome $b \in [d_B]$ to $\perp$ with probability $q_B(b|y)$

- Effective behavior given $\eta_A, \eta_B, \mathbf{q}$

$$\tilde{\mathbf{p}} = \Omega_{\eta_A\eta_B}(\mathbf{p}) = \eta_A\eta_B\mathbf{p} + \eta_A(1-\eta_B)\mathbf{p}^A + (1-\eta_A)\eta_B\mathbf{p}^B + (1-\eta_A)(1-\eta_B)\mathbf{q},$$

where $\mathbf{p} \equiv p(ab|xy)$, $\mathbf{p}^A \equiv \{p_A(a|x)q_B(b|y)\}$, $\mathbf{p}^B \equiv \{q_A(a|x)p_B(b|y)\}$



$q(ab|xy) = q_A(a|x)q_B(b|y) = \delta_{a,a_x}\delta_{b,b_y}$

Maximum loophole-free nonlocality

- Effect of imperfect detectors on the value of Bell inequalities

$$\beta(\tilde{\mathbf{p}}) = \eta_A\eta_B\beta(\mathbf{p}) + \eta_A(1-\eta_B)\beta(\mathbf{p}^A) + (1-\eta_A)\eta_B\beta(\mathbf{p}^B) + (1-\eta_A)(1-\eta_B)\beta(\mathbf{q})$$

- **Loophole-free violation:**

$$\beta(\tilde{\mathbf{p}}) > \beta_{\mathscr{L}}$$

Objective: Find quantum strategies that yield the **maximum loophole-free violation**

$$\max_{\mathbf{p}\in\mathscr{Q}}\{\beta(\Omega_{\eta_A,\eta_B,\mathbf{q}}(\mathbf{p})) - \beta_{\mathscr{L}}\}$$

Lemma: Tilted Bell inequalities

- For any η_A, η_B, and any Bell inequality $\beta(\mathbf{p}) \leq \beta_{\mathscr{L}}$, the optimal quantum strategies that yield the maximum loophole-free violation of the Bell inequality are those that maximally violate a tilted Bell inequality of the form,

$$\beta_{\eta_A\eta_B}(\mathbf{p}) = \beta(\mathbf{p}) + \frac{1-\eta_B}{\eta_B}\sum_{a,x} c_a^x p_A(a|x) + \frac{1-\eta_A}{\eta_A}\sum_{b,y} c_b^y p_B(b|y) \leq \beta_{\mathscr{L}}(\eta_A, \eta_B)$$

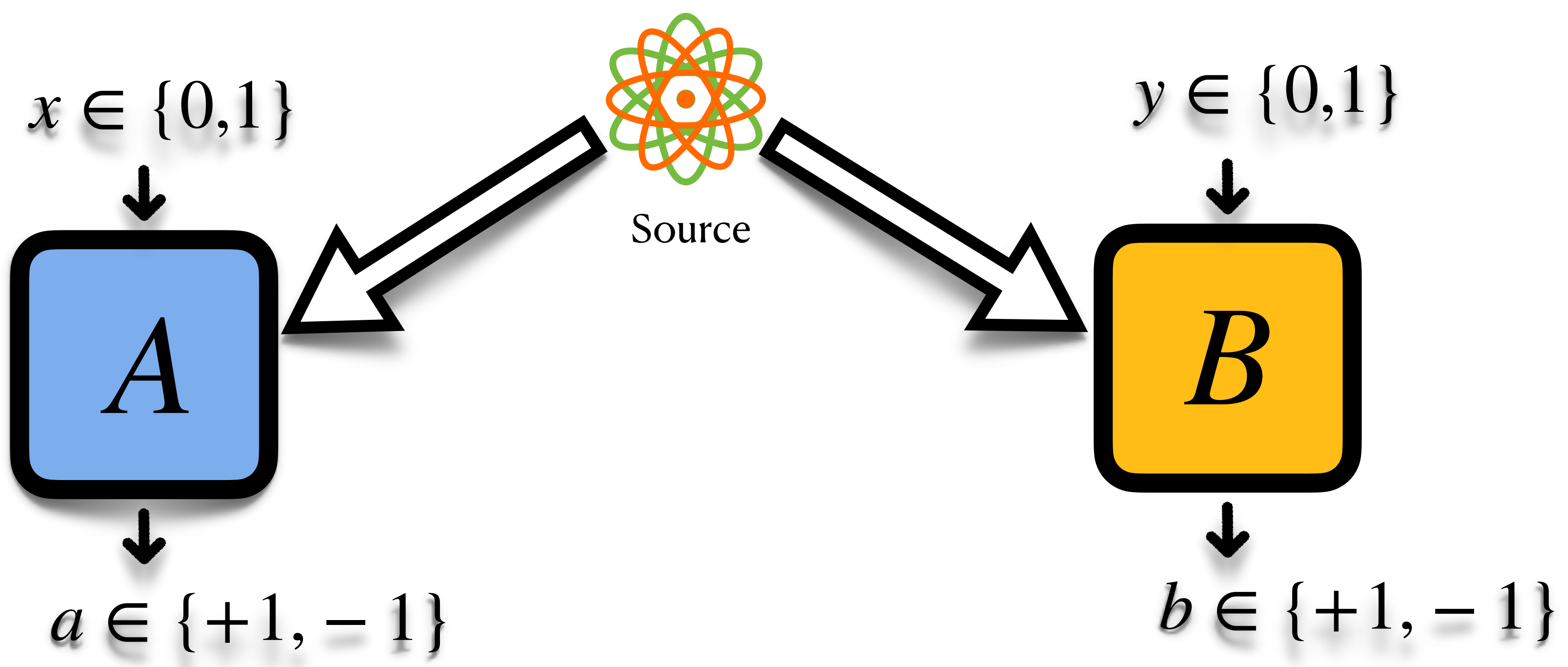
where $\beta_{\mathscr{L}}(\eta_A, \eta_B) \leq \frac{\beta_{\mathscr{L}}}{\eta_A\eta_B} - \frac{1-\eta_A}{\eta_A}\frac{1-\eta_B}{\eta_B}\left(\sum_{x,y} c_{a_x b_y}^{x,y}\right)$, $q_A(a|x) = \delta_{a,a_x}, q_B(b|y) = \delta_{b,b_y}, c_a^x = \sum_y c_{ab_y}^{xy}, c_b^y = \sum_x c_{a_x b}^{xy}$.

- The loophole-free value $\beta(\tilde{\mathbf{p}})$ is

$$\beta(\tilde{\mathbf{p}}) = \eta_A\eta_B\beta_{\eta_A\eta_B}(\mathbf{p}) + (1-\eta_A)(1-\eta_B)\left(\sum_{x,y} c_{a_x b_y}^{x,y}\right)$$

Example: The simplest Bell scenario

- CHSH Bell experiment and the **CHSH inequality:**



$$C(\mathbf{p}) = \sum_{x,y} (-1)^{x\cdot y} \langle A_x B_y \rangle \leq 2$$

Effective violation of CHSH inequality in the presence of imperfect detectors with a local assignment strategy $\mathbf{q}$:

$$C(\tilde{\mathbf{p}}) = C(\Omega_{\eta_A,\eta_B,\mathbf{q}}(\mathbf{p})) = \eta_A\eta_B C(\mathbf{p}) + (1-\eta_A)(1-\eta_B)C(\mathbf{q}) + \eta_A(1-\eta_B)C(\mathbf{p}^A) + (1-\eta_A)\eta_B C(\mathbf{p}^B) > 2.$$

