Analog information decoding of bosonic-LDPC codes

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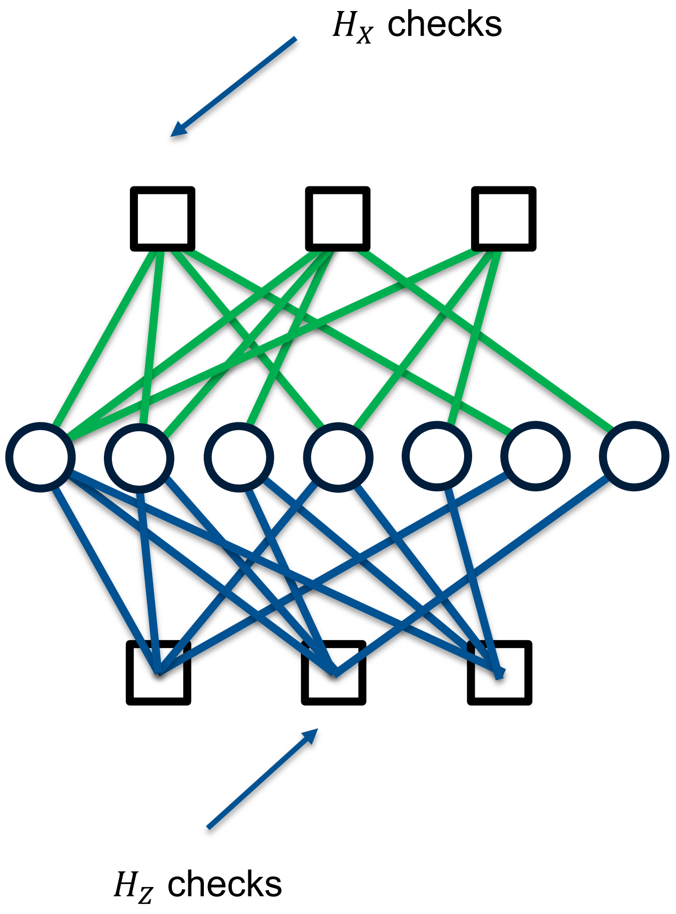
PRX Quantum 5 020349 (arXiv:2311.01328)

Discrete variable quantum codes

- LDPC quantum code - defined by two sparse matrices
  - H_X: detects Z-errors
  - H_Z: detects X-errors
- Each row = support of an X/Z operator
  - $H_X H_Z^T = 0$
- Codespace = +1 eigenspace of all checks
- Tanner graph
  - Bipartite graph whose incidence matrix is H_X/H_Z
- [[7,1,3]]-Steane code:

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$



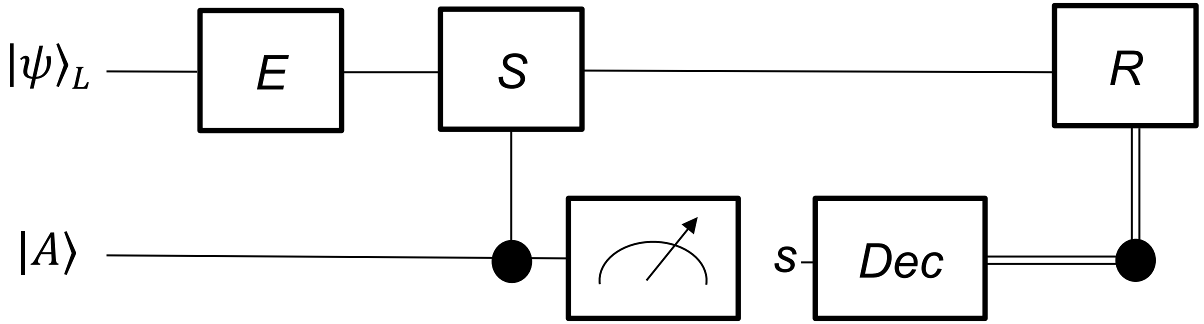
Decoding QLDPC codes

- Syndrome extraction
  - Input: logical state $E|\psi\rangle_L$, ancillas $|A\rangle$
  - Measure checks to obtain syndrome *s*
  - Typically $O(d)$ rounds necessary