- **Objective:** Find quantum strategies that yield the maximum loophole-free violation of the CHSH inequality

$$\max_{\mathbf{p}\in\mathcal{Q}} \{C(\tilde{\mathbf{p}}) - 2\}$$

Maximum loophole-free nonlocality in the CHSH scenario

- Consider the deterministic strategy

$$q(a|x) = \delta_{a,+1}, q(b|y) = \delta_{b,+1} \ \forall x, y, \in \{0,1\}$$

- The useful Lemma yields the following doubly-tilted CHSH inequality

$$C_{\eta_A\eta_B}(\mathbf{p}) = C(\mathbf{p}) + \frac{2}{\eta_B}(1-\eta_B)\langle A_0\rangle + \frac{2}{\eta_A}(1-\eta_A)\langle B_0\rangle \leq 2[\frac{1}{\eta_A} + \frac{1}{\eta_B} - 1]\,.$$

The loophole-free value of the CHSH functional is $C(\tilde{\mathbf{p}}) = \eta_A\eta_B C_{\eta_A,\eta_B}(\mathbf{p}) + 2(1-\eta_A)(1-\eta_B)\,.$

- Consequently, for any given η_A, η_B, the maximum loophole-free violation of CHSH inequality $\max_{\mathbf{p}\in\mathcal{Q}}\{C(\tilde{\mathbf{p}}) - 2\}$ corresponds to the maximum violation of the doubly-tilted CHSH inequality

$$C_{\alpha,\beta}(\mathbf{p}) = C(\mathbf{p}) + \alpha\langle A_0\rangle + \beta\langle B_0\rangle \leq 2 + \alpha + \beta\,.$$

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The loophole-free value of the CHSH functional is $C(\tilde{\mathbf{p}}) = \eta_A\eta_B C_{\eta_A,\eta_B}(\mathbf{p}) + 2(1-\eta_A)(1-\eta_B)\,.$

- Consequently, for any given η_A, η_B, the maximum loophole-free violation of CHSH inequality $\max_{\mathbf{p}\in\mathcal{Q}}\{C(\tilde{\mathbf{p}}) - 2\}$ corresponds to the maximum violation of the doubly-tilted CHSH inequality

$$C_{\alpha,\beta}(\mathbf{p}) = C(\mathbf{p}) + \alpha\langle A_0\rangle + \beta\langle B_0\rangle \leq 2 + \alpha + \beta\,.$$

Maximum loophole-free nonlocality in the CHSH scenario

- **Observation:** A quantum loophole-free violation of the CHSH inequality $C(\tilde{\mathbf{p}}) > 2$ is not possible if the detection efficiencies η_A, η_B fail to satisfy,

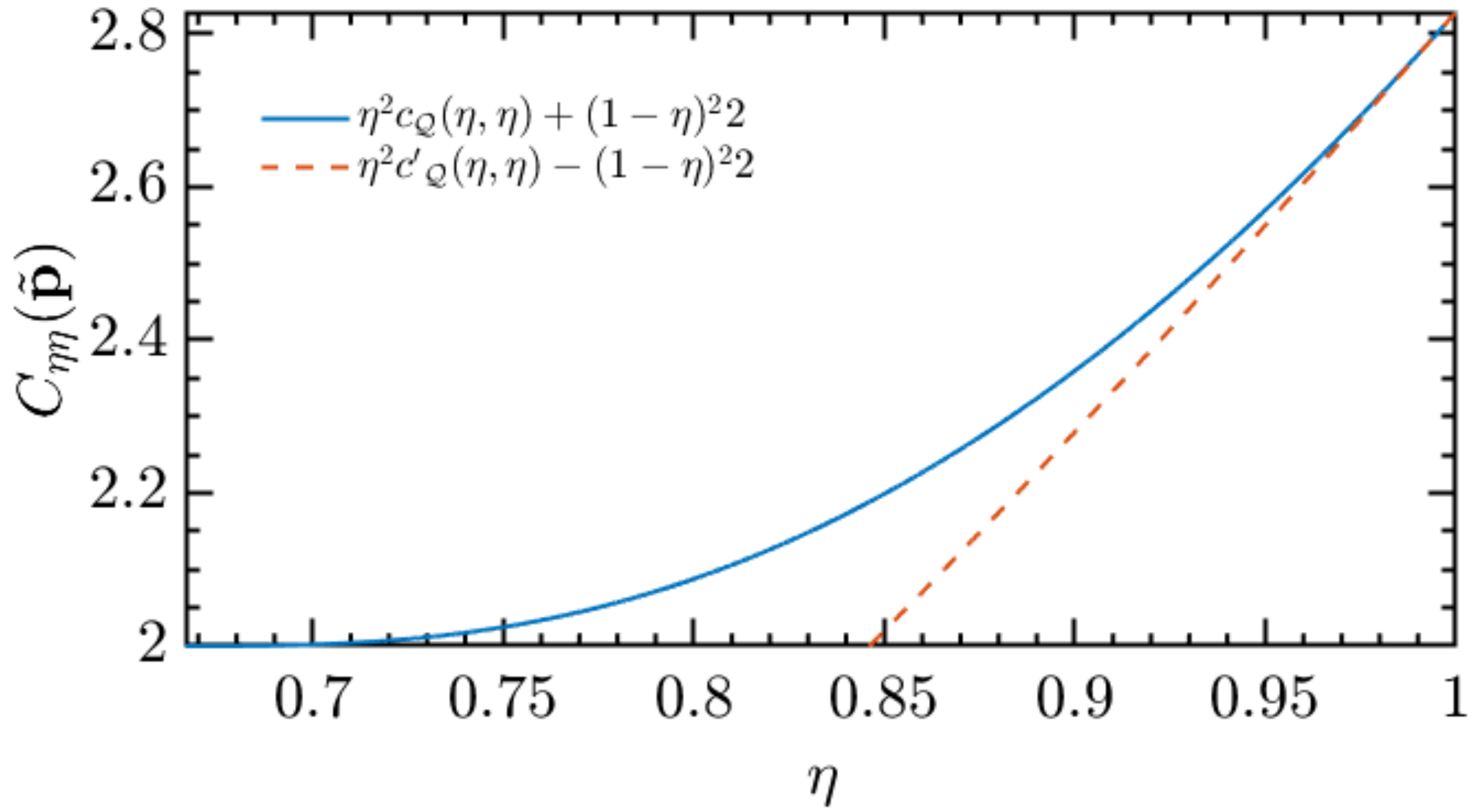
$$\eta_B > \frac{\eta_A}{3\eta_A - 1}$$

- Retrieving the exact expression for the maximum violation of the doubly-tilted CHSH inequalities as a function of η_A, η_B, the traditional methods, such as the NPA hierarchy and SOS decomposition method, turned out to be intractable.
- Nevertheless, via Jordan's Lemma-based proof technique, we obtain analytical self-testing statements entailing the analytical expression for maximum quantum violation $c_{\mathcal{Q}}(\eta_A, \eta_B)$, demonstrating that the optimal strategies are unique up to local isometries.

Maximum loophole-free nonlocality in the CHSH scenario

- Optimality of local assignment strategy
  Up to local relabelling there is one additional family of doubly-tilted CHSH inequalities,

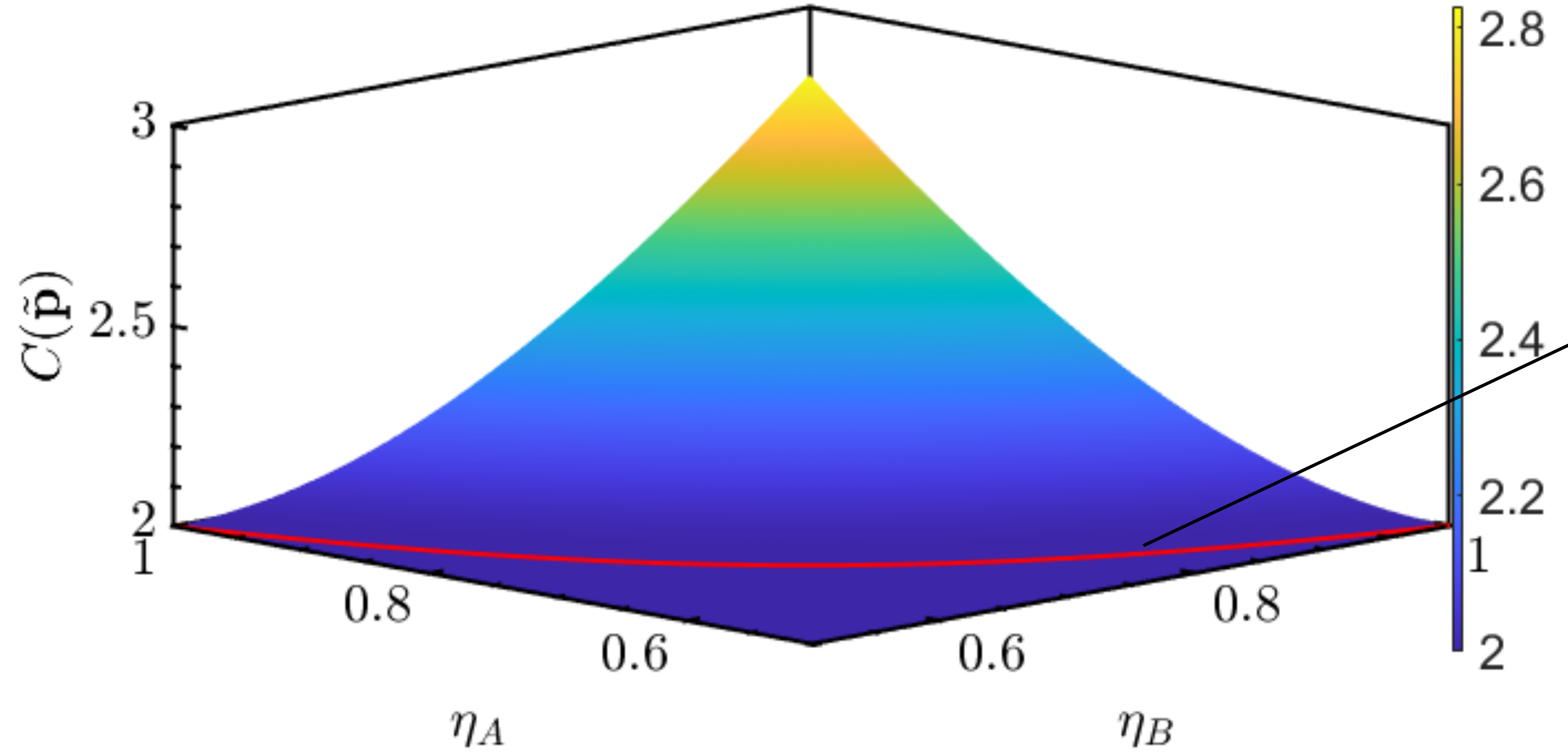
$$C'_{\eta_A\eta_B}(\mathbf{p}) = C(\tilde{\mathbf{p}}) + \frac{2}{\eta_B}(1-\eta_B)\langle A_0\rangle - \frac{2}{\eta_A}(1-\eta_A)\langle B_0\rangle \leq 2\left[1 - \frac{1}{\eta_A} - \frac{1}{\eta_B}\right] = c'_{\mathscr{L}}(\eta_A, \eta_B)\,.$$



comparing the maximum effective violation of the CHSH inequality (solid blue line) with the assignment strategy $\perp \to +1$, and the maximum effective violation of the CHSH inequality with the other assignment strategy $\perp_A \to +1$, $\perp_B \to -1$ (dashed orange curve).

Maximum loophole-free nonlocality in the CHSH scenario

- Maximum loophole-free violation of the CHSH inequality



Observation: For a loophole-free violation,

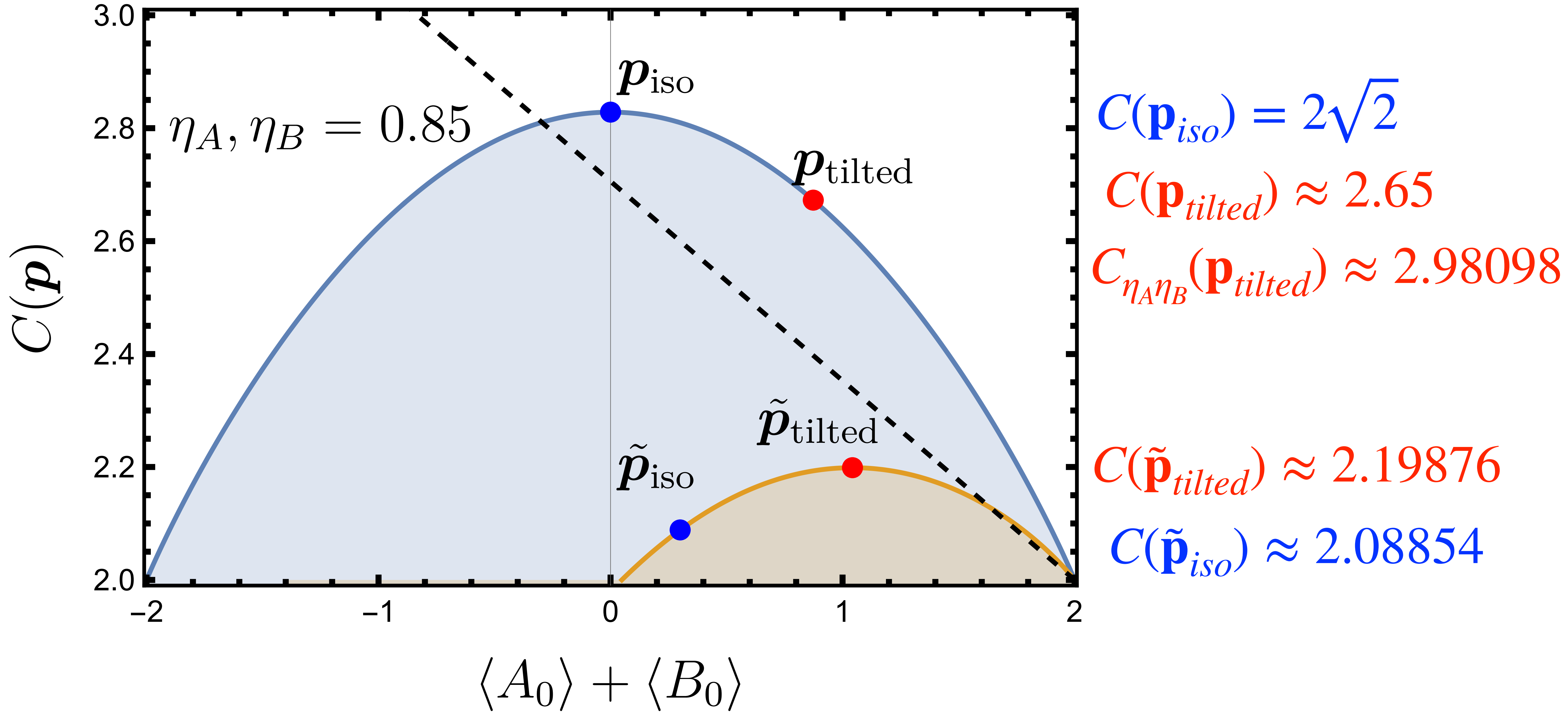
$$\eta_B > \frac{\eta_A}{3\eta_A - 1}$$

$$\alpha + \beta \leq 2$$

A plot of the maximum loophole-free violation of the CHSH inequality, $C(\tilde{\mathbf{p}})$, against detection efficiencies $\eta_A, \eta_B \in [\frac{1}{2}, 1]$, where we used the analytical expression for maximum quantum violation of the doubly-tilted CHSH inequality $c_{\mathcal{Q}}(\eta_A, \eta_B)$. The solid red line represents Bob's critical detection efficiency $\eta_B^* = \frac{\eta_A}{3\eta_A - 1}$, below which a loophole-free quantum violation of the CHSH inequality is not possible.