- Decoder
  - Find recovery R given *s*
  - Apply correction to the state: $RE|\psi\rangle_L$
  - Successful if $RE|\psi\rangle_L = |\psi\rangle_L$

- Algebraically:
  - 'syndrome equation'

    $$s = He$$
  - Find min. wt. r: $s = Hr$



Belief propagation

- goal: given $\boldsymbol{s}$, find a minimum-weight estimate $\hat{e}$ satisfying
  $$\hat{e} = \mathrm{ARGMAX}_e \, Pr(e|\boldsymbol{s})$$

- Marginalisation problem on each bit – exhaustive bit-wise computation:
  $$Pr(e_1) = \sum_{e_2} \sum_{e_3} \dots \sum_{e_n} P(e_1, e_2, \dots, e_n|\boldsymbol{s})$$

- Exploit structure of distribution by factorization from Tanner graph (check matrix):
  $$Pr(e_1) = \sum_{e_4} \sum_{e_5} \dots \sum_{e_n} f(e_4, \dots, e_n) \dots \sum_{e_2} f(e_2|\boldsymbol{s}) \sum_{e_3} f(e_1, e_2 e_2|\boldsymbol{s})$$

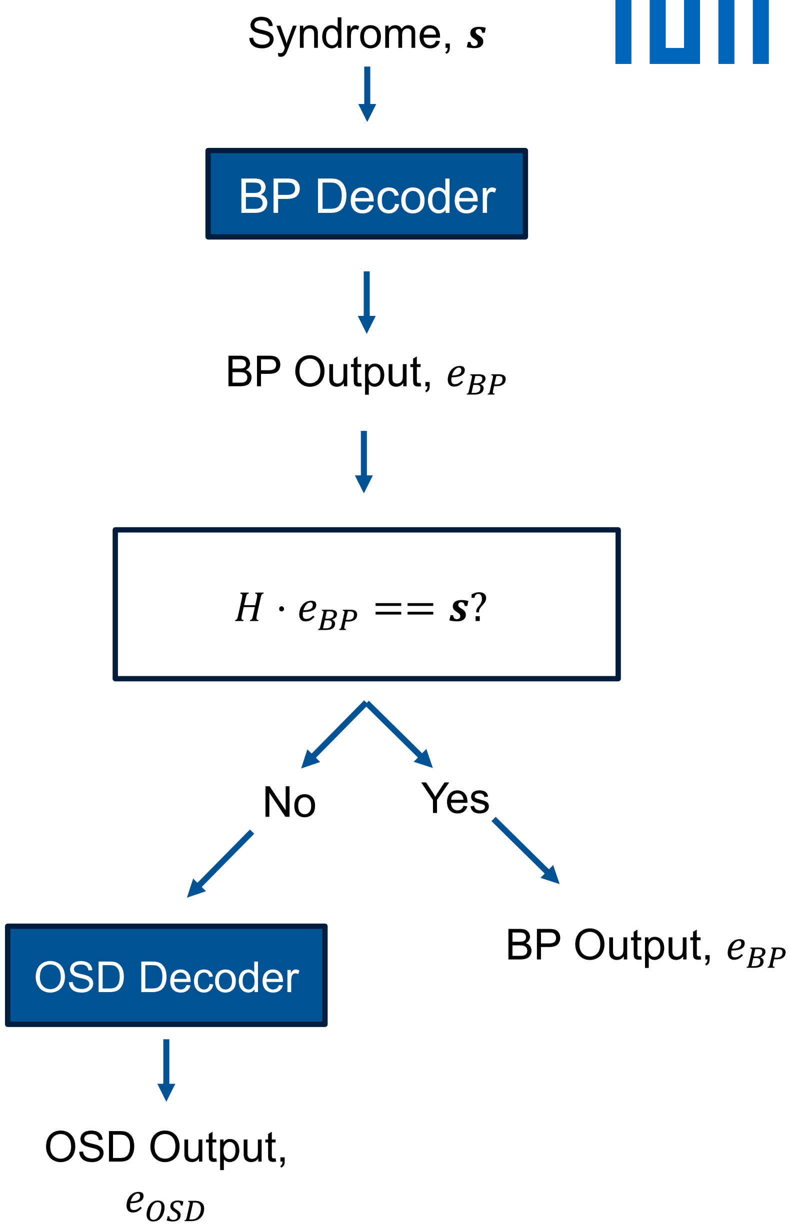
- "Iterative message passing"