Maximum loophole-free nonlocality in the CHSH scenario

- Effect of inefficient detectors on nonlocal correlations



The blue region represents the set of quantum correlations $\mathbf{p} \in \mathcal{Q}$ in ideal conditions. With the detection efficiencies $\eta_A = \eta_B = 0.85$ and the local assignment strategy $q_A(a|x) = \delta_{a,0}, q_B(b|y) = \delta_{b,0}$, the effective quantum correlations $\tilde{\mathbf{p}} = \Omega_{\eta_A \eta_B}(\mathbf{p})$ are constrained to the smaller orange subset.

Self-testing of Bell inequalities

- The most accurate form of certification of quantum devices!

- **Self-testing statement**: Any quantum strategy $(\{M_a^x\}, \{N_b^y\}, \rho_{AB})$ attaining the maximum violation $\beta(\mathbf{p}) = \beta_{\mathcal{Q}}$ of a Bell inequality must be equivalent to the self-tested optimal quantum strategy $(\{\Pi_a^x\}, \{\Pi_b^y\}, \psi_{AB})$, up to auxiliary degrees of freedom and local unitary transformations,

$$\beta(\mathbf{p}) = \beta_{\mathcal{Q}} \implies (\{\Pi_a^x\}, \{\Pi_b^y\}, \psi_{AB})$$

(up to local isometries)

Determined by the Bell functional and optimal measurements $\{\Pi_a^x\}, \{\Pi_b^y\}$

Self-testing of CHSH inequalities tilted for an imperfect detector

- Asymmetrically tilted CHSH inequalities

$$C_\alpha(\mathbf{p}) = \sum_{x,y} (-1)^{x \cdot y} \langle A_x B_y \rangle + \alpha \langle A_0 \rangle \leq 2 + \alpha$$

where $\alpha = \frac{2}{\eta_B}(1 - \eta_B)$, $\eta_B \in (1/2, 1]$, $\eta_A = 1$.

- Self-testing statement:

$$c_{\mathcal{Q}}(\alpha) = \sqrt{8 + 2\alpha^2} \implies \begin{pmatrix} \{\hat{A}_0 = \sigma_z, \hat{A}_1 = \sigma_x\} \\ \{\hat{B}_0 = \cos\mu\,\sigma_z + \sin\mu\,\sigma_x, \hat{B}_1 = \cos\mu\,\sigma_z - \sin\mu\,\sigma_x\} \\ |\psi\rangle = \cos\theta\,|00\rangle + \sin\theta\,|11\rangle) \end{pmatrix}$$

where $\alpha = 2/\sqrt{1 + 2\tan^2 2\theta}$, $\tan(\mu) = \sin(2\theta)$.

- Proof via the Sum of Squares (SOS) decomposition method.

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Self-testing of CHSH inequalities tilted for an imperfect detector

- These decompositions are then used to prove that $c_{\mathcal{Q}}(\alpha)$ self-tests the optimal strategy.

$$(\{A_x\}, \{B_y\}, \psi_{AB})$$

- For any quantum strategy $(\{A'_x\}, \{B'_y\}, \psi'_{AB})$, the SOS decomposition implies $P_i|\psi'\rangle = 0$ for all i, such that there exists operators $\{\hat{Z}_A, \hat{X}_A, \hat{Z}_B, \hat{X}_B\}$ satisfying,

$$\hat{Z}_A|\psi'\rangle = \hat{Z}_B|\psi'\rangle, \quad \sin\theta \hat{X}_A(\mathbf{1} + \hat{Z}_B)|\psi'\rangle = \cos\theta \hat{X}_A(\mathbf{1} - \hat{Z}_A)|\psi'\rangle\,.$$

- This implies the existence of local isometries, Φ_A and Φ_B, mapping any optimal strategy $(\{A'_x\}, \{B'_y\}, \psi'_{AB})$ to the reference strategy $(\{A_x\}, \{B_y\}, \psi_{AB})$

$$\Phi_A \otimes \Phi_B(|\psi'\rangle) = |\psi\rangle \otimes |\text{junk}\rangle, \quad \Phi_A \otimes \Phi_B(\hat{A}'_x \otimes \hat{B}'_y|\psi'\rangle) = \hat{A}_x \otimes \hat{B}_y|\psi\rangle \otimes |\text{junk}\rangle,$$

where $|\text{junk}\rangle$ represents the arbitrary state of additional degrees of freedom on which the measurements act trivially.

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Self-testing of CHSH inequalities tilted for imperfect detectors

Self-testing statement:

The maximum quantum violation $c_{\mathcal{Q}}(\alpha, \alpha)$ of the symmetrically $(\alpha = \beta)$ tilted CHSH inequality is the largest root of the degree 4 polynomial,

$$f(\lambda) = \lambda^4 + (4 - \alpha^2)\lambda^3 + \left(\frac{11}{4}\alpha^4 - 12\alpha^2 - 4\right)\lambda^2 + (2\alpha^6 - \alpha^4 - 20\alpha^2 - 32)\lambda + 5\alpha^6 - 21\alpha^4 + 16\alpha^2 - 32.$$

$C_{\alpha\alpha}(\mathbf{p}) = c_{\mathcal{Q}}(\alpha, \alpha)$ self-tests a two-qubit quantum strategy with optimal ($*$) local observables of the form (from **Jordan's Lemma**),

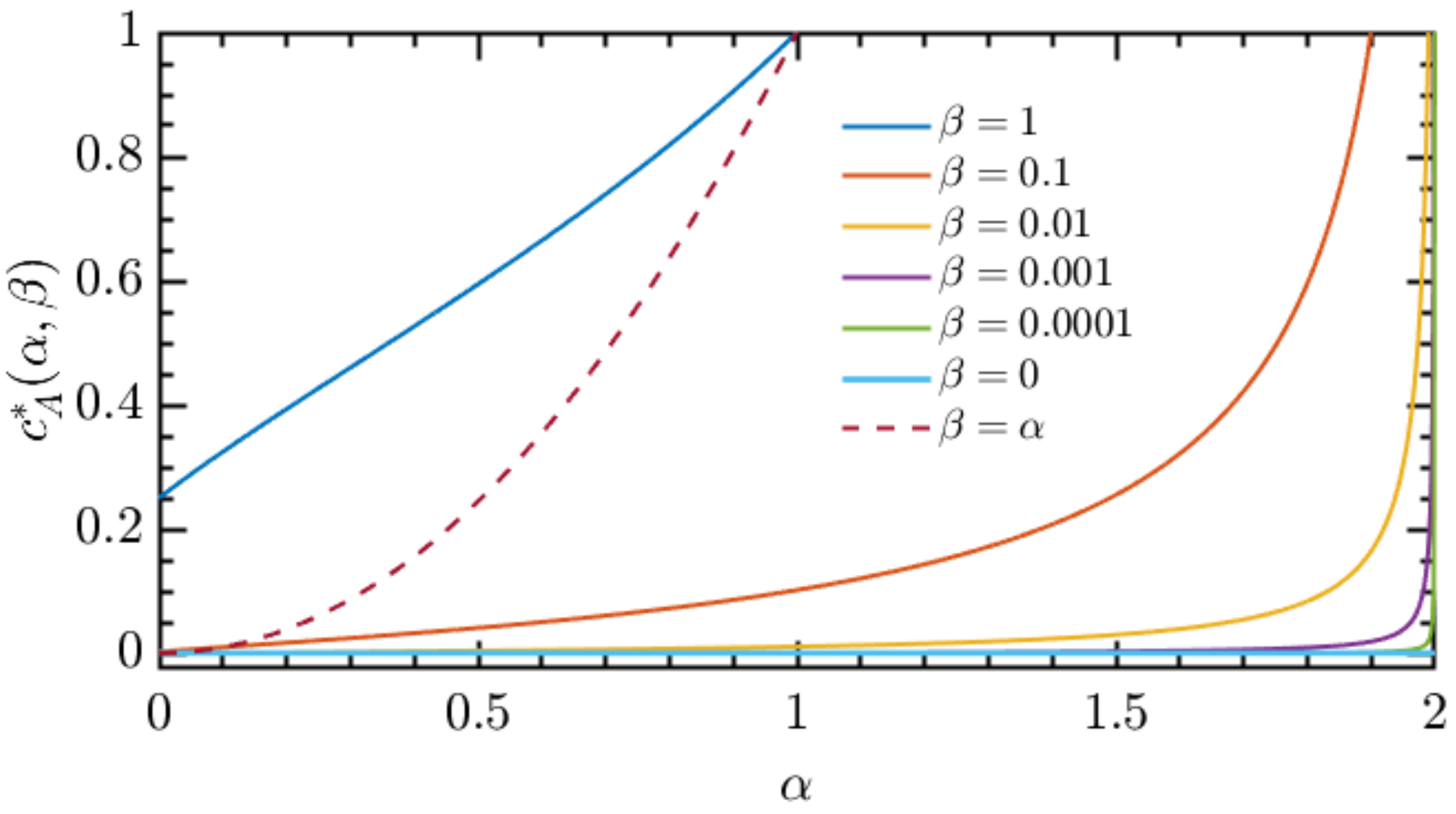
$$\hat{A}_0 = \sigma_Z \,,\ \hat{A}_1 = c_A\,\sigma_Z + s_A\,\sigma_X, \qquad \hat{B}_0 = \sigma_Z \,,\ \hat{B}_1 = c_B\,\sigma_Z + s_B\,\sigma_X,$$