BP+OSD

- BP may not converge
  - Degeneracy of codes (“split-beliefs”)

$$H\hat{e} \neq \boldsymbol{s}$$

- OSD ~ inversion decoder to obtain a low-weight correction $H\hat{e}' = \boldsymbol{s}$

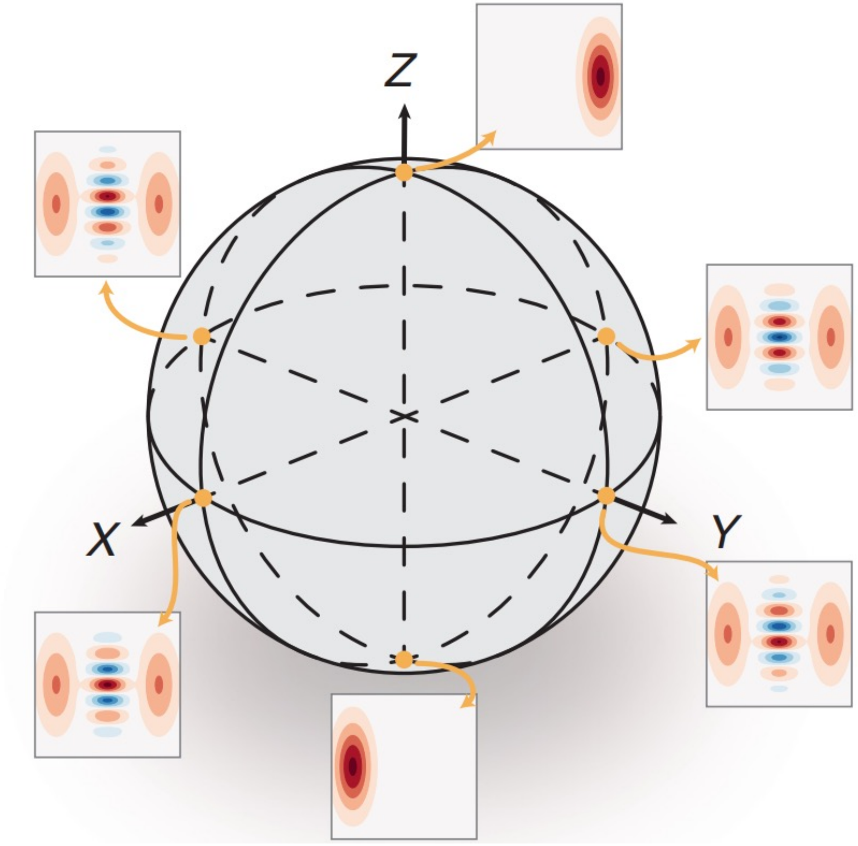


Continuous variable quantum codes

- (CV) cat qubits
  - Information is encoded in a subspace of the Hilbert space of a QHO

- Basis: coherent states $\{|\alpha\rangle, |-\alpha\rangle\}$
  - $|+\rangle_{cat} \sim |\alpha\rangle + |-\alpha\rangle$
  - $|-\rangle_{cat} \sim |\alpha\rangle - |-\alpha\rangle$

- Logical computational basis states are
  - $|0\rangle_{cat} \sim |+\rangle_{cat} + |-\rangle_{cat} \sim |+\alpha\rangle + O(e^{-2|\alpha|^2})$
  - $|1\rangle_{cat} \sim |+\rangle_{cat} - |-\rangle_{cat} \sim |-\alpha\rangle + O(e^{-2|\alpha|^2})$