such that the optimal cosines are equal, i.e., $c^*(\alpha) = c_A^*(\alpha) = c_B^*(\alpha) \in [0,1]$ and satisfy the relation,

$$c^*(\alpha) = \frac{1}{8}\left[3\alpha^2 - 4 + \sqrt{16 + 9\alpha^4 + 8\alpha^2(2c_{\mathcal{Q}}(\alpha, \alpha) - 1)}\right]$$

Self-testing of CHSH inequalities tilted for imperfect detectors

- Self-testing of partially incompatible observables



optimal cosines of Alice $c_A^*(\alpha, \beta)$ self-tested by the maximum quantum violation $C_{\alpha\beta}(\mathbf{p}) = c_{\mathcal{Q}}(\alpha, \beta)$ of the doubly-tilted CHSH inequalities.

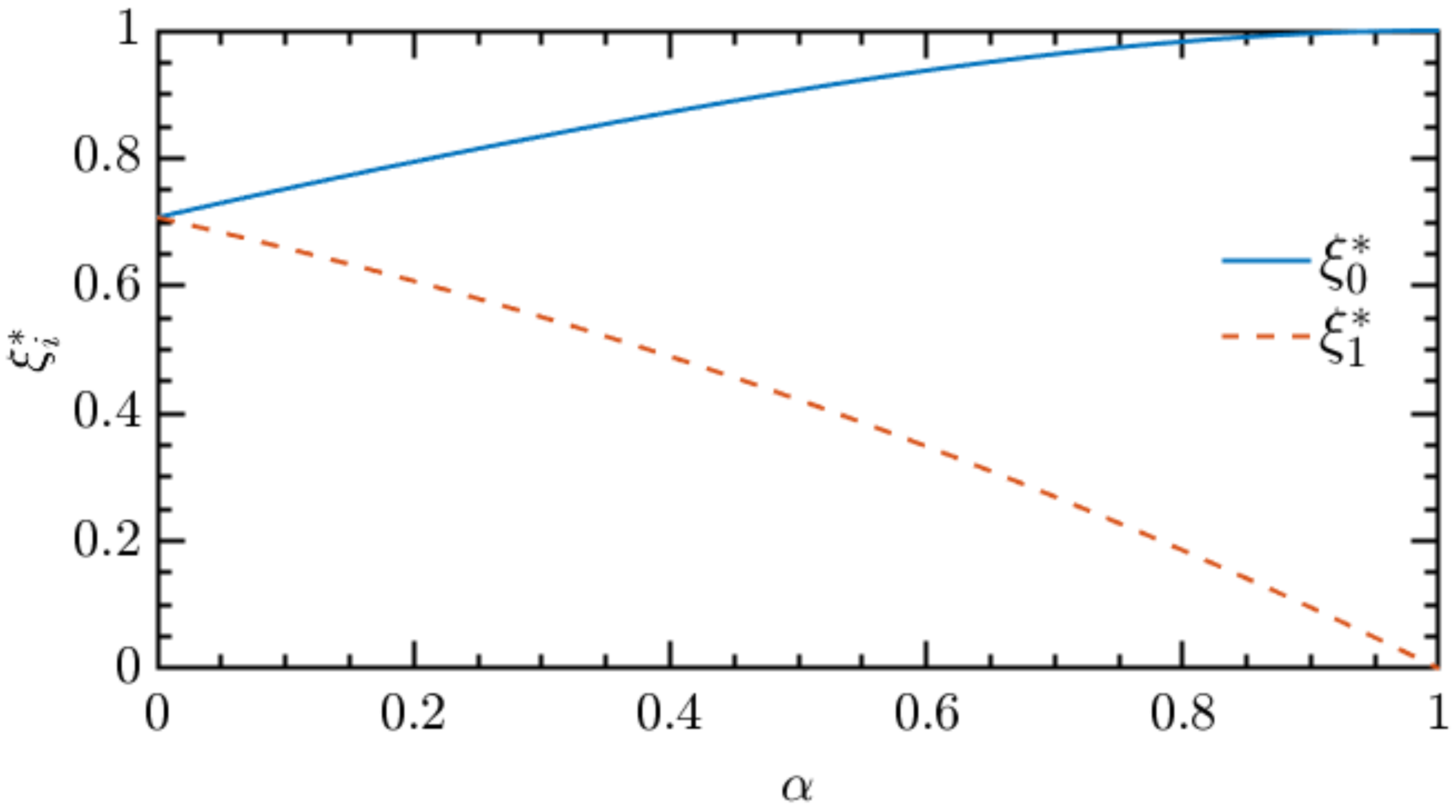
Self-testing of CHSH inequalities tilted for imperfect detectors

- The optimal state $|\psi\rangle$ the eigenvector corresponding to the non-degenerate maximum eigenvalue of the Bell operator

$$\hat{C}_{\alpha\alpha} = \sum_{x,y} (-1)^{x\cdot y} \hat{A}_x \otimes \hat{B}_y + \alpha(\hat{A}_0 \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \hat{B}_0)\,.$$

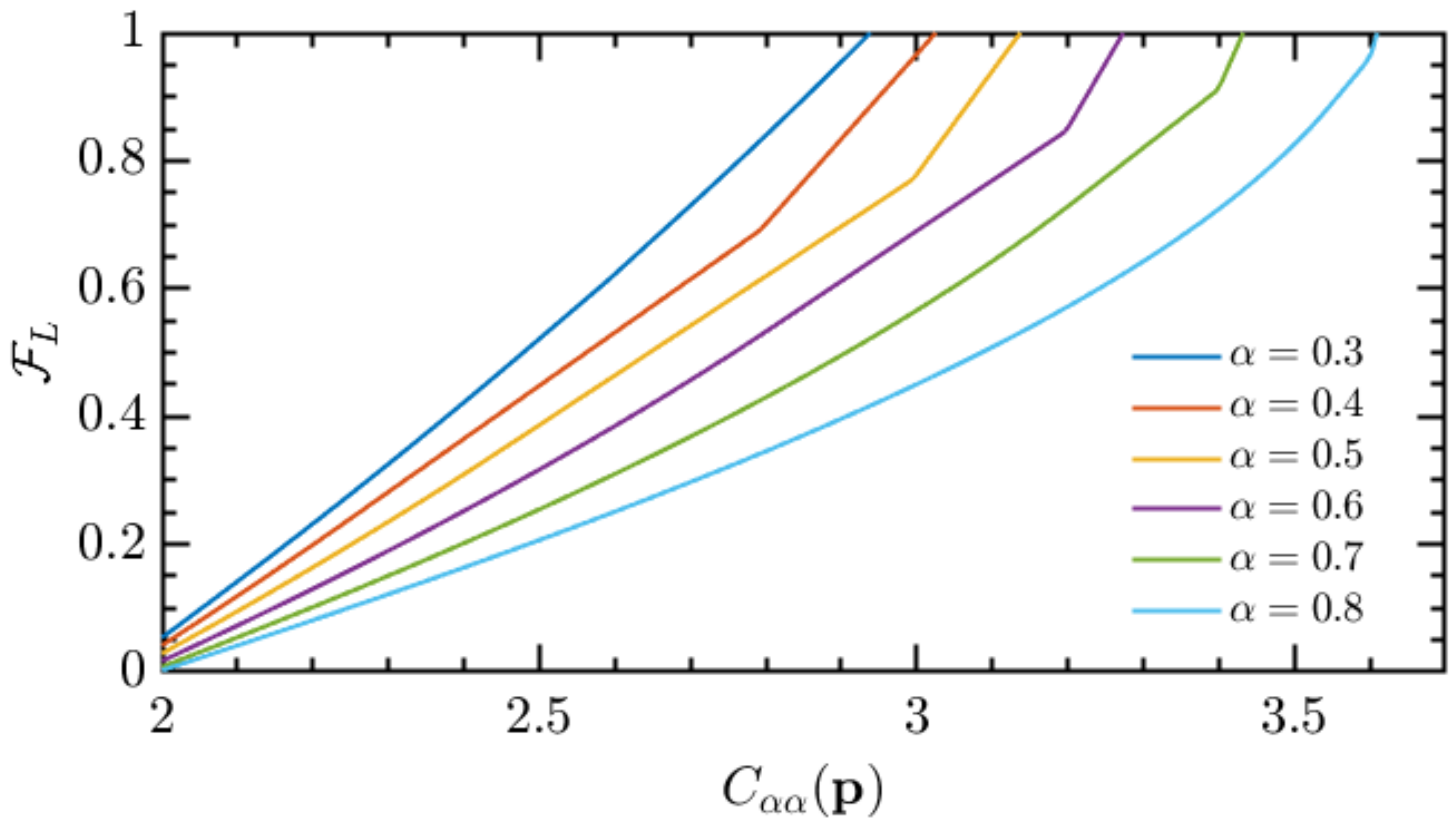
- Self-testing of non-maximally entangled states:

  A plot of the Schmidt coefficients $\xi_i^*$of the optimal non-maximally entangled quantum state. Notice, as $\alpha = \beta \to 1$, the optimal state becomes almost product.



Robust Self-Testing

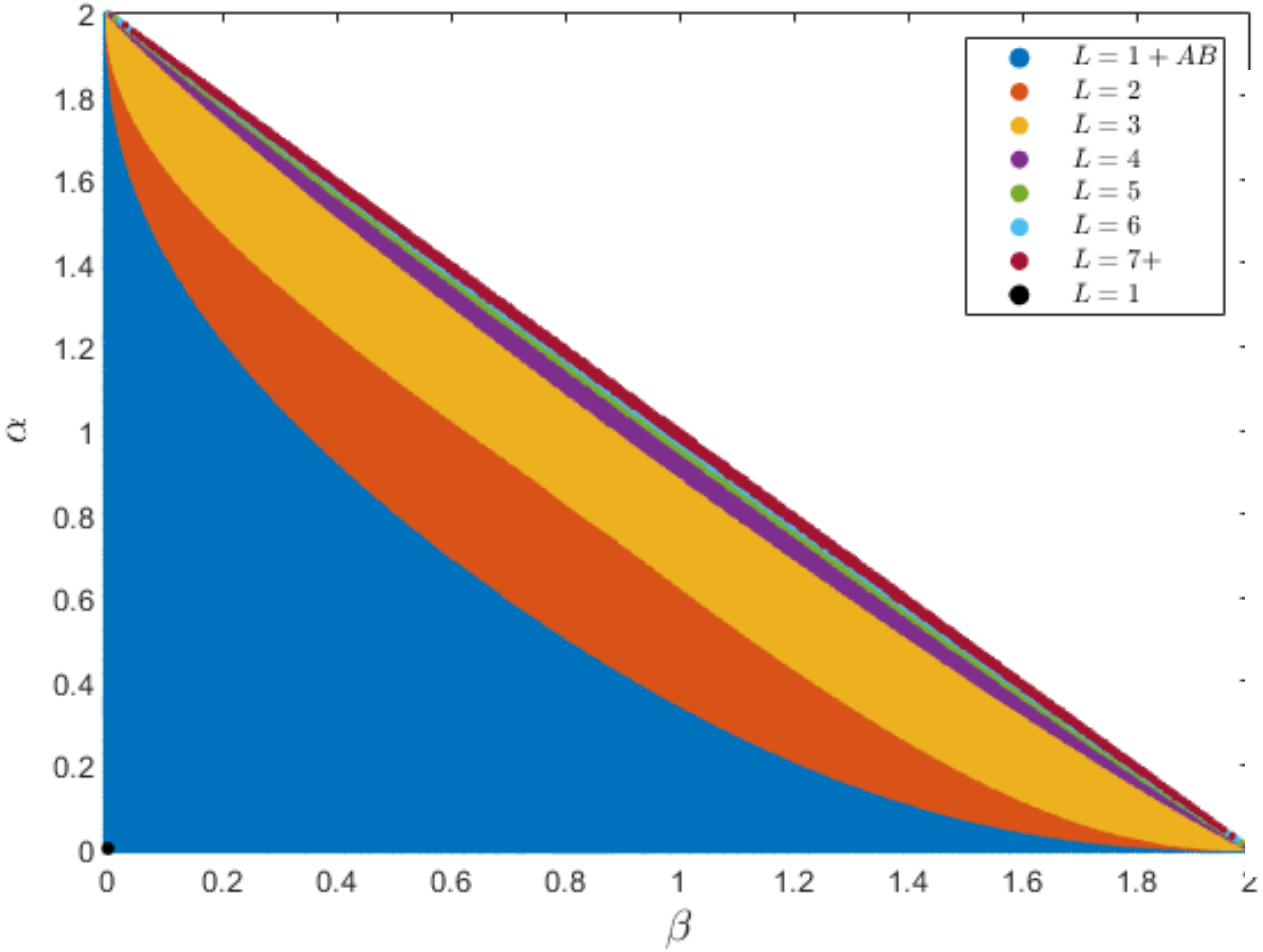
- Robustness of the self-testing statements