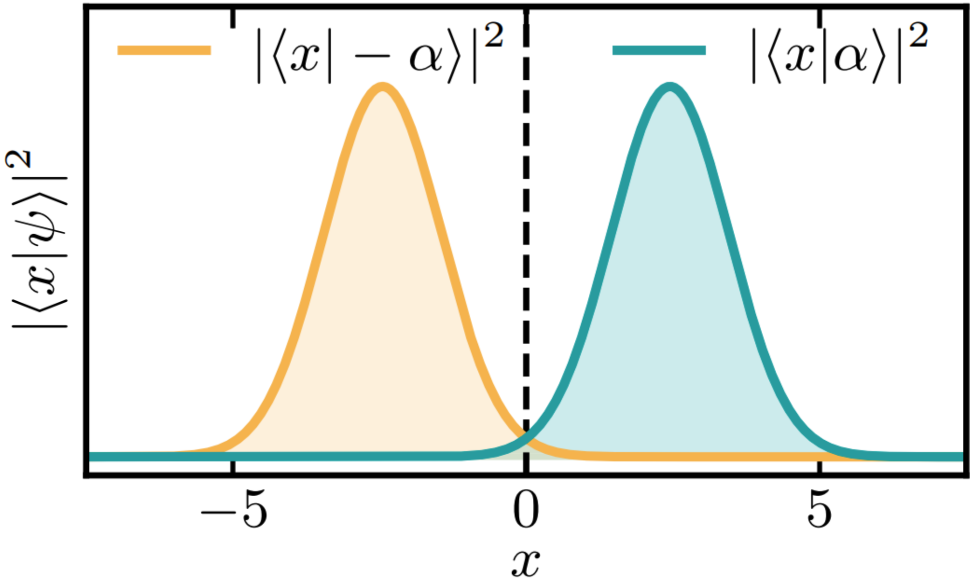


Cat qubits – noise bias and analog syndromes

- The wave function of a coherent state is a Gaussian centered at α
  - Depending on readout value, we can assign an error probability

- Through engineered protection mechanisms
  - X error rate $\exp\big(O(|\alpha|^2)\big)$
  - Z error rate $O(|\alpha|^2)$
- effective biased-noise channel

- Concatenate with outer QLDPC code to obtain a bosonic-LDPC code

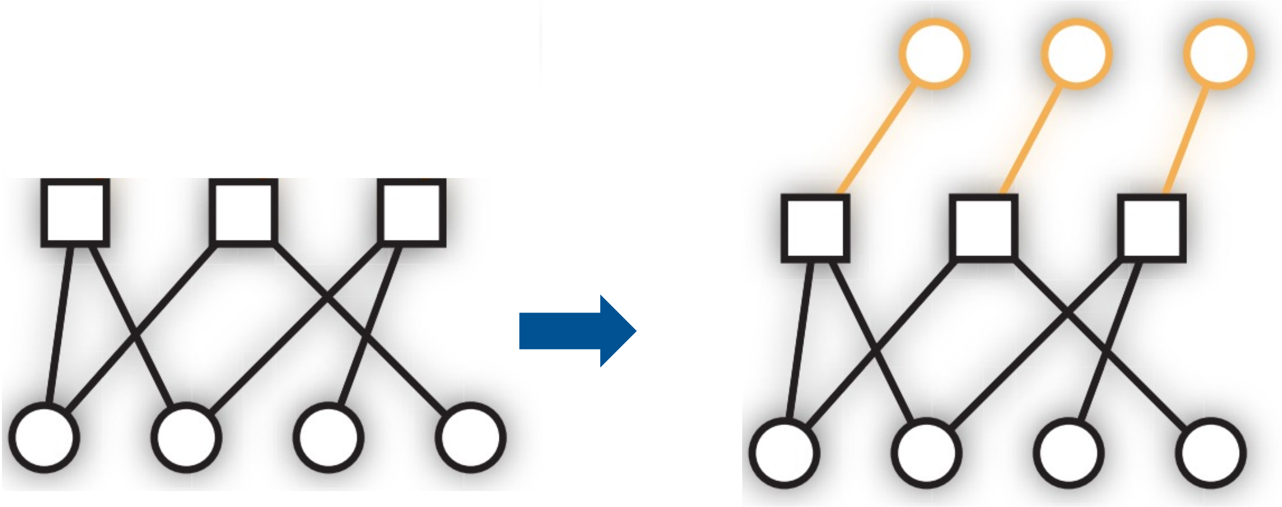


Analog Tanner graph decoding (ATD)