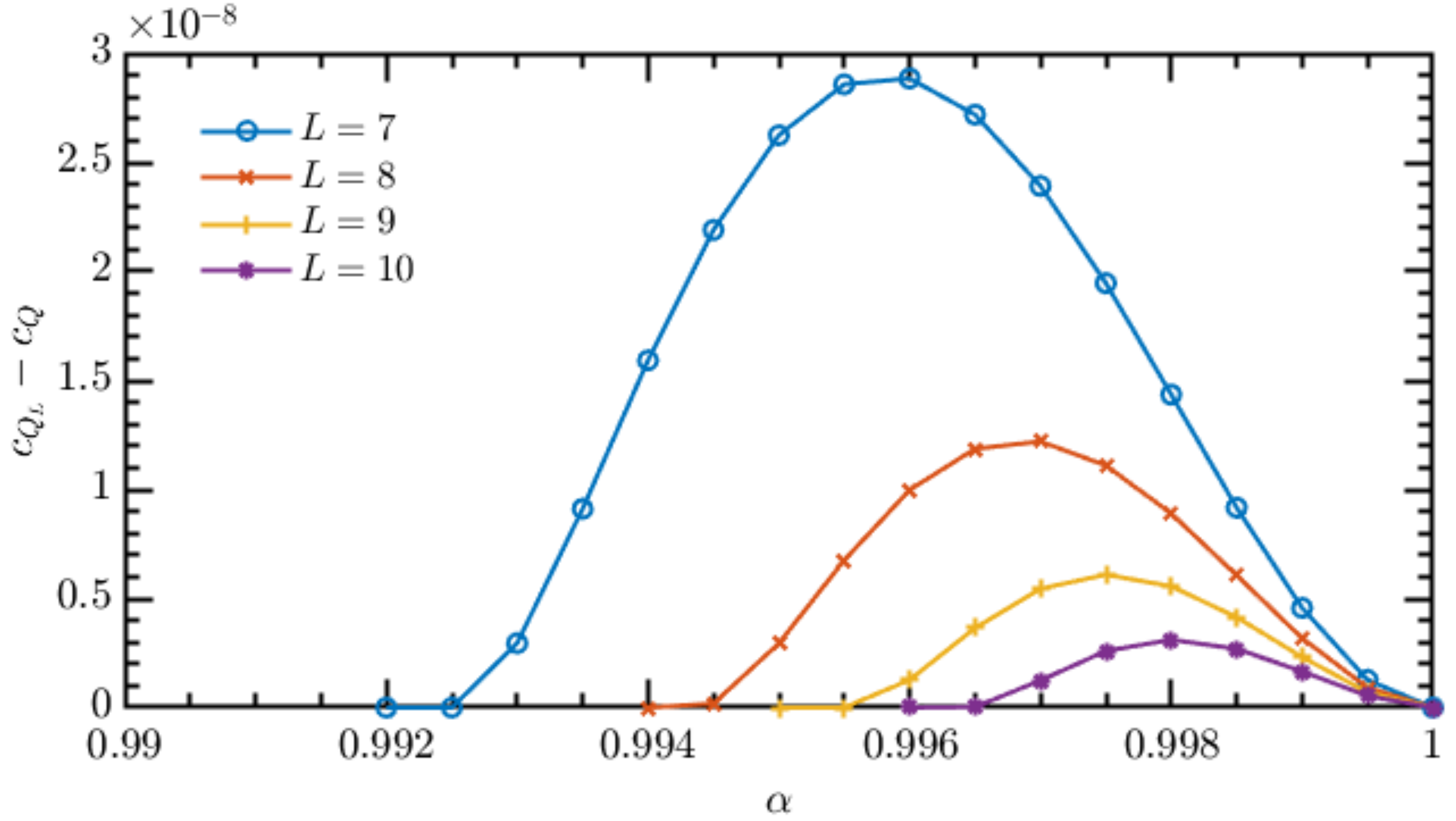


Lower bounds $\mathscr{F}_L^*$ on the minimum quantum fidelity $\mathscr{F}_L^* \leq \mathscr{F}^*$ from the level $L = 3$ of NPA hierarchy between the actual state and the optimal self-testing state against the violation $C_{\alpha\alpha}(\mathbf{p})$ of the symmetrically $(\alpha = \beta)$ tilted CHSH inequality for tilting parameters $\alpha \in \{0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$.