- Add further analog nodes to Tanner graph

- $H^A := (H \mid I_m)$
  - → full rank

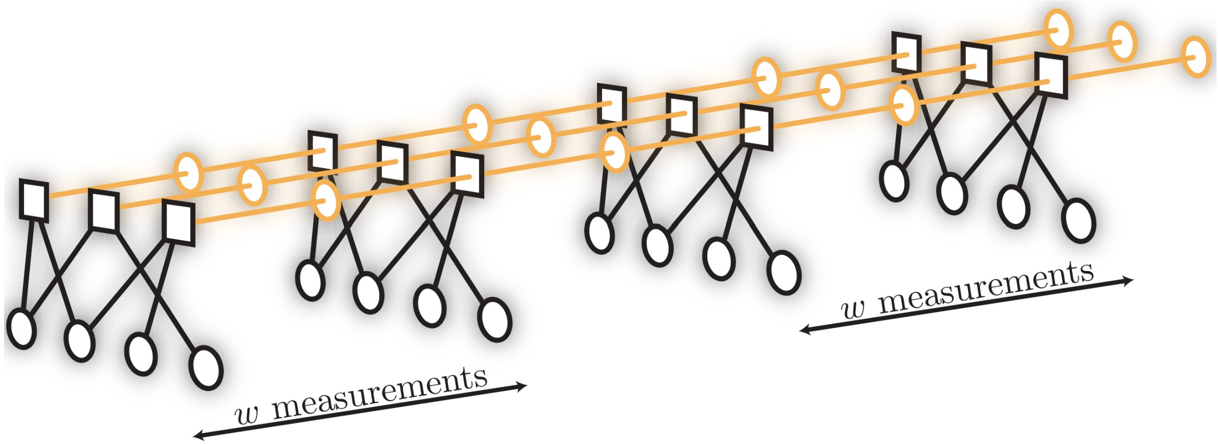
- ~ BP+OSD on the ATG



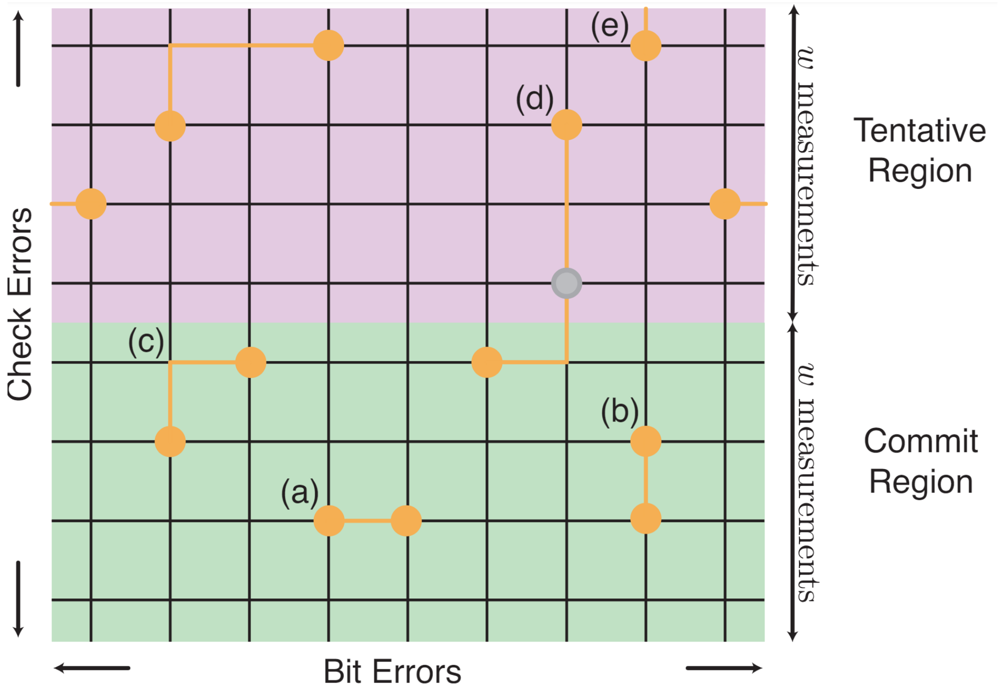
ATD: time-domain decoding

- Multiround ATG:

$$H^{3D} := \begin{pmatrix} H & 0 & 0 & \dots & \mathbb{1}_m & & & & \\ & H & 0 & \dots & \mathbb{1}_m & \mathbb{1}_m & & & \\ & & H & 0 & \dots & \mathbb{1}_m & \mathbb{1}_m & & \\ & & & \ddots & & \ddots & & & \\ & & & H & 0 & 0 & & \mathbb{1}_m & \mathbb{1}_m \end{pmatrix}$$



- == $H^A \otimes$ repetition code
- #repetitions $w \sim d$
- Overlapping window decoding using ATD



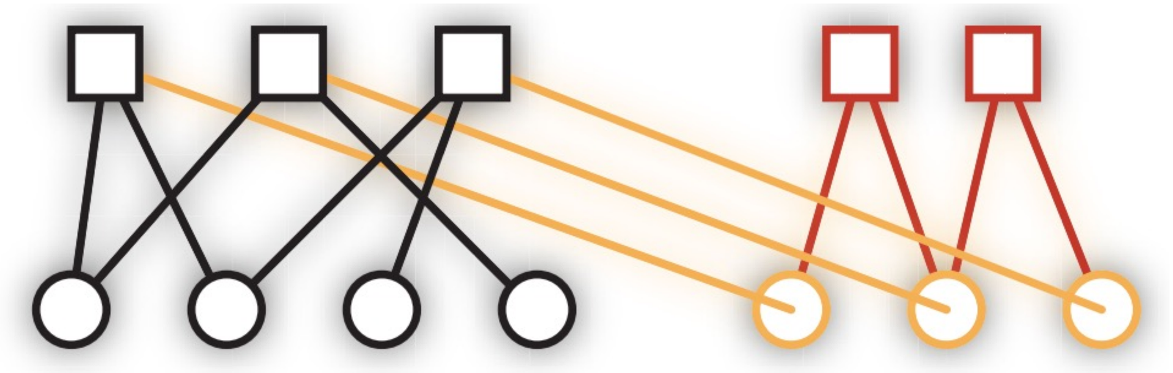
Single shot decoding with analog information

- Some codes have a single-shot property
  - O(1) many repetitions of syndrome are enough to achieve fault-tolerance