Self-testing of CHSH inequalities tilted for imperfect detectors

- Exploding NPA levels in the simplest Bell scenario

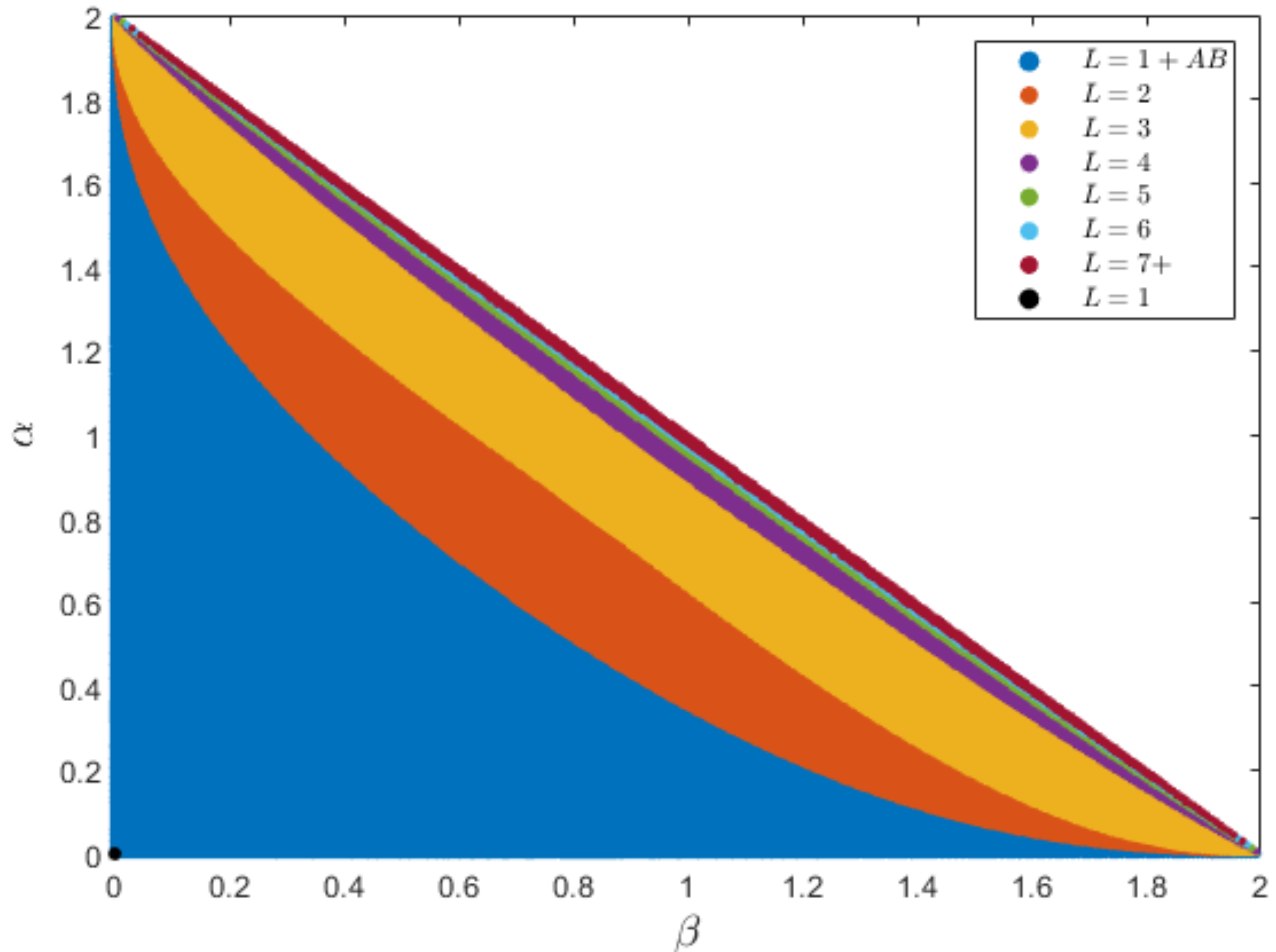


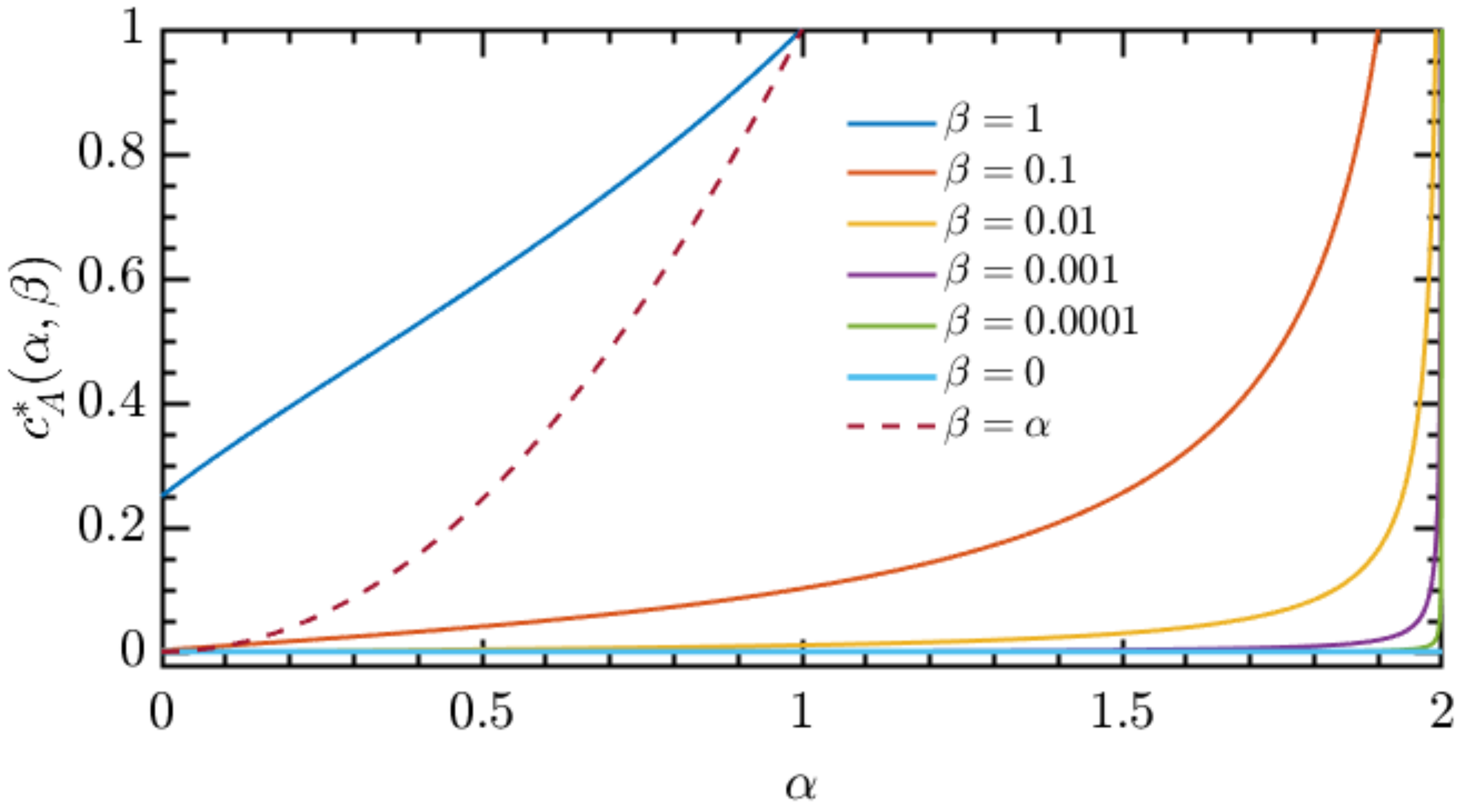


- Self-testing via SOS decompositions method is analytically intractable!

Complexity related to compatibility?

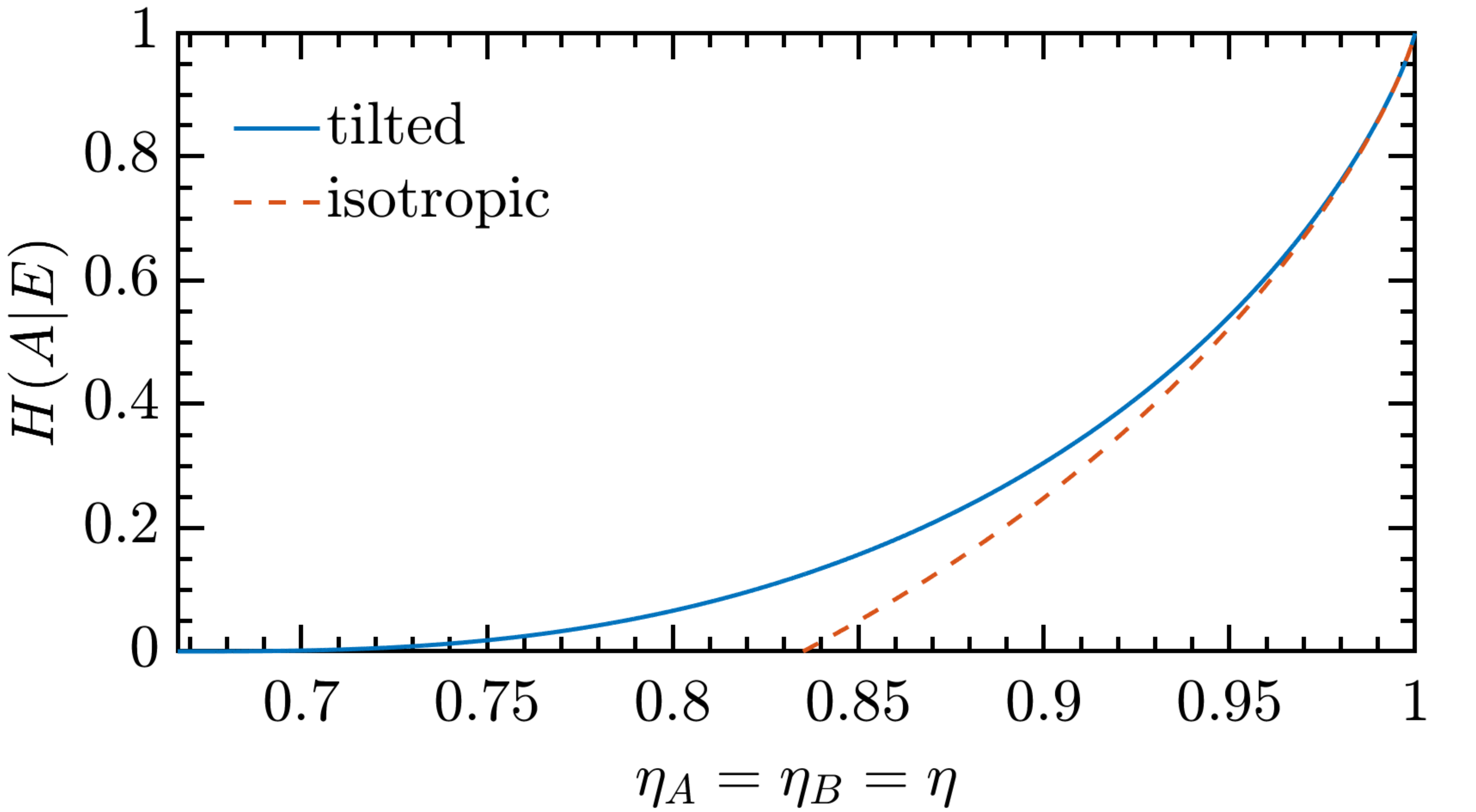
- Notice, that in contrast to the **asymmetrically tilted case** $\beta = 0$ wherein Alice's optimal cosine $c_A^*(\alpha, \beta = 0)$ stays constant with α and **level 1+AB is enough**, for the general case, **whenever** $\beta > 0$, Alice's optimal measurements change with α, and tend towards compatible measurements as $\alpha \to 2 - \beta$, where **the NPA levels explode**.





Towards optimal DIQKD with imperfect detectors

- **Objective:** To device optimal protocols for DIQKD given efficiencies $\eta_A, \eta_B \in [0,1]$



Advantage of tilted strategies obtained in this work over the isotropic strategy in DIQKD

Thank you!

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- We would like to thank Tamás Vértesi, Marcin Pawłowski, Jedrzej Kaniewski, Cosmo Lupo, Marcin Wieśniak and Marek Żukowski for insightful discussions.
- E.P. acknowledges support by NCN SONATA-BIS grant No. 2017/26/E/ST2/01008.