- Single-shot ATG
  - $H^M := \begin{pmatrix} H & I_m \\ 0 & M \end{pmatrix}$

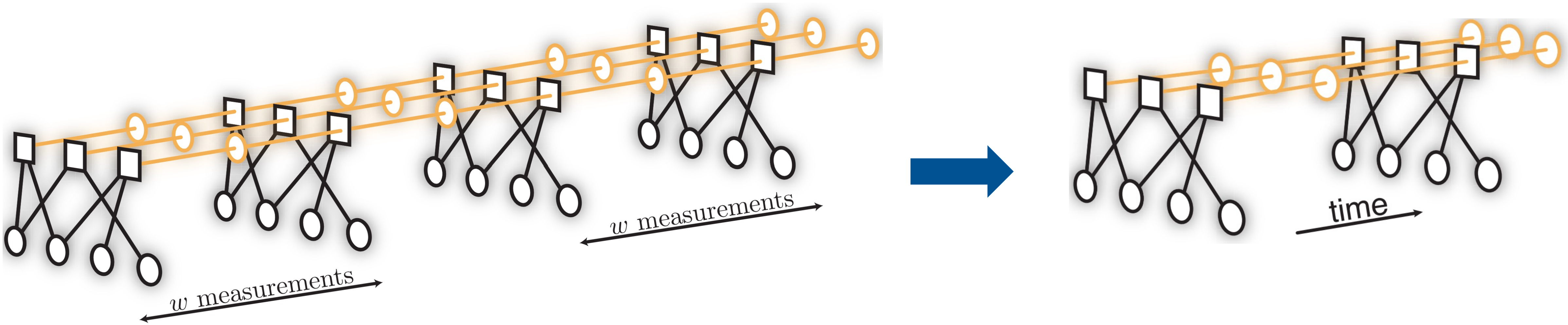
- Additional nodes for analog syndrome

- Additional checks ‘metachecks’ for single-shot decoding

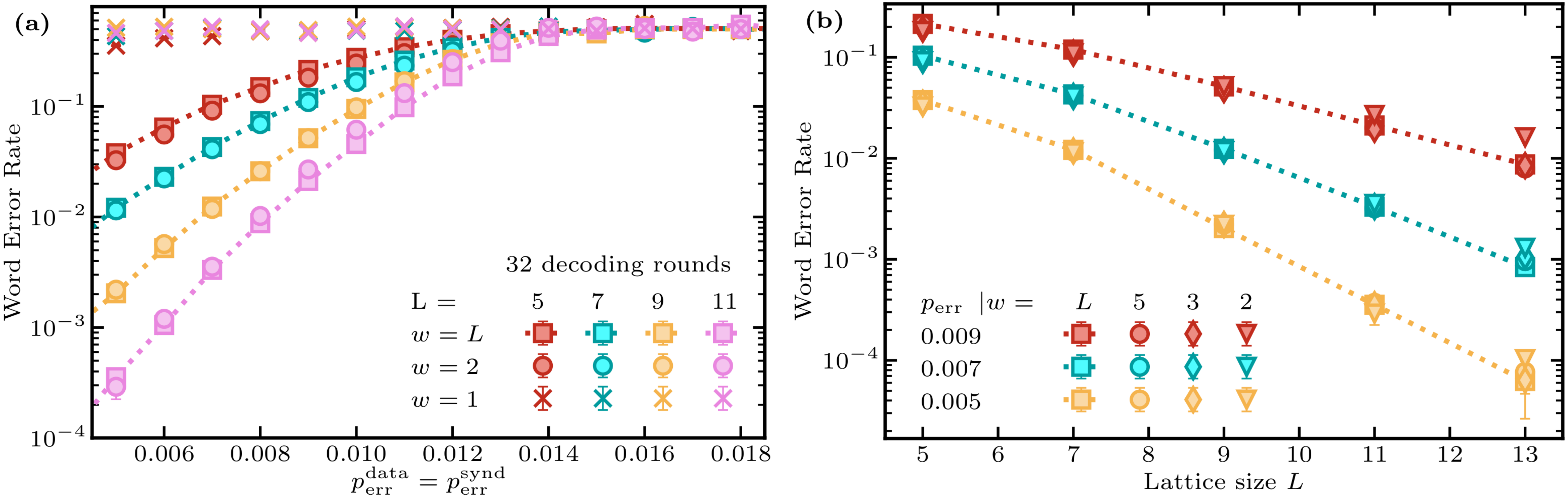


Quasi-single shot protocol

- $Q(H_X, H_Z)$, H_X is single-shot
  - e.g., 3D toric code
- Using ATD we can reduce the number of syndrome rounds needed
- We observe a small number *w* (independent of *d*) suffices



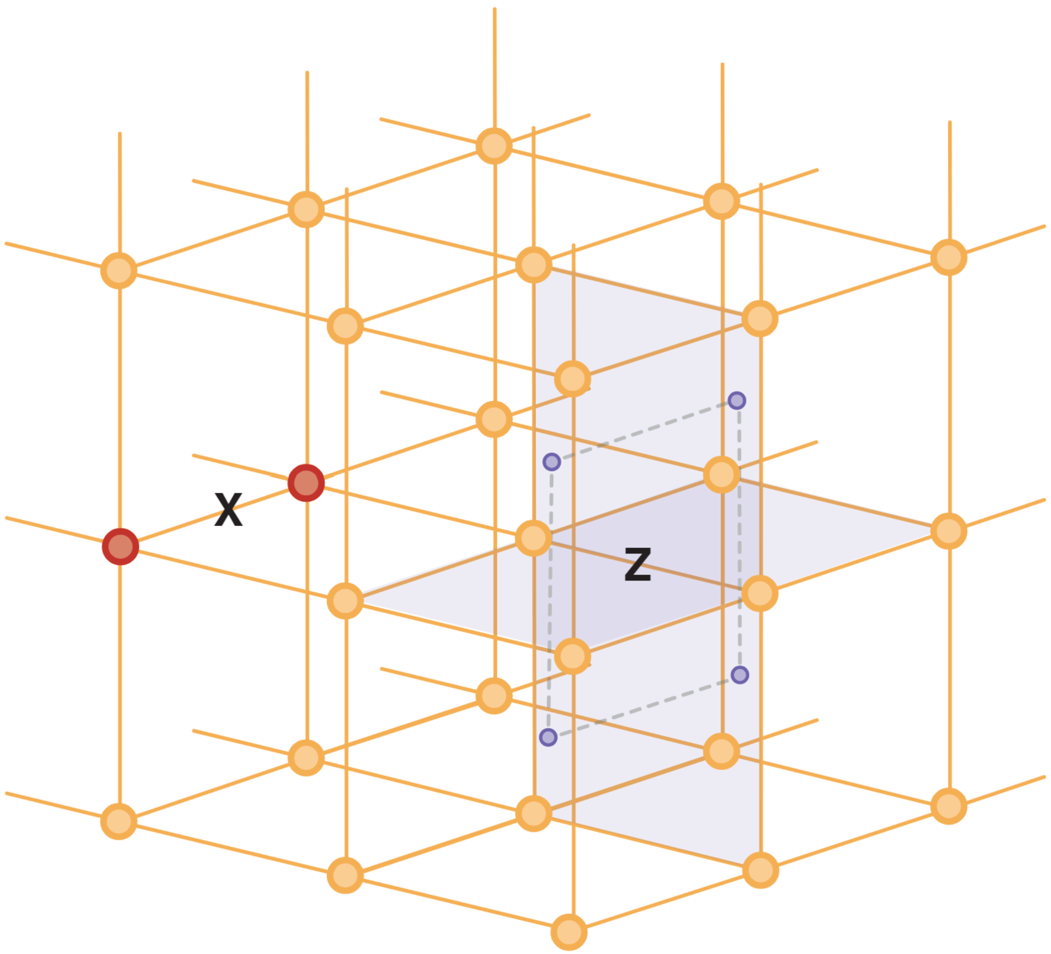
w-QSS protocol simulations



Summary and outlook

- We show how to use analog information in bosonic-LDPC decoding
  - OWD, single-shot using analog-information
- Inspired by concatenated bosonic-LDPC architecture
  - Also applicable to GKP-concatenated code and “soft-info” for DV codes
- QSS protocol allows to reduce the overhead

- Future work
  - Cats on LSD
    - combine ATD and localized statistics decoding (arXiv:2406.18655)
  - Circuit-noise & hardware tailoring



Thanks

- Cheers to my collaborators
  - Timo Hillmann
  - Joschka Roffe
  - Jens Eisert
  - Robert Wille

- Check out MQT.QECC and LDPCv2
  - Open-source software is important

